New Trends In High Energy Physics

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Symplectic Field Theory of the Galilean Covariant Scalar and Spinor Represantations

Gustavo Xavier Antunes Petronilo, A.E. Santana, S. Ulhoa

International Center of Physics
Institute of Physics - University of Brasília, DF, Brazil

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Sumário

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Motivation

- Wigner Function $\rightarrow$ Landau Problem
  - Galilean Covariance
  - Star Product
  - Symplectic Represantations [1]

[1] Oliveira, M. D., et al. "Symplectic quantum mechanics." Annals of Physics 312.2 (2004): 492-510.

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Galilei Group

$$\begin{aligned} \boldsymbol{x} &\rightarrow \text{Space (Euclidian)} \\ t &\rightarrow \text{time} \end{aligned}$$

Galilei Transformations

$$\begin{cases} \boldsymbol{x}' = R\boldsymbol{x} + \boldsymbol{v}t + \boldsymbol{a} \\ t' = t + b \end{cases}$$

$$\begin{aligned} R &\rightarrow \text{rotations (3 parameters)} \\ \boldsymbol{v} &\rightarrow \text{Galilei's boosts(3 parameters)} \\ \boldsymbol{a} &\rightarrow \text{translations (3 parameters)} \\ b &\rightarrow \text{clock synchronization (1 parameter)} \end{aligned}$$

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Galilei Group

We denote the transformations of Galilei by

$$G(\boldsymbol{x}, t) = (\boldsymbol{x}', t'),$$

where G is given by

$$G = (b, \boldsymbol{a}, \boldsymbol{v}, R),$$

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Galilei Group

$$\begin{aligned} G_1 &= (b_1, \boldsymbol{a}_1, \boldsymbol{v}_1, R_1) \\ G_2 &= (b_2, \boldsymbol{a}_2, \boldsymbol{v}_2, R_2) \end{aligned}$$

G_2G_1, results in a composition law given by

$$\begin{aligned} G_1G_2 &= (b_1 + b_2, \boldsymbol{a}_2 + R_2\boldsymbol{a}_1 + b_1\boldsymbol{v}_2, \boldsymbol{v}_2, R_2\boldsymbol{v}_1, R_2R_1). \\ G_2G_1 &= G \text{ (is also a Galilei's trnsformation).} \end{aligned}$$

Identity transformation

$$E = (0, 0, 0, 1)$$

Inverse

$$G^{-1} = (-b, -R^{-1}(\boldsymbol{a} - b\boldsymbol{v}), -R^{-1}\boldsymbol{v}, R^{-1})$$

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Galilean Covariance

Consider a particle free of mass m, the ratio of dispersion is given by

$$\mathbf{p}^2 - 2mE = 0$$

We can then define a 5-vector, $p^\mu = (p_x, p_y, p_z, m, E) = (p^i, m, E)$, with $i = 1, 2, 3$.

Thus, to define a scalar product of the type $p_\mu p_\nu g^{\mu\nu} = \mathbf{p}^2 - 2mE = k^2$

$$p_\mu p_\nu g^{\mu\nu} = p_i p_i - p_4 p_5 - p_5 p_4 = \mathbf{p}^2 - 2mE = k^2,$$

where $g^{\mu\nu}$ is the metric of Space to be constructed, and $p_\nu g^{\mu\nu} = p^\mu$.

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Galilean Covariance

Let q^μ the set of canonical coordinates associated with p^μ, we write $q^\mu = (\mathbf{q}, q^4, q^5)$.

- $\mathbf{q}$ is the canonical coordinate associated with $\mathbf{p}$;
- q_4 is the canonical coordinate associated with E, and thus can be considered as the time coordinate;
- q_5 is the canonical coordinate associated with m explicitly given in terms of $\mathbf{q}$ e q^4
  $q^\mu q_\mu = q_\mu q_\nu g^{\mu\nu} = \mathbf{q} - 2q_4 q_5 = s^2$. Since $p^\mu p_\mu = 0$, we have to take $s = 0$
  $q^5 = \dfrac{\mathbf{q}^2}{2t}$; or infinitesimally, we obtain $\delta q^5 = v \cdot \delta \frac{\mathbf{q}}{2}$.
  Therefore, the fifth component is defined by velocity.

