Non-Abelian axal anomaly, low energy theorem and decays of pseudoscalar mesons

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- η and η' mesons are known to be deeply related to Abelian and non-Abelian axial anomalies.
- We generalize the exact anomaly sum rules to the case of non-Abelian axial anomaly and
  apply the results to the processes of η and η' radiative decays.

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Outline

Axial anomaly

In QCD, for a given flavor q, the divergence of the axial current $J_{\mu 5}^{(q)} = \bar{q}\gamma_\mu\gamma_5 q$ acquires both electromagnetic and strong anomalous terms:

$$\partial_\mu J_{\mu 5}^{(q)} = m_q \bar{q}\gamma_5 q + \frac{e^2}{8\pi^2} e_q^2 N_c F\tilde{F} + \frac{\alpha_s}{4\pi} G\tilde{G}, \tag{1}$$

An octet of axial currents

$$J_{\mu 5}^{(a)} = \sum_q \bar{q}\gamma_5\gamma_\mu \frac{\lambda^a}{\sqrt{2}} q$$

Singlet axial current $J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s)$:

$$\partial^\mu J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(m_u \overline{u}\gamma_5 u + m_d \overline{d}\gamma_5 d + m_s \overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi} C^{(0)} N_c F\tilde{F} + \frac{\sqrt{3}\alpha_s}{4\pi} G\widetilde{G}, \tag{2}$$

The diagonal components of the octet of axial currents
$J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$,
$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s)$
acquire an electromagnetic anomalous term only:

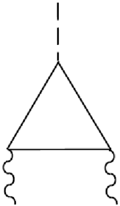
$$\partial^\mu J^{(3)}_{\mu 5} = \frac{1}{\sqrt{2}}(m_u\overline{u}\gamma_5 u - m_d\overline{d}\gamma_5 d) + \frac{\alpha_{em}}{2\pi}C^{(3)}N_c F\tilde{F}, \quad (3)$$

$$\partial^\mu J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(m_u\overline{u}\gamma_5 u + m_d\overline{d}\gamma_5 d - 2m_s\overline{s}\gamma_5 s) + \frac{\alpha_{em}}{2\pi}C^{(8)}N_c F\tilde{F}. \quad (4)$$

The electromagnetic charge factors $C^{(a)}$ are

$$C^{(3)} = \frac{1}{\sqrt{2}}(e_u^2 - e_d^2) = \frac{1}{3\sqrt{2}},$$
$$C^{(8)} = \frac{1}{\sqrt{6}}(e_u^2 + e_d^2 - 2e_s^2) = \frac{1}{3\sqrt{6}},$$
$$C^{(0)} = \frac{1}{\sqrt{3}}(e_u^2 + e_d^2 + e_s^2) = \frac{2}{3\sqrt{3}}. \quad (5)$$

Anomaly sum rule for the singlet axial current



The matrix element for the transition of the axial current $J_{\alpha 5}$ with momentum $p = k + q$ into two real or virtual photons with momenta k and q is:

$$e^2 T_{\alpha\mu\nu}(k,q) = \int d^4x d^4y e^{(ikx+iqy)} \langle 0| T\{J_{\alpha 5}(0) J_\mu(x) J_\nu(y)\}|0\rangle; \qquad (6)$$

Kinematics:

$$k^2 = 0, Q^2 = -q^2$$

Anomalous axial-vector Ward identity for the singlet component of axial current:

$$p_\alpha T^{\alpha\mu\nu} = 2mG\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma + \frac{C_0 N_c}{2\pi^2}\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma + N(p^2, q^2, k^2)\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma, \tag{7}$$

where $2mG\epsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma = \langle 0|\sum_{q=u,d,s} m_q \bar{q}\gamma_5 q|\gamma\gamma\rangle$,

$$\langle 0|\frac{\sqrt{3}\alpha_s}{4\pi}G\tilde{G}|\gamma(k)\gamma(q)\rangle = e^2 N(p^2, k^2, q^2)\epsilon^{\mu\nu\rho\sigma}k_\mu q_\nu \epsilon_\rho^{(k)}\epsilon_\sigma^{(q)}, \tag{8}$$

$$\langle 0|F\tilde{F}|\gamma(k)\gamma(q)\rangle = 2\epsilon^{\mu\nu\rho\sigma}k_\mu q_\nu \epsilon_\rho^{(k)}\epsilon_\sigma^{(q)}. \tag{9}$$

