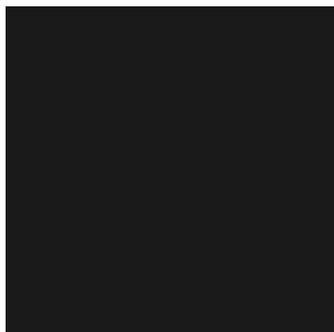




# Late stage Universe: cosmological models with interacting and non-interacting perfect fluids and scalar fields



Alvina Burgazli  
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New Trends in HEP, Odesa, 2019

# Outline

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**The scalar field coupled to inhomogeneities may exist and can provide the late cosmic acceleration.**

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We investigate the conditions under which a scalar field can become coupled, and show that, at the background level, such coupled scalar field behaves as a two component perfect fluid: a network of frustrated cosmic strings with EoS parameter  $w = -1/3$  and a cosmological constant.

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The potential of this scalar field is very flat at the present time. Hence, the coupled scalar field can provide the late cosmic acceleration.

The fluctuations of the energy density and pressure of this field are concentrated around the galaxies screening their gravitational potentials.

Therefore, such scalar fields can be regarded as coupled to the inhomogeneities.

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The potential of this scalar field is constant at present time. Hence, the coupled scalar field can be considered as a perfect fluid.

The fluctuations of this field are concentrated around the galaxies.

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**Coupled scalar fields in the late Universe: The mechanical approach and the late cosmic acceleration**

Alvina Burgazli<sup>1</sup>, Alexander Zhuk<sup>1</sup>, João Morais<sup>2</sup>,  
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JCAP (arXiv:1512.03819)

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**Rigorous theoretical constraint on constant negative EoS parameter  $\omega$  and its effect for the late Universe**

Alvina Burgazli<sup>a,1</sup>, Maxim Eingorn<sup>b,2,3,4</sup>, Alexander Zhuk<sup>c,2</sup>

The European Physical Journal C  
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# The great challenge for modern cosmology



Photo: Roy Kaltschmidt. Courtesy:  
Lawrence Berkeley National Laboratory

**Saul Perlmutter**



Photo: Belinda Pratten, Australian  
National University

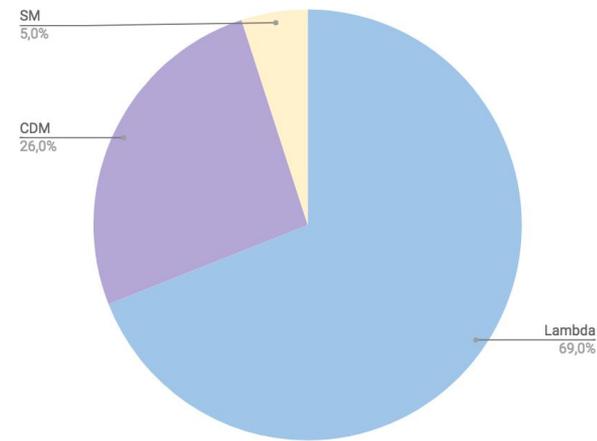
**Brian P. Schmidt**



Photo: Homewood Photography

**Adam G. Riess**

# Concordance cosmology

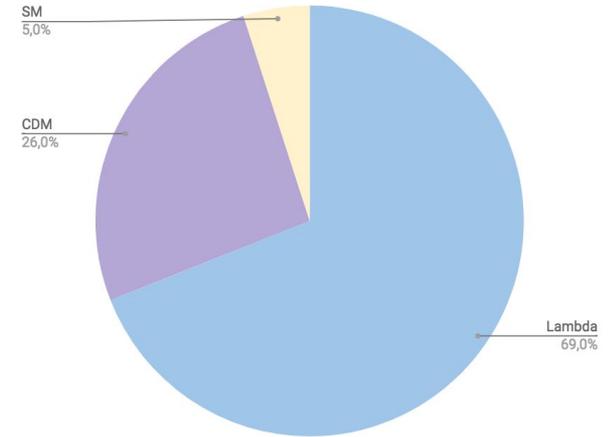


Planck 2015 Results.  
XIII. Cosmological parameters.

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The nature of the cosmological constant is, however, still unclear.

- There are, in fact, about 120 orders of magnitude between its observed value and the theoretically expected one, which is related to the vacuum energy density.

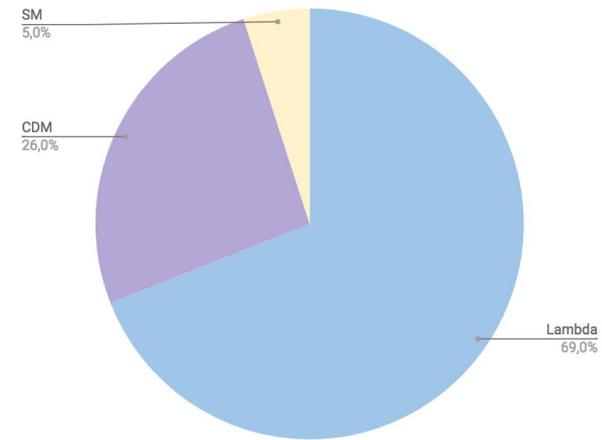


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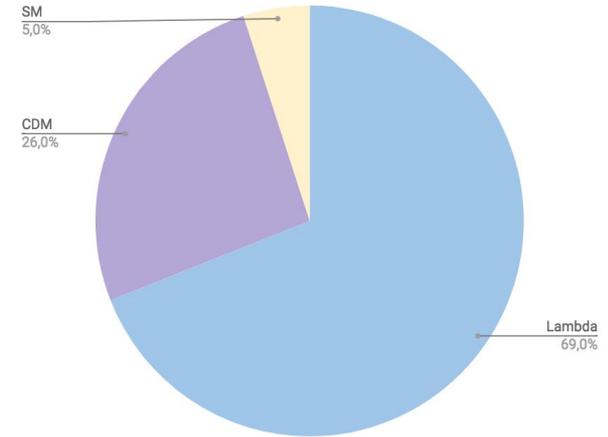