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Galilean Covariance

That can be seen as a special case of a vector product in G denoted as

$$(x|y) = g^{\mu\nu} x_\mu y_\nu = \sum_{i=1}^{3} x_i y_i - x_4 y_5 - x_5 y_4,$$

where $x^4 = y^4 = t$, $x^5 = \dfrac{x^2}{2t}$ e $y^5 = \dfrac{y^2}{2t}$

we can introduce the metric

$$(g_{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}.$$

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Hilbert Space and Sympletic Structure

Consider $\mathcal{G}$ an analytical manifold where each point is specified by coordinates q_μ, with $\mu = 1, 2, 3, 4, 5$ and metric specified by (9). The coordinates of each point in the cotangent-bundle $T^*\mathcal{G}$ will be denoted by (q_μ, p_μ). The Space $T^*\mathcal{G}$ is equipped with a symplectic structure via a 2-form.

$$\omega = dq^\mu \wedge dp_\mu$$

called the symplectic form (sum over repeated indices is assumed). We consider the following bidifferential operator on $C^\infty(T^*\mathcal{G})$ functions,

$$\Lambda = \frac{\overleftarrow{\partial}}{\partial q^\mu}\frac{\overrightarrow{\partial}}{\partial p_\mu} - \frac{\overleftarrow{\partial}}{\partial p^\mu}\frac{\overrightarrow{\partial}}{\partial q_\mu}$$

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Hilbert Space and Sympletic Structure

such that for C^∞ functions, $f(q,p)$ and $g(q,p)$, we have

$$\omega(f\Lambda, g\Lambda) = f\Lambda g = \{f,g\}$$

where

$$\{f,g\} = \frac{\partial f}{\partial q^\mu}\frac{\partial g}{\partial p_\mu} - \frac{\partial f}{\partial p^\mu}\frac{\partial g}{\partial q_\mu}$$

is the poison bracket and $f\Lambda$ and $g\Lambda$ are two vector fields given by $h\Lambda = X_h = -\{h,\}$.
The Space $T^*\mathcal{G}$ endowed with this symplectic structure is called the phase-space, and will be denoted by Γ.

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Hilbert Space and Sympletic Structure

To associate the Hilbert Space with the phase-space Γ, we will consider the set of complex functions of integrable square, $\phi(q,p)$ in Γ, such that

$$\int dpdq\ \phi^{\dagger}(q,p)\phi(q,p) < \infty$$

is called the real bilinear form. In this case $\phi(q,p) = \langle q,p|\phi\rangle$ is written with the aid of

$$\int dpdq\, |q,p\rangle\langle q,p| = 1$$

where $\langle\phi|$ is the dual vector of $|\phi\rangle$.
This symplectic Hilbert Space is denoted by $H(\Gamma)$.

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Symplectic Quantum Mechanics and the Galilei Group

Consider the unit transformations $U:\mathcal{H}(\Gamma) \to \mathcal{H}(\Gamma)$ such that $\langle\psi_1|\psi_2\rangle$ is invariant. Using the Λ operator, we define a mapping $e^{i\frac{\Lambda}{2}} = \star:\Gamma \times \Gamma \to \Gamma$ called as Moyal (or star) product, defined by.

$$f \star g \quad = f(q,p)\mathrm{exp}\left[\frac{i}{2}\left(\frac{\overleftarrow{\partial}}{\partial q^\mu}\frac{\overrightarrow{\partial}}{\partial p_\mu} - \frac{\overleftarrow{\partial}}{\partial p^\mu}\frac{\overrightarrow{\partial}}{\partial q_\mu}\right)\right]g(q,p)$$

where $\hbar = 1$.