The VVA triangle graph amplitude presented as a tensor decomposition:

$$
\begin{aligned}
T_{\alpha\mu\nu}(k,q) \;&=\; F_1\, \varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\, \varepsilon_{\alpha\mu\nu\rho}q^\rho \\
&+\; F_3\, k_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\, q_\nu \varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\
&+\; F_5\, k_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\, q_\mu \varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma ,
\end{aligned} \tag{10}
$$

$F_j = F_j(p^2, k^2, q^2; m^2)$, $p = k + q$.
In the kinematical configuration with one real photon ($k^2 = 0$) the anomalous Ward identity can be rewritten in terms of form factors F_j as follows ($N(p^2, q^2) \equiv N(p^2, q^2, k^2 = 0)$):

$$
(q^2 - p^2)F_3 - q^2 F_4 = 2mG + \frac{C_0 N_c}{2\pi^2} + N(p^2, q^2). \tag{11}
$$

– G, F_3, F_4 can be rewritten as dispersive integrals without subtractions. [Horejsi, Teryaev '94]
– N : rewrite it in the form with one subtraction,

$$
N(p^2, q^2) = N(0, q^2) + p^2 R(p^2, q^2), \tag{12}
$$

where the new form factor R can be written as an unsubtracted dispersive integral.

The imaginary part of AWI (11) w.r.t. p^2 (s in the complex plane) reads

$$(q^2 - s) ImF_3 - q^2 ImF_4 = 2m ImG + s ImR. \quad (13)$$

– Divide every term of Eq. (13) by $(s - p^2)$ and integrate:

$$\frac{1}{\pi}\int_0^\infty \frac{(q^2 - s)ImF_3}{s - p^2} ds - \frac{q^2}{\pi}\int_0^\infty \frac{ImF_4}{s - p^2} ds = \frac{1}{\pi}\int_0^\infty \frac{2mImG}{s - p^2} ds + \frac{1}{\pi}\int_0^\infty \frac{sImR}{s - p^2} d \quad (14)$$

– After transformation and making use of the dispersive relations for the form factors F_3, F_4, G, R:

$$(q^2 - p^2)F_3 - \frac{1}{\pi}\int_0^\infty ImF_3 ds - q^2 F_4 = 2mG + p^2 R + \frac{1}{\pi}\int_0^\infty ImR ds. \quad (15)$$

Comparing (15) with (11) we arrive at the anomaly sum rule for the singlet current:

$$\frac{1}{\pi}\int_0^\infty ImF_3 ds = \frac{C_0 N_c}{2\pi^2} + N(0, q^2) - \frac{1}{\pi}\int_0^\infty ImR(s, q^2) ds \quad (16)$$

ASR and meson contributions

Saturating the l.h.s. of (16) with resonances according to global quark-hadron duality, we write out the first resonances' contributions explicitly, while the higher states are absorbed by the integral with a lower limit s_0,

$$\Sigma f_M^0 F_{M\gamma}(q^2) + \frac{1}{\pi}\int_{s_0}^{\infty} ImF_3 ds = \frac{C_0 N_c}{2\pi^2} + N(0,q^2) - \frac{1}{\pi}\int_0^{\infty} ImR(s,q^2)ds, \tag{17}$$

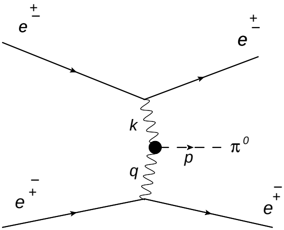
where

$$\int d^4x e^{ikx}\langle M(p)|T\{J_\mu(x)J_\nu(0)\}|0\rangle = e^2\epsilon_{\mu\nu\rho\sigma}k^\rho q^\sigma F_{M\gamma}(q^2)\ , \tag{18}$$

$$\langle 0|J_{\alpha 5}^{(a)}(0)|M(p)\rangle = ip_\alpha f_M^a\ . \tag{19}$$

- "Continuum threshold" $s_0(q^2)$ [KOT'11],[Oganesian,Pimikov,Stefanis,Teryaev'15]. $s_0 \gtrsim 1$ GeV^2.
- If one saturates with resonances the last term in the ASR: the glueball-like states.