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Alternative models using scalar fields to explain the DE:

- quintessence ( $-1 < \omega < 0$ )
- phantom ( $\omega < -1$ )
- quintom ( $\omega = -1$  crossing)

$\omega =$   
const?

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MA was generalised to the case of cosmological models with different perfect fluids which can play the role of dark energy and dark matter. Fluctuations of these additional perfect fluids also form their own inhomogeneities. In the mechanical approach, it is supposed that the velocities of displacement of such inhomogeneities is of the order of the peculiar velocities of inhomogeneities of dust-like matter, i.e. they are non-relativistic.

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  - the potential of such scalar field is very flat;
- They allow to compute the peculiar velocities at the first order approximation.

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- deep inhomogeneities (as compared with the background level) can be considered as dust-like matter
- The coupled scalar field behaves (at the background level) as a two-component perfect fluid; (as compared with the background level) the potential of such scalar field is very flat;
- the scalar field fluctuations are absent, its energy density and pressure fluctuate due to the interaction of the gravitational potential and with the scalar field background.

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Works perfectly:

- The fluctuations of the energy density of the scalar field are concentrated around the galaxies, screening their gravitational potentials.
- The two-component perfect fluid (dark energy and dark matter) behaves (at the background level) as a dust-like matter.
- The potential of such scalar field is absent.
- the scalar field fluctuations are absent at the first order approximation.

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- We obtain the expressions for the gravitational potential for flat, open and closed topologies of the Universe.
- An important property of this potential is that its averaged (over the volume of the Universe) value is equal to zero, as it should be.

MA works perfectly: models with different perfect fluids which can play the role of dark matter. Fluctuations of these additional fluids also form their own inhomogeneities. In the mechanical approach, it is supposed that the velocities of displacement of such inhomogeneities is of the order of the peculiar velocities of inhomogeneities of dust-like matter, i.e. they are non-relativistic.

# Background equations

Friedman-Lemaître-Robertson-Walker background metric:

$$ds^2 = a^2 \left( d\eta^2 - \delta_{\alpha\beta} dx^\alpha dx^\beta \right)$$

scale factor  $a(\eta)$       conformal time      comoving spatial coordinate  
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$$S_\phi = \int d^4x \sqrt{-g} \left( \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right), \quad T_\nu^\mu(\phi) = g^{\mu\lambda} \partial_\nu \phi \partial_\lambda \phi - \delta_\nu^\mu \left( \frac{1}{2} g^{\lambda\rho} \partial_\lambda \phi \partial_\rho \phi - V(\phi) \right)$$

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The Friedmann and Raychaudhuri equations for our cosmological model:

$$\mathcal{H}^2 = \frac{1}{3} a^2 \kappa \left[ \bar{\varepsilon}_{\text{dust}} + \bar{\varepsilon}_{\text{rad}} + \frac{1}{2} (\phi'_c)^2 / a^2 + V(\phi_c) \right] - \mathcal{K}, \quad \mathcal{H}' = \frac{1}{3} a^2 \kappa \left[ -\bar{\varepsilon}_{\text{rad}} - \frac{1}{2} \bar{\varepsilon}_{\text{dust}} - (\phi'_c)^2 / a^2 + V(\phi_c) \right]$$

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For the considered model, Einstein equations are reduced (after linearising the system of 3 equations) to:

$$\begin{aligned} \Delta \Phi - 3\mathcal{H}(\Phi' + \mathcal{H}\Phi) + 3\mathcal{K}\Phi &= \frac{\kappa}{2} a^2 (\delta \varepsilon_{\text{dust}} + \delta \varepsilon_{\text{rad}}) \\ &- \frac{\kappa}{2} \left[ (\phi'_c)^2 \Phi - \phi'_c \varphi' - a^2 \frac{dV}{d\phi}(\phi_c) \varphi \right], \\ \partial_i \Phi' + \mathcal{H} \partial_i \Phi &= \frac{\kappa}{2} \phi'_c \partial_i \varphi, \end{aligned}$$

$$\begin{aligned} &\frac{2}{a^2} \left[ \Phi'' + 3\mathcal{H}\Phi' + \Phi \left( 2\frac{a''}{a} - \mathcal{H}^2 - \mathcal{K} \right) \right] \\ &= \kappa \left( \delta p_{\text{rad}} - \frac{1}{a^2} (\phi'_c)^2 \Phi + \frac{1}{a^2} \phi'_c \varphi' - \frac{dV}{d\phi}(\phi_c) \varphi \right) \end{aligned}$$