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Symplectic Quantum Mechanics and the Galilei Group

The generators of U can be introduced by the following (Moyal-Weyl) star-operators:

$$\widehat{F} = f(q,p)\star = f\left(q^\mu + \frac{i}{2}\frac{\partial}{\partial p_\mu}, p^\mu - \frac{i}{2}\frac{\partial}{\partial q_\mu}\right).$$

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Symplectic Quantum Mechanics and the Galilei Group

To construct a representation of Galilei algebra in $\mathcal{H}$, we define the following operators,

$$\begin{aligned} \widehat{P}^{\mu} &= p^{\mu}\star = p^{\mu} - \frac{i}{2}\frac{\partial}{\partial q_{\mu}}, \\ \widehat{Q}^{\mu} &= q\star = q^{\mu} + \frac{i}{2}\frac{\partial}{\partial p_{\mu}}. \end{aligned}$$

and

$$\widehat{M}_{\nu\sigma} = M_{\nu\sigma}\star = \widehat{Q}_{\nu}\widehat{P}_{\sigma} - \widehat{Q}_{\sigma}\widehat{P}_{\nu}.$$

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Hilbert Space and Sympletic Structure

From this set of unitary operators we obtain, after some simple calculations, the following set of commutations relations,

$$
\begin{aligned}
\left[\widehat{M}_{\mu\nu}, \widehat{M}_{\rho\sigma}\right] &= -i(g_{\nu\rho}\widehat{M}_{\mu\sigma} - g_{\mu\rho}\widehat{M}_{\nu\sigma} + g_{\mu\sigma}\widehat{M}_{\nu\rho} - g_{\mu\sigma}\widehat{M}_{\nu\rho}), \\
\left[\widehat{P}_{\mu}, \widehat{M}_{\rho\sigma}\right] &= -i(g_{\mu\rho}\widehat{P}^{\sigma} - g_{\mu\sigma}\widehat{P}^{\rho}), \\
\left[\widehat{P}_{\mu}, \widehat{P}_{\sigma}\right] &= 0.
\end{aligned}
$$

A Casimir invariant of this algebra is $\widehat{P}^{\mu}\widehat{P}_{\mu}$.

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Hilbert Space and Sympletic Structure

Consider a vector $q^{\mu} \in G$ that obeys the set of linear transformations of the type

$$\bar{q}^{\mu} = G^{\mu}{}_{\nu} q^{\nu} + a^{\mu}.$$

A particular case of interest of these transformation, given by

$$\begin{aligned} \bar{q}^{i} &= R^{i}_{j} q^{j} + v^{i} q^{4} + a^{i} \\ \bar{q}^{4} &= q^{4} + a^{4} \\ \bar{q}^{5} &= q^{5} - (R^{i}_{j} q^{j}) v_{i} + \frac{1}{2} \mathbf{v}^{2} q^{4}. \end{aligned}$$

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Hilbert Space and Sympletic Structure

In the matricial form, the homogeneous transformations are written as

$$G^{\mu}{}_{\nu} = \begin{pmatrix} R^1_1 & R^1_2 & R^1_3 & v^i & 0 \\ R^2_1 & R^2_2 & R^2_3 & v^2 & 0 \\ R^3_1 & R^3_2 & R^3_3 & v^3 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ v_i R^i{}_j & v_i R^i{}_2 & v_i R^i{}_3 & \frac{\mathbf{v}^2}{2} & 1 \end{pmatrix}.$$

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Hilbert Space and Sympletic Structure

We can write down the generators as

$$\begin{aligned}
\widehat{J}_i &= \frac{1}{2}\epsilon_{ijk}\widehat{M}_{jk} \\
\widehat{K}_i &= \widehat{M}_{5i} \\
\widehat{C}_i &= \widehat{M}_{4i} \\
\widehat{D}_i &= \widehat{M}_{5i}
\end{aligned}$$

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Hilbert Space and Sympletic Structure

With the commutation relations, considering non-vanish ones, they are rewritten as

$$\left[\widehat{J}_i, \widehat{J}_j\right] = i\epsilon_{ijk}\widehat{J}_k$$

$$\left[\widehat{J}_i, \widehat{C}_j\right] = i\epsilon_{ijk}\widehat{C}_k$$

$$\left[\widehat{D}, \widehat{K}_i\right] = i\widehat{K}_i$$

$$\left[\widehat{P}_4, \widehat{D}\right] = i\widehat{P}_4$$

$$\left[\widehat{P}_i, \widehat{K}_j\right] = i\delta_{ij}\widehat{P}_5$$

$$\left[\widehat{P}_4, \widehat{K}_i\right] = i\widehat{P}_i$$

$$\left[\widehat{D}, \widehat{P}_5\right] = i\widehat{P}_5$$

$$\left[\widehat{J}_i, \widehat{K}_j\right] = i\epsilon_{ijk}\widehat{K}_k$$

$$\left[\widehat{K}_i, \widehat{C}_j\right] = i\delta_{ij}\widehat{D} + i\epsilon_{ijk}J_k$$