Transition form factors



Form factors $F_{M\gamma}$ of the transitions $\gamma\gamma^* \to M$ (M=π^0, η, η'):

$$\int d^4x e^{ikx} \langle M(p)| T\{J_\mu(x) J_\nu(0)\}|0\rangle = \epsilon_{\mu\nu\rho\sigma} k^\rho q^\sigma F_{M\gamma} \quad (20)$$

Kinematics: $k^2 = 0$, $-q^2 \equiv Q^2 > 0$ (space-like region), $q^2 > 0$ (time-like region)

Mixing

$$\langle 0|J^{(a)}_{\alpha 5}(0)|M(p)\rangle = ip_\alpha f^a_M \tag{21}$$

Octet-singlet basis (of currents):

$$J^{(8)}_{\mu 5} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s),\, J^{(0)}_{\mu 5} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s). \tag{22}$$

Matrix of decay constants (30)

$$\mathbf{F} = \begin{pmatrix} f^8_\eta & f^8_{\eta'} \\ f^0_\eta & f^0_{\eta'} \end{pmatrix} \tag{23}$$

- *Octet-singlet (SU(3)) mixing scheme*: $f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'} = 0$.

$$\mathbf{F} = \begin{pmatrix} f_8 \cos\theta & f_8 \sin\theta \\ -f_0 \sin\theta & f_0 \cos\theta \end{pmatrix}. \tag{24}$$

For quark-flavour basis one explores the definitions of axial currents with decoupled light and strange quark composition:

$$J^q_{\mu 5} = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d),\ \ J^s_{\mu 5} = \bar{s}\gamma_\alpha\gamma_5 s, \tag{25}$$

$$\begin{pmatrix} J^8_{\mu 5} \\ J^0_{\mu 5} \end{pmatrix} = \mathbf{V}(\alpha)\begin{pmatrix} J^q_{\mu 5} \\ J^s_{\mu 5} \end{pmatrix},\ \mathbf{V}(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \tag{26}$$

where $\tan\alpha = \sqrt{2}$.

- *Quark-flavour mixing scheme*: [Feldmann,Kroll,Stech'97]

$$f^q_\eta f^s_\eta + f^q_{\eta'} f^s_{\eta'} = 0.$$

$$\mathbf{F_{qs}} = \begin{pmatrix} f_q\cos\phi & f_q\sin\phi \\ -f_s\sin\phi & f_s\cos\phi \end{pmatrix}. \tag{27}$$

Low-energy theorem

The matrix element $\langle 0|G\tilde{G}(p)|\gamma(k)\gamma(q)\rangle$?

- No rigorous calculation from the QCD.
- Possible to estimate it in the limit $p^\mu = 0$. [Shifman'88].

We consider the case of two real photons ($q^2 = k^2 = 0$). Supposing that there are no massless particles in the singlet channel in the chiral limit (i.e. *no admixture of the* η):

$$\lim_{p\to 0} p^\mu \langle 0|J_{\mu 5}(p)|\gamma\gamma\rangle = 0,$$

$$\langle 0|\partial^\mu J_{\mu 5}|\gamma\gamma\rangle = 0.$$

Using the explicit expression for the divergence of axial current in the chiral limit (put $m_q = 0$), one can relate the matrix elements of $\langle 0|G\tilde{G}|\gamma\gamma\rangle$ and $\langle 0|F\tilde{F}|\gamma\gamma\rangle$ in the considered limits.

- Mixing: η *spoils the theorem!*

Low-energy theorem for mixing states

Take into account mixing.

$$J_{\mu 5}^{(x)} = aJ_{\mu 5}^{(0)} + bJ_{\mu 5}^{(8)}, \;\; \langle 0|J_{\mu 5}^{(x)}|\eta\rangle = 0. \tag{28}$$

$$J_{\mu 5}^{(x)} = b(J_{\mu 5}^{(8)} - \frac{f_\eta^8}{f_\eta^0} J_{\mu 5}^{(0)}), \tag{29}$$

$$\langle 0|J_{\mu 5}^{(i)}(0)|M(p)\rangle = ip_\mu f_M^i. \tag{30}$$

The current (29) gives no massless poles in the matrix element $\langle 0|J_{\mu 5}^{(x)}|\gamma\gamma\rangle$ even in the chiral limit, and therefore

$$\lim_{p\to 0}\langle 0|\partial_\mu J_{\mu 5}^{(x)}(p)|\gamma\gamma\rangle = 0. \tag{31}$$