## Spatial distributions of $\delta\epsilon_{\text{rad}}$ is defined by $\Phi$

After substitution of the second Einstein equation, usage of some well-known relations and help of the background Friedmann and Raychaudhuri equations and the equation of motion, the third Einstein equation takes the form:

$$\Phi \left[ -\frac{2}{3}a^2\kappa\bar{\epsilon}_{\text{rad}} - \frac{1}{2}a^2\kappa\bar{\epsilon}_{\text{dust}} \right] = \kappa\frac{a^2}{2}\delta p_{\text{rad}}$$

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$$\delta p_{\text{rad}} = -\Phi\bar{\epsilon}_{\text{dust}} = -\Phi\frac{\bar{\rho}c^2}{a^3} = \frac{1}{3}\delta\epsilon_{\text{rad}}$$

Since the definition for  $\delta\epsilon_{\text{rad}}$ , taking into account the resulted second and third Einstein equations, some algebra and after substitution  $\Phi = \Omega/a$ , where  $\Omega$  is a function of  $\mathbf{a}$  and the spatial coordinates, the first Einstein equation reads:

$$\frac{\Delta\Omega}{a} - \frac{\kappa}{2}\frac{\delta\rho c^2}{a} = \frac{\Omega}{a} \left[ -3\mathcal{K} - \frac{\kappa}{2}(\phi'_c)^2 \right] + \frac{d\Omega}{da} \left[ 3\mathcal{H}^2 + \mathcal{H}' - \mathcal{H}\frac{\phi''_c}{\phi'_c} + a^2\frac{dV}{d\phi}(\phi_c)\frac{1}{\phi'_c}\mathcal{H} \right] + \frac{d^2\Omega}{da^2}a\mathcal{H}^2$$

Then, we are looking for the solutions of this equation which have a Newtonian limit near gravitating masses.

## The coupled scalar field can provide the late cosmic acceleration

In the flat  $K=0$  topology, if the dust-like matter is described by the discrete distributed gravitating sources (galaxies) with masses  $m_i$  and the rest-mass density  $\rho = \sum_i m_i \delta(\mathbf{r} - \mathbf{r}_i)$ , the grav. potential is:

$$\Phi = -\frac{G_N}{c^2} \frac{1}{a} \sum_i \frac{m_i}{|\mathbf{r} - \mathbf{r}_i|} = -\frac{G_N}{c^2} \sum_i \frac{m_i}{|\mathbf{R} - \mathbf{R}_i|}$$

It also demonstrates that  $\Phi \sim 1/a$ . After  $\phi'_c = \beta = \text{const}$ , we get that  $\phi_c = \beta\eta + \gamma$ ,  $\gamma = \text{const}$

Then the eq. of motion:  $2\frac{a'}{a}\beta + a^2\frac{dV}{d\phi}(\phi_c) = 2\frac{a'}{a}\beta + a^2\frac{V'}{\beta} = 0$ ,

$$\Rightarrow V = \frac{\beta^2}{a^2} + V_\infty, \quad V_\infty = \text{const}$$

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**The scalar field behaves as a two-component perfect fluid**

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For the background energy density and pressure:  $\bar{\epsilon}_\varphi = \frac{3}{2} \frac{\beta^2}{a^2} + V_\infty$ ,  $\bar{p}_\varphi = -\frac{1}{2} \frac{\beta^2}{a^2} - V_\infty$

We derived a specific form for the time dependence of the background scalar field and for its potential so that the scalar field is consistent with such a coupling to the inhomogeneities.

The fluctuations of the scalar field are absent but the fluctuations of the energy density and pressure are non-zero. These fluctuations are concentrated around galaxies, in full agreement with the coupling condition.

## The coupled scalar field can provide the late cosmic acceleration

In the flat  $K=0$  topology, if the dust-like matter is described by the discrete distributed gravitating sources (galaxies) with masses  $m_i$  and the rest-mass density  $\rho = \sum_i m_i \delta(\mathbf{r} - \mathbf{r}_i)$ , the grav

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It also demonstrates that  $\Phi \sim 1/a$  for  $\rho = \text{const}$ , we get that  $\phi_c = \beta\eta + \gamma$ ,  $\gamma = \text{const}$

Then the eq. of motion:  $2\frac{a'}{a}\beta + a^2 \frac{dV}{d\phi}(\phi_c) = 2\frac{a'}{a}\beta + a^2 \frac{V'}{\beta} = 0,$

$$\Rightarrow V = \frac{\beta^2}{a^2} + V_\infty, \quad V_\infty = \text{const}$$

For the background energy den

We derived a specific form for that the scalar field is consistent

$$V \simeq V_\infty + \frac{\beta^2}{a_0^2} \left( \frac{\phi_c}{A} \right)^2 = V_\infty \left[ 1 + \frac{2\Omega_\beta}{3\Omega_\lambda} \left( \frac{\phi_c}{A} \right)^2 \right]$$

The fluctuations of the scalar field are absent but the fluctuations of the energy density and pressure are non-zero. These fluctuations are concentrated around galaxies, in full agreement with the coupling condition.