$$\left[\widehat{C}_i, \widehat{D}\right] = i\widehat{C}_i$$

$$\left[\widehat{J}_i, \widehat{P}_j\right] = i\epsilon_{ijk}\widehat{P}_k$$

$$\left[\widehat{P}_i, \widehat{C}_j\right] = i\delta_{ij}\widehat{P}_4$$

$$\left[\widehat{P}_5, \widehat{C}_i\right] = i\widehat{P}_i$$

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Hilbert Space and Sympletic Structure

This relations form an algebra which has as subalgebra the Lie algebra of Galilei group in the case of $\mathcal{R}^3 \times t$, considering J_i the generators of rotations and C_i of the pure Galilei transformations, P_μ the spacial and temporal translations and D of the kind temporal dilation (which we will not discuss here). The commutation of K_i and P_i is naturally non-zero in this context, being P_5 will be related with mass.

The invariants of this algebra are

$$
\begin{aligned}
I_1 &= \widehat{P}_\mu \widehat{P}^\mu \\
I_2 &= \widehat{P}_5
\end{aligned}
$$

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Hilbert Space and Sympletic Structure

Using the Casimir invariants I_1 and I_2 and applying in Ψ,

$$\widehat{P}_\mu \widehat{P}^\mu \Psi = k^2 \Psi$$
$$\widehat{P}_5 \Psi = -m\Psi$$

From this we obtain

$$\left(p^2 - i\mathrm{p}\cdot\nabla - \frac{1}{4}\nabla^2 - k^2\right)\Psi = 2\left(p_4 - \frac{i}{2}\partial_t\right)\left(p_5 - \frac{i}{2}\partial_5\right)\Psi,$$

with $widehat P^\mu = p^\mu \star = p^\mu - \frac{i}{2}\frac{\partial}{\partial q_\mu}$. A solution for this equation is

$$\Psi = e^{-2i[(p_5+m)q_5+(p_4+E)t]}\Phi(q,p).$$

Thus,

$$\frac{1}{2m}\left(p^2 - i\mathbf{p}\cdot\nabla - \frac{1}{4}\nabla^2\right)\Phi = \left(E + \frac{k^2}{2m}\right)\Phi,$$

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Hilbert Space and Sympletic Structure

This equation, and its complex conjugate, can also be obtained by the Lagrangian density in phase-Space (we use $d^{\mu} = d/dq_{\mu}$)

$$\begin{aligned}\mathcal{L}_0 &= \partial^{\mu}\Psi(q,p)\partial\Psi^{*}(q,p) + \frac{i}{2}p^{\mu}[\Psi(q,p)\partial^{\mu}\Psi^{*}(q,p) \\ &- \Psi^{*}(q,p)\partial^{\mu}\Psi(q,p)] + \left[\frac{p^{\mu}p_{\mu}}{4} - k^2\right]\Psi = 0.\end{aligned}$$

The association of this representation with the Wigner formalism is given by

$$f_w(q,p) = \Psi(q,p) \star \Psi^{\dagger}(q,p)$$

where $f_w(q,p)$ is the Wigner function.

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Spin 1/2 Symplectic Representaion

In order to study the representations of spin particles $1/2$, we will introduce the $\gamma^\mu \widehat{P}_\mu$, where $\widehat{P}_\mu = p_\mu - \frac{i}{2}\partial_\mu$ in such a way that acting on the 5-spinor in the phase-space $\Psi(q,p)$, we have

$$\left(\gamma^\mu \widehat{P}_\mu - k\right)\Psi(p,q) = 0$$

or

$$\gamma^\mu \left(p_\mu - \frac{i}{2}\partial_\mu\right)\Psi(p,q) = k\Psi(p,q)$$

Which is the galilean covariant Pauli-Schrödinger equation in phase-space.