In the chiral limit, at $p^\mu = 0$:

$$\langle 0|\frac{\sqrt{3}\alpha_s}{4\pi} G\tilde{G}|\gamma\gamma\rangle = \frac{N_c}{f_\eta^8}(f_\eta^0 C^{(8)} - f_\eta^8 C^{(0)})\langle 0|\frac{\alpha_e}{2\pi} F\tilde{F}|\gamma\gamma\rangle. \tag{32}$$

$$N(0,0,0) = \frac{N_c}{2\pi^2 f_\eta^8}(f_\eta^0 C^{(8)} - f_\eta^8 C^{(0)}). \tag{33}$$

Hadron contributions and analysis of the ASR

$$\Sigma f_M^0 F_{M\gamma}(q^2) + \frac{1}{\pi}\int_{s_0}^{\infty} ImF_3 ds = \frac{C_0 N_c}{2\pi^2} + N(0,q^2) - \frac{1}{\pi}\int_0^{\infty} ImR(s,q^2)ds$$

The first hadron contributions to the ASR: η and η'. For real photons, the transition form factors determine the 2-photon decay amplitudes A_M $(M = \eta, \eta')$:

$$A_M \equiv F_{M\gamma}(0) = \sqrt{\frac{64\pi\Gamma_{M\to 2\gamma}}{e^4 m_M^3}}. \tag{34}$$

The ASR for the octet channel [KOT'12] for real photons:

$$f_\eta^8 A_\eta + f_{\eta'}^8 A_{\eta'} = \frac{1}{2\pi^2} N_c C^{(8)}. \tag{35}$$

The ASR in the singlet channel:

$$f_\eta^0 A_\eta + f_{\eta'}^0 A_{\eta'} = \frac{1}{2\pi^2} N_c C_0 + B_0 + B_1, \qquad (36)$$

where

$$B_0 \equiv N(0,0,0),\ B_1 \equiv -\frac{1}{\pi}\int_0^\infty ImR(s)ds - \frac{1}{\pi}\int_{s_0}^\infty ImF_3 ds. \qquad (37)$$

- The B_0 term stands for a subtraction constant in the dispersion representation of gluon anomaly;
- The B_1 term consists of two parts: spectral representation of gluon anomaly and the integral covering higher resonances. The latter is proportional to α_s^2: F_3 is described by a triangle graph (no α_s corrections) plus diagrams with additional boxes ($\propto \alpha_s^2$ for the first box term). The α_s^2 suppression of the box graph contribution is due to $s > s_0 \gtrsim 1$ GeV^2.
- In the case of both real photons in the chiral limit the triangle amplitude is zero ($\propto q^2$). So, B_1 is represented by the integral with the lower limit $s_0 \sim 1$ GeV^2 and is suppressed at least as α_s^2 on the scale of 1 GeV^2.

Combining ASRs for the octet and singlet channels, we obtain the 2-photon decay amplitudes:

$$A_\eta = \frac{1}{\Delta}\left(\frac{N_c}{2\pi^2}(C^{(8)} f^0_{\eta'} - C^{(0)} f^8_{\eta'}) - (B_0 + B_1) f^8_{\eta'}\right), \qquad (38)$$

$$A_{\eta'} = \frac{1}{\Delta}\left(\frac{N_c}{2\pi^2}(C^{(0)} f^8_{\eta} - C^{(8)} f^0_{\eta}) + (B_0 + B_1) f^8_{\eta}\right), \qquad (39)$$

where $\Delta = f^8_\eta f^0_{\eta'} - f^8_{\eta'} f^0_\eta$.
Making use of the result of the LET for B_0:

$$A_\eta = \frac{N_c C^{(8)}}{2\pi^2 f^8_\eta} - \frac{B_1 f^8_{\eta'}}{\Delta}, \qquad (40)$$

$$A_{\eta'} = \frac{B_1 f^8_\eta}{\Delta}. \qquad (41)$$

Note, that low energy theorem leads to the cancellation of the photon anomaly term with subtraction part of gluon anomaly B_0 in (39), so the amplitude $\eta' \to \gamma\gamma$ (in the chiral limit) is entirely determined by B_1, i.e., predominantly by the spectral part of the gluon anomaly.