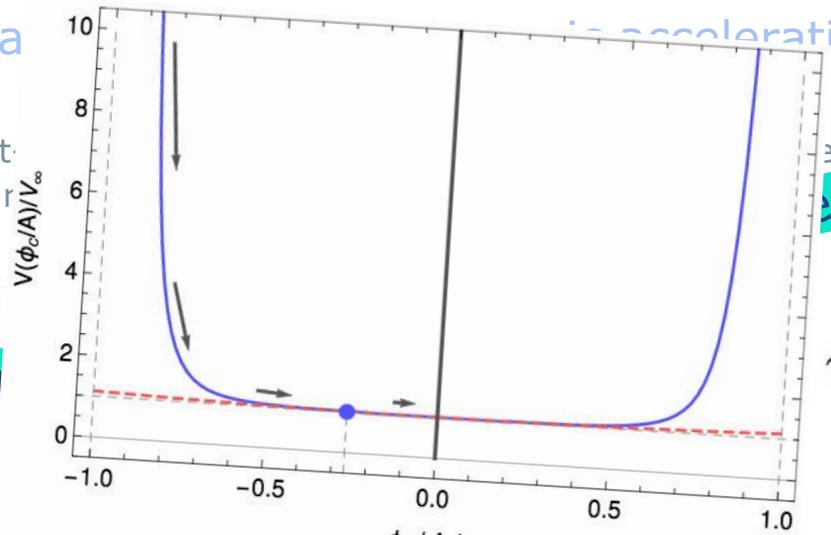
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It also demonstrates that  $\Phi \sim 1/a$

Then the eq. of motion:  $2\frac{\ddot{\phi}}{a} + a^2$



ed gravitating sources  
perfect fluid

$\gamma = \text{const}$

$$\Rightarrow V = \frac{v^2}{a^2} + V_\infty; \quad v_\infty = \frac{\phi_c / \Delta\phi_c}{\dots}$$

For the background energy den

We derived a specific form for that the scalar field is consistent

$$V \simeq V_\infty + \frac{\beta^2}{a_0^2} \left( \frac{\phi_c}{A} \right)^2 = V_\infty \left[ 1 + \frac{2\Omega_\beta}{3\Omega_\lambda} \left( \frac{\phi_c}{A} \right)^2 \right]$$

The fluctuations of the scalar field are absent but the fluctuations of the energy density and pressure are non-zero. These fluctuations are concentrated around galaxies, in full agreement with the coupling condition.

## Conclusions

- Considered scalar field behaves as a two component perfect fluid.
- Its potential should be very flat at the present time.
- The fluctuations of this field are absent but the fluctuations of the energy density and pressure are non-zero. These fluctuations are concentrated around galaxies, in full agreement with the coupling condition.

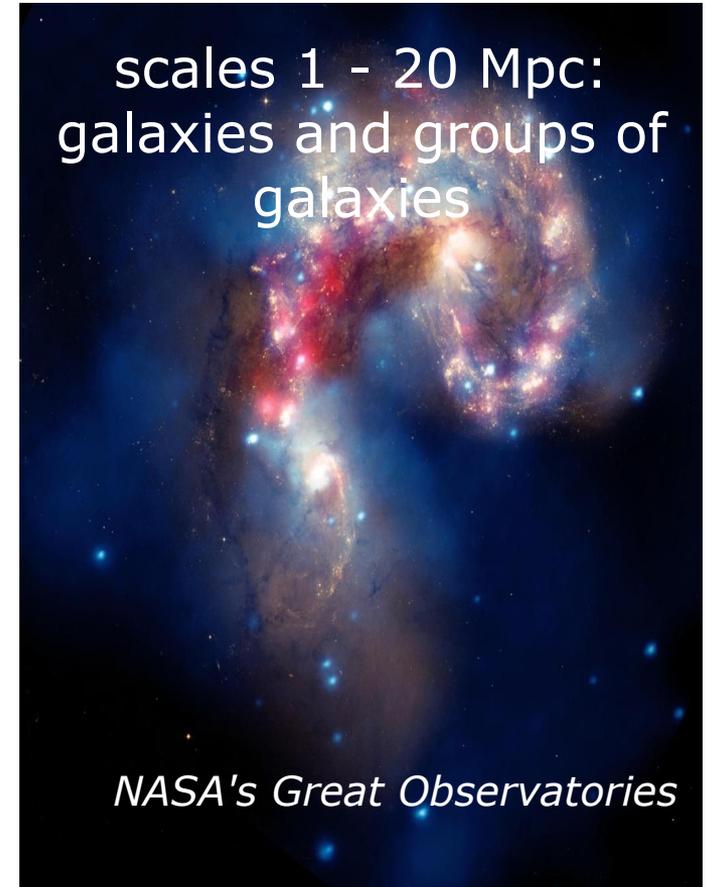
The coupled scalar fields may exist under the conditions mentioned above and can provide the late cosmic acceleration.