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Spin 1/2 Symplectic Representaion

Consequently the mass layer condition is obtained by following the usual steps.

$$(\gamma^\mu \widehat{P}_\mu)(\gamma^\nu \widehat{P}_\nu)\Psi(q,p) = k^2\Psi(q,p),$$

therefore

$$\gamma^\mu \gamma^\nu (\widehat{P}_\mu \widehat{P}_\nu) = k^2 = \widehat{P}^\mu \widehat{P}_\nu,$$

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Spin 1/2 Symplectic Representaion

since $\widehat{P}_\mu \widehat{P}_\nu = \widehat{P}_\nu \widehat{P}_\mu$, we have

$$\frac{1}{2}(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu)\widehat{P}_\mu \widehat{P}_\nu = \widehat{P}^\mu \widehat{P}_\nu,$$

so

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

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Spin 1/2 Symplectic Representaion

This equation can be derive from the Lagrangian density for spin 1/2 particles in phase-Space, which is given by

$$\mathcal{L} = -\frac{i}{4}\Big((\partial_\mu \bar{\Psi})\gamma^\mu \Psi - \bar{\Psi}(\gamma^\mu \partial_\mu \Psi)\Big) - (k - \gamma^\mu p_\mu)\Psi\bar{\Psi}.$$

where $\bar{\Psi} = \Psi^\dagger \zeta$, with

$$\zeta = -\frac{i}{\sqrt{2}}\{\gamma^4 + \gamma^5\}$$

In the case of Pauli-Schrödinger in phase-space equation the association with the Wigner function is given by

$$f_w = \Psi \star \bar{\Psi},$$

New Trends in High Energy Physics

Electromagnetic Interactions

Let us examine the gauge symmetries in phase-space demanding the invariance of the Lagrangian by a local gauge transformation given by $e^{\Lambda(q,p)}\Psi$. This leads to the minimal coupling,

$$\widehat{P}_\mu \Psi \rightarrow \left(\widehat{P}_\mu - e\widehat{A}_\mu\right)\Psi = \left(p_\mu - \frac{i}{2}\partial_\mu - e\widehat{A}_\mu\right)\Psi,$$

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Electromagnetic Interactions

This describes an electron in an external field, with the Pauli-Schrödinger equation in phase-space given by

$$\left[\gamma^{\mu}\left(p_{\mu}-\frac{i}{2}\partial_{\mu}-e\widehat{A}_{\mu}\right)-k\right]\Psi=0.$$

In order to illustrate such result, lets consider a electron in a external field given by $\widehat{A}_{\mu}(\widehat{\mathbf{A}},\widehat{A}_4,\widehat{A}_5)$, with $\widehat{A}_4=-\phi$ and $\widehat{A}_5=0$.

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Electromagnetic Interactions

Considering the following represantation of γ^μ matrices

$$\gamma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix}, \qquad \gamma^4 = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 0 & -\sqrt{2} \\ 0 & 0 \end{pmatrix}.$$

where σ^i are the Pauli matrices and $\sqrt{2}$ is the identity matrix 2x2 multiplied by $\sqrt{2}$. We can rewrite the object Ψ, as

$$\Psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix},$$

where φ and χ are 2-spinors dependent on $x^\mu; \mu = 1, ..., 5$.

New Trends in High Energy Physics
Electromagnetic Interactions

Thus, in the representation where $k = 0$,

$$\boldsymbol{\sigma} \cdot \left(\mathbf{p} - \frac{i}{2}\partial_q - e\widehat{\mathbf{A}} \right) \varphi - \sqrt{2} \left(p_5 - \frac{i}{2}\partial_5 \right) \chi = 0,$$

$$\sqrt{2} \left(p_4 - \frac{i}{2}\partial_t - e\phi \right) \varphi - \boldsymbol{\sigma} \cdot \left(\mathbf{p} - \frac{i}{2}\partial_q - e\widehat{\mathbf{A}} \right) \chi = 0.$$

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Electromagnetic Interactions