Numerical analysis

Gluon anomaly term contributions for different sets of meson decay constants

	$\begin{pmatrix} f_\eta^8 & f_{\eta'}^8 \\ f_\eta^0 & f_{\eta'}^0 \end{pmatrix} \frac{1}{f_\pi}$	$B_0 \times 10^2$	$B_1 \times 10^2$	$(B_0 + B_1) \times 10^2$
[KOT'12], free analysis	$\begin{pmatrix} 1.11 & -0.42 \\ 0.16 & 1.04 \end{pmatrix}$	-5.55	4.91	-0.64
[KOT'12], OS mix. sch.	$\begin{pmatrix} 0.85 & -0.22 \\ 0.20 & 0.81 \end{pmatrix}$	-5.36	3.84	-1.53
[KOT'12], QF mix. sch.	$\begin{pmatrix} 1.38 & -0.63 \\ 0.18 & 1.35 \end{pmatrix}$	-5.58	6.39	0.81
[Escribano,Frere'05], free analysis	$\begin{pmatrix} 1.39 & -0.59 \\ 0.054 & 1.29 \end{pmatrix}$	-5.77	5.86	0.095
[Feldmann,Kroll'98], QF mix. sch.	$\begin{pmatrix} 1.17 & -0.46 \\ 0.19 & 1.15 \end{pmatrix}$	-5.51	5.47	-0.047

- The contribution of gluon anomaly and higher order resonances (expressed by $B_0 + B_1$ term) to the 2-photon decay amplitudes appears to be rather small numerically in comparison with the contribution of electromagnetic anomaly $(1/2\pi^2)N_c C^{(0)} \simeq 0.058$.
- B_0 and B_1 enter the ASR with different signs and almost cancel each other, giving only a small total contribution to the two-photon decay widths of the η and η'.

Conclusions

- Employing the dispersive approach to axial anomaly in the singlet current, we obtained the sum rule with photon and gluon anomaly contributions.
- The contributions of gluon and electromagnetic parts of axial anomaly in the $\eta(\eta') \to \gamma\gamma$ decays have been evaluated using the ASR for the singlet axial current.
- LET was generalized to mixed states and an estimation for the subtraction constant of the gluon anomaly contribution in the dispersive form of axial anomaly was obtained.

Thank you for your attention!

η/η' ratio in heavy ion collisions

$\langle 0 \mid G\widetilde{G} \mid \eta(\eta') \rangle$ enter J/Ψ decays:

$$R_{J/\Psi} = \frac{\Gamma(J/\Psi \to \eta'\gamma)}{\Gamma(J/\Psi \to \eta\gamma)} = \left| \frac{\langle 0 \mid G\widetilde{G} \mid \eta' \rangle}{\langle 0 \mid G\widetilde{G} \mid \eta \rangle} \right|^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3, \tag{42}$$

$p_{\eta(\eta')} = M_{J/\Psi}(1 - m^2_{\eta(\eta')}/M^2_{J/\Psi})/2$. [Novikov et al. '80]
Can be evaluated in terms of the decay constants:

$$R_{J/\Psi} = \left(\frac{f^8_{\eta'} + \sqrt{2} f^0_{\eta'}}{f^8_\eta + \sqrt{2} f^0_\eta} \right)^2 \left(\frac{m_{\eta'}}{m_\eta} \right)^4 \left(\frac{p_{\eta'}}{p_\eta} \right)^3. \tag{43}$$

$$R_{J/\Psi} = 4.67 \pm 0.15, \quad \left(\frac{p_{\eta'}}{p_\eta} \right)^3 \sim 0.81$$

(used as an additional constraint in [KOT'12])
Similarly, ratio of production of η/η' from gluons (CGC) in HIC: no kinematical factor.

η/η' ratio in heavy ion collisions

Possible sources of $G\tilde{G}$:
– rotating gluon-dominated plasma [Torrieri'18, *"η' Production in Nucleus-Nucleus collisions as a probe of chiral dynamics"*, suggested η'/π^0 as a probe – we use η/η'],
– self-dual fields [Nedelko et al.]
– inclusive process $(G\tilde{G})^2 \sim G^4$.

Multihadron production in HIC – universal thermal pattern with $T \sim 160 - 170$ MeV for hadron abundances and transverse momentum spectra $\rightarrow$ Less η' than η.
Direct gluonic production should dominate at larger transverse momentum. We expect growth of the ratio η'/η at larger transverse momentum. Detailed calculations are still required.