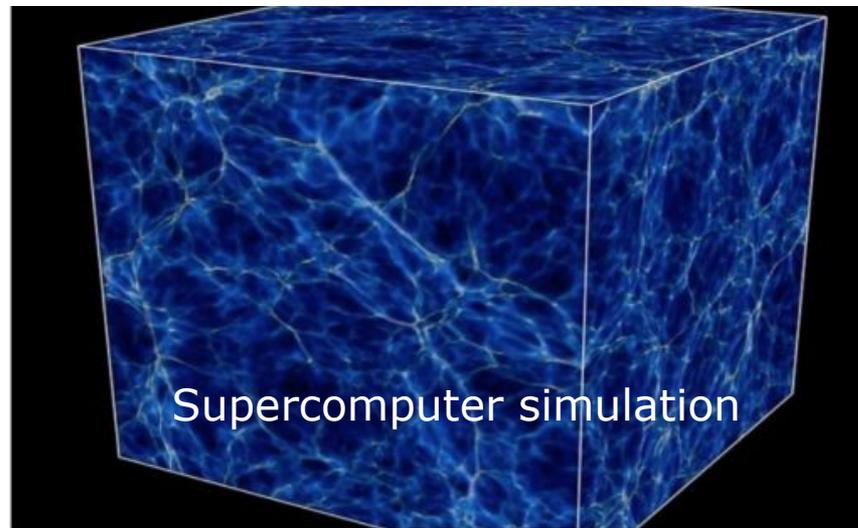
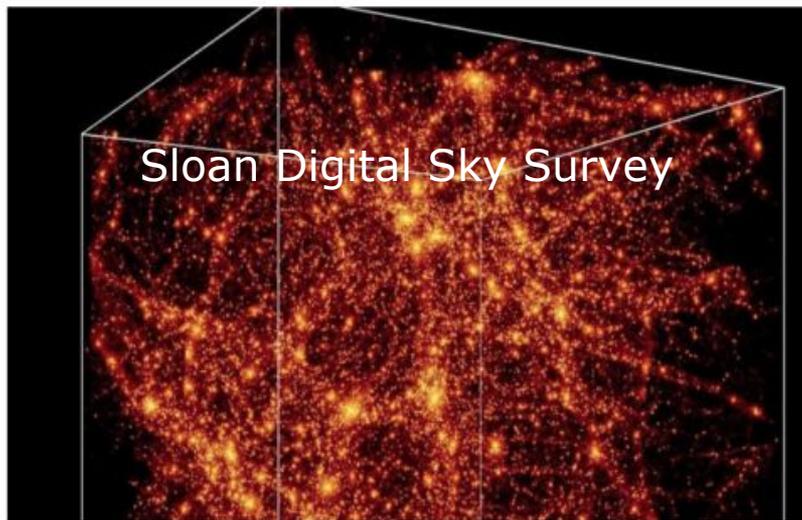
Thank you for your attention!

Alvina Burgazli  
aburgazli@gmail.com

# Inhomogeneous Universe



## Scales 50-200 Mpc: filaments and voids



Newtonian potential  $\Phi \sim 1/R$  is used to simulate the structure formation.

Up to which scales can we use the Newtonian potential?

# Cosmological Principle (CP)

————→ Physical background for the CP:

CP is the notion that the spatial distribution of matter in the Universe is homogeneous and isotropic when viewed on large enough scales

However, recent observations (XXI Century):

There are cosmic structures with  $l \gg 300\text{Mpc}$  .