Solving the coupled equations we get an equation for φ and χ, given by

$$
\begin{aligned}
\left[\boldsymbol{\sigma}\cdot\left(\mathbf{p}-\frac{i}{2}\partial_q-e\widehat{\mathbf{A}}\right)\right]^2\varphi &= 2\left(p_4-\frac{i}{2}\partial_t-e\phi\right)\left(p_5-\frac{i}{2}\partial_5\right)\varphi\\
\left[\boldsymbol{\sigma}\cdot\left(\mathbf{p}-\frac{i}{2}\partial_q-e\widehat{\mathbf{A}}\right)\right]^2\chi &= 2\left(p_4-\frac{i}{2}\partial_t-e\phi\right)\left(p_5-\frac{i}{2}\partial_5\right)\chi
\end{aligned}
$$

with

$$
\begin{aligned}
f_w &= \Psi\star\bar{\Psi},\\
&= i\varphi\star\chi^\dagger-i\chi\star\varphi^\dagger
\end{aligned}
$$

New Trends in High Energy Physics
Electromagnetic Interactions

Replacing the eigenvalues of $\widehat{P}_4$ and $\widehat{P}_5$, we have

$$\begin{aligned}
\left[\frac{1}{2m}\left(\boldsymbol{\sigma}\cdot\left(\mathbf{p}-\frac{i}{2}\partial_q - e\widehat{\mathbf{A}}\right)\right)^2 + e\phi\right]\varphi &= E\varphi \\
\left[\frac{1}{2m}\left(\boldsymbol{\sigma}\cdot\left(\mathbf{p}-\frac{i}{2}\partial_q - e\widehat{\mathbf{A}}\right)\right)^2 + e\phi\right]\chi &= E\chi
\end{aligned}$$

Which is the non-covariant form of the time independent Pauli-Schrödinger equation in phase-space.

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Electromagnetic Interactions

Laundau levels

$$E = \frac{eB}{m}\left(n + \frac{1}{2} - \frac{s}{2}\right) - \frac{k^2}{2m}$$

where $s = \pm 1$. and, again

$$\begin{aligned} f_w &= \Psi \star \bar{\Psi}, \\ &= i\varphi \star \chi^\dagger - i\chi \star \varphi^\dagger \end{aligned}$$

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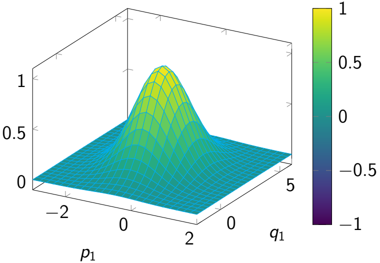
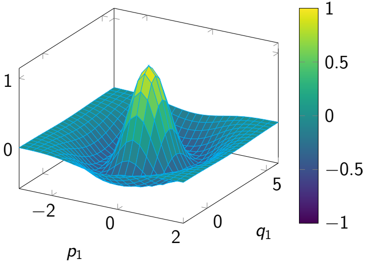
Electromagnetic Interactions

Quasi-amplitudes of probability





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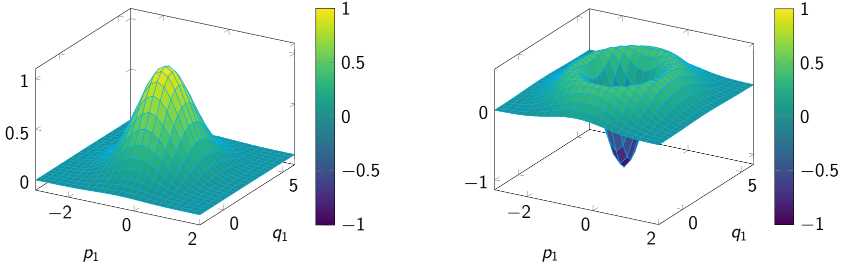
Electromagnetic Interactions

Wigner Functions



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Concluding Remarks

We study the spin $1/2$ particle equation, the Pauli-Schrödinger equation, in the context of Galiean covariance and then construct a phase-space formalism using such covariance.
We construct the formalism of the quantum mechanics of the Galilean covariant phase-space and we arrive at the representations of the spin 0 and spin $1/2$ equations, where for the spin equation $1/2$, the Dirac-like equation, we study the electron in an external field and with the supposed solution we were able to recover the Pauli-Schrödinger equation (written its non-covariant form) in phase-space.

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The Brazilian universities and research centers are under attack, with cutting of budget, by the present days Brazilian Federal government. Due to this situation works like this may be impossible.

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Thank you!