# The largest cosmic structures



$l \approx 423 \text{ Mpc}$

Giant filament  
consisting of a number  
of superclusters

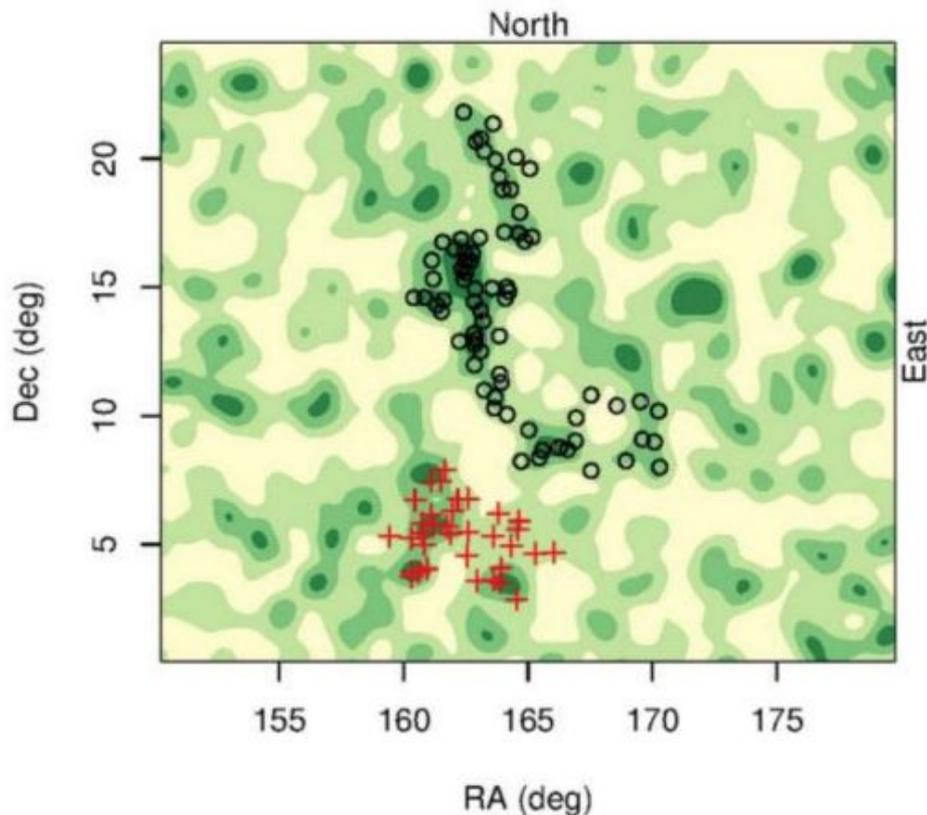
# Large Quasar Groups

2. Clowes-Campusano LQG:  
34 quasars (red crosses)

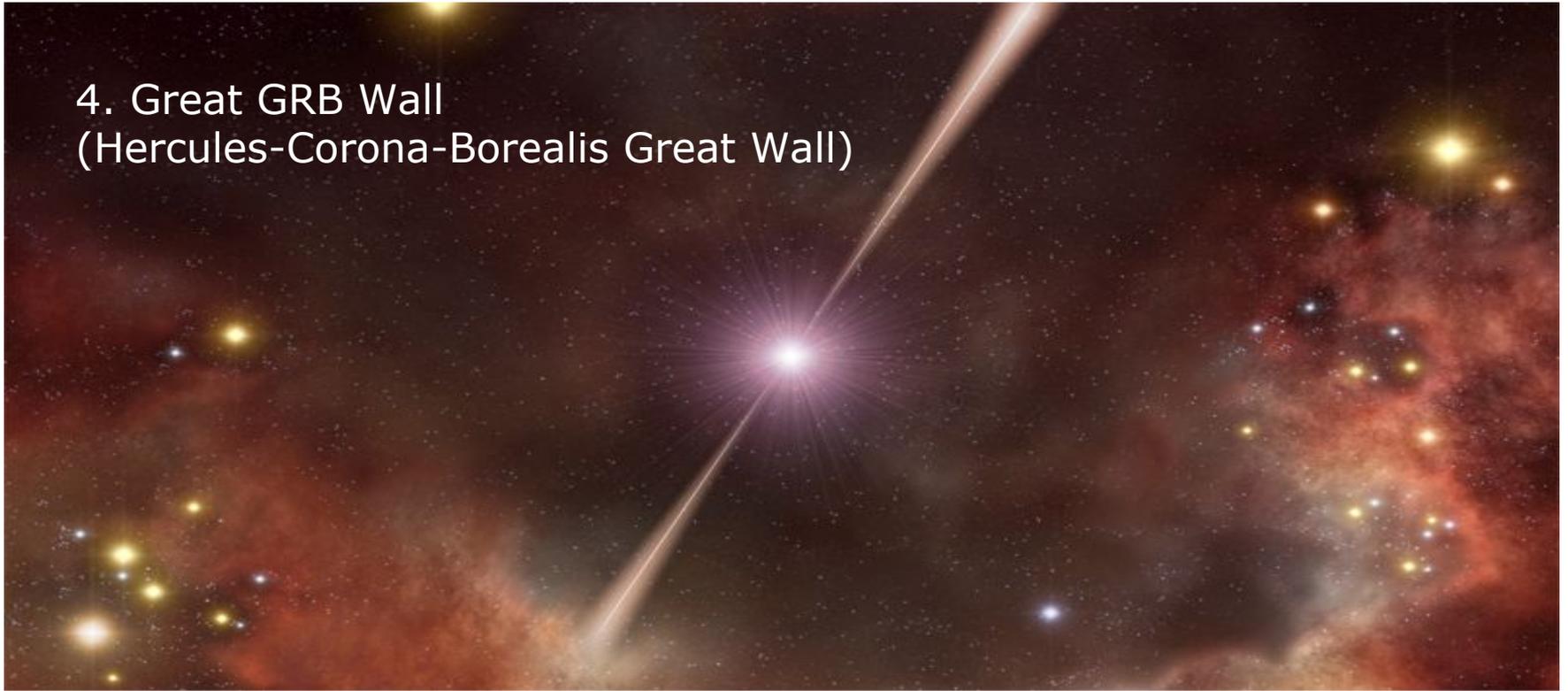
$$l \approx 613 \text{ Mpc}$$

3. Huge LQG:  
73 quasars (black rings)

$$l \approx 1226 \text{ Mpc}$$



#### 4. Great GRB Wall (Hercules-Corona-Borealis Great Wall)



A region of the sky seen in the data set mapping of gamma-ray bursts (GRBs) that has been found to have an unusually higher concentration of similarly distanced GRBs than the expected average distribution.

# These huge structures are the great challenge!

- Can we explain such big structures?
- Do larger structures exist?
- From which scales does the cosmological principle start to work?
- Can we use the Newtonian potential for the structure formation simulation on any cosmological scale?



## Two main distinct approaches to structure growth investigation

<p><b>relativistic perturbation theory</b></p>	<p><b>N-body simulations generally based on Newtonian cosmological approximation</b></p>
<p><b>Keywords</b></p>	
<p><b>early Universe; linearity; large scales</b></p>	<p><b>late Universe; nonlinearity; small scales</b></p>
<p><b>fails in describing nonlinear dynamics at small distances</b></p>	<p><b>do not take into account relativistic effects becoming non-negligible at large distances</b></p>

# Origin of cosmic structures

Cosmic structures are formed (grow up) due to the gravitational interaction



**Gravitational potential**

$$\Phi, \nabla\Phi \neq 0$$

## Newtonian mechanics

Poisson eq.  $\Delta\Phi = \frac{4\pi G}{c^2} \rho \equiv \frac{\kappa}{2} \rho c^2$ ,  $\rho$  – rest-mass density

$$\rho = \sum_n m_n \delta(\vec{R} - \vec{R}_n)$$

Neumann-Seeliger paradox

$$\Phi(R) \sim \sum_n \frac{m_n}{|\vec{R} - \vec{R}_n|}$$

This sum is divergent in the case of infinite number of sources

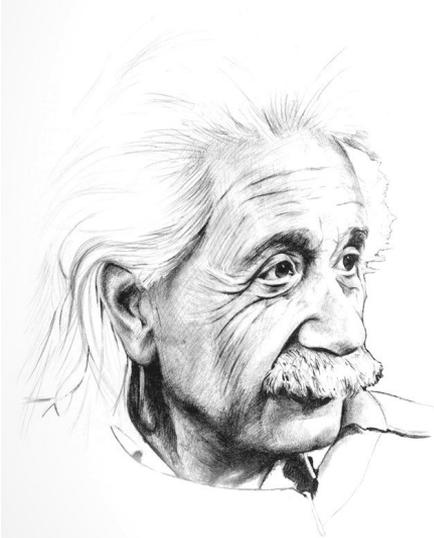


# The Neumann-Seeliger paradox

should be resolved within the framework of General Relativity (taking into account relativistic effects)



Einstein equations



# Background equations

Background FLRW metric (homogeneous and isotropic):

$$ds^2 = a^2 \left( d\eta^2 - \delta_{\alpha\beta} dx^\alpha dx^\beta \right)$$

scale factor  $a(\eta)$       conformal time      comoving spatial coordinate  
 physical distance:  $R = ar$

Background matter (CDM:  $\bar{p} = 0$ ):

energy density  $\bar{\varepsilon} = \bar{\rho}c^2 = \frac{\bar{\rho}_c c^2}{a^3}$ ,       $\bar{\rho}_c = \text{const}$  ← comoving rest mass density

# Friedmann equations

in the framework of the pure  $\Lambda$ CMD model (with a negligible radiation contribution):

$$\frac{3\tilde{H}^2}{a^2} = \kappa\bar{\mathcal{E}} + \Lambda$$

**energy density of  
nonrelativistic  
pressureless matter**

$$\frac{2\tilde{H}' + \tilde{H}^2}{a^2} = \Lambda$$

**cosmological  
constant**

**overline:** average value ;  
**prime:** derivative with  
respect to  $\eta$

$$\tilde{H} \equiv \frac{a'}{a} \quad \kappa \equiv 8\pi G_N / c^4$$

**Newtonian  
gravitational constant**



# Newtonian approximation in cosmology

without relativistic effects (see e.g. Peebles, Gorbunov & Rubakov)

$$\Delta_c \Phi = \frac{4\pi G}{c^2} a^2 \delta\rho = \frac{4\pi G}{c^2} \frac{\delta\rho_c}{a}, \quad \delta\rho_c = \rho_c - \bar{\rho}_c, \quad \Delta_c = a^2 \Delta$$

Solution:

$$\rho_c = \sum_n m_n \delta(\vec{r} - \vec{r}_n)$$

$$\Phi \sim \int d\vec{r}' \frac{\rho_c|_{\vec{r}=\vec{r}'} - \bar{\rho}_c}{|\vec{r} - \vec{r}'|}, \quad -\nabla\Phi \sim \int d\vec{r}' \frac{\rho_c|_{\vec{r}=\vec{r}'}}{|\vec{r} - \vec{r}'|^3} (\vec{r} - \vec{r}') \sim \sum_n \frac{m_n}{|\vec{r} - \vec{r}_n|^3} (\vec{r} - \vec{r}_n)$$

are not well-defined!

Can we resolve both of these problems:

- to define an upper limit on the size of cosmic structures;
- to obtain well-defined form of the gravitational potential?

Solution: **relativistic effects!**

# Gravitational potential in general relativity

Theory of scalar perturbations

Perturbed FLRW metric:  $ds^2 = a^2 \left[ (1 + 2\Phi) d\eta^2 - (1 - 2\Phi) \delta_{\alpha\beta} dx^\alpha dx^\beta \right]$

gravitational potential

Linearized Einstein equations ( $\Phi \ll 1$ ):

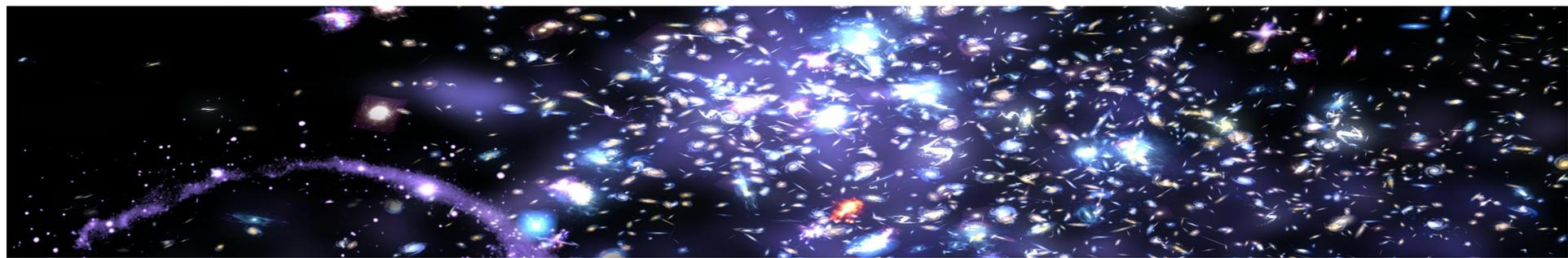
$$G_0^0 = \kappa T_0^0 + \Lambda \quad \Rightarrow \quad \Delta_c \Phi - 3\tilde{H}(\Phi' + \tilde{H}\Phi) = \frac{1}{2} \kappa a^2 \delta T_{0(\text{CDM})}^0$$

$$G_\alpha^0 = \kappa T_\alpha^0 + \Lambda \quad \Rightarrow \quad \frac{\partial}{\partial x^\alpha} (\Phi' + \tilde{H}\Phi) = \frac{1}{2} \kappa a^2 \delta T_{\alpha(\text{CDM})}^0$$

$$' \equiv \frac{d}{d\eta}, \quad \tilde{H} = \frac{da}{d\eta} \frac{1}{a}$$

$$G_\beta^\alpha = \kappa T_\beta^\alpha + \Lambda \quad \Rightarrow \quad \Phi'' + 3\tilde{H}\Phi' + (2\tilde{H}' + \tilde{H}^2)\Phi = 0$$

We consider CDM as a set of point-like inhomogeneities (e.g. galaxies, groups and clusters of galaxies).



Energy-momentum tensor (EMT) of inhomogeneities (e.g. Landau&Lifshitz):

$$T^{ik} = \sum_n \frac{m_n c^2}{(-g)^{1/2} [\eta]} \frac{dx_n^i}{d\eta} \frac{dx_n^k}{d\eta} \frac{1}{ds_n / d\eta} \delta(\mathbf{r} - \mathbf{r}_n)$$

$$\tilde{v}_n^\alpha \equiv \frac{dx_n^\alpha}{d\eta} = \frac{a}{c} \frac{dx_n^\alpha}{dt} = \frac{av_n^\alpha}{c} = \frac{v_{phn}^\alpha}{c}, \quad \alpha = 1, 2, 3 \quad \leftarrow \text{comoving peculiar velocity}$$

$$\rho_c = \sum_n m_n \delta(\mathbf{r} - \mathbf{r}_n) \equiv \sum_n \rho_{cn} \quad \leftarrow \text{comoving rest-mass density}$$

EMT perturbations:

Due to explicit dependence  $T_0^0$   
on  $g_{ik}$  and  $\rho$

$$\delta T_{0(\text{CDM})}^0 \approx \frac{\delta \rho_c c^2}{a^3} + \frac{3\bar{\rho}_c c^2}{a^3} \Phi, \quad \delta \rho_c = \rho_c - \bar{\rho}_c - \text{can be} \gg \bar{\rho}_c$$

effects of nonlinearity:

$$\delta T_\alpha^0 = -\frac{c^2}{a^3} \sum_n m_n \delta(\vec{r} - \vec{r}_n) \tilde{v}_n^\alpha = -\frac{c^2}{a^3} \sum_n \rho_{cn} \tilde{v}_n^\alpha, \quad \delta T_\beta^\alpha = 0$$

effective velocity potential:

$$\sum_n \rho_{cn} \tilde{v}_n \equiv \nabla \Xi + \text{curl}$$

$$\frac{\partial}{\partial x^\beta} (\Phi' + \tilde{H}\Phi) = \frac{1}{2} \kappa a^2 \delta T_{\beta(\text{CDM})}^0 \longrightarrow \Phi' + \tilde{H}\Phi = -\frac{\kappa c^2}{2a} \Xi$$

$$\Delta_c \Phi - 3\tilde{H}(\Phi' + \tilde{H}\Phi) = \frac{1}{2} \kappa a^2 \delta T_{0(\text{CDM})}^0$$

## Helmholtz (not Poisson!) equation

$$\Delta_c \Phi - \frac{a^2}{\lambda^2} \Phi = \frac{\kappa c^2}{2a} \delta \rho_c - \frac{3\kappa c^2 \tilde{H}}{2a} \Xi \quad \Xi = \frac{1}{4\pi} \sum_n m_n \frac{(\vec{r} - \vec{r}_n) \vec{v}_n}{|\vec{r} - \vec{r}_n|^3}$$

$$\lambda = \left[ \frac{3\kappa}{2} \bar{\varepsilon} \right]^{-1/2} = \sqrt{\frac{2a^3}{3\kappa \bar{\rho}_c c^2}} = \sqrt{\frac{2c^2}{9H_0^2 \Omega_M} \left( \frac{a}{a_0} \right)^3}, \quad \Omega_M \equiv \frac{\kappa \bar{\rho}_c c^4}{3H_0^2 a_0^3}$$

$\lambda$  defines the range of the Yukawa interaction!

At present time  $\lambda_0 \approx 3700 \text{ Mpc}$

Cosmological screening (i.e. finite  $\lambda$ ) is the effect of the background:  $\bar{\varepsilon} \neq 0$

$\frac{1}{3}$ 

for CDM

Newton potential

Yukawa potential

$$\Phi = \frac{\kappa \bar{\rho}_c c^2 \lambda^2}{2a a^2} - \frac{\kappa c^2}{8\pi a} \sum_n \frac{m_n}{|\vec{r} - \vec{r}_n|} \exp(-q_n)$$

$$+ \frac{3\kappa c^2}{8\pi a} \tilde{H} \sum_n \frac{m_n \left[ \tilde{v}_n (\vec{r} - \vec{r}_n) \right]}{|\vec{r} - \vec{r}_n|} \frac{1 - (1 + q_n) \exp(-q_n)}{q_n^2}$$

$$\vec{q}_n \equiv \frac{a}{\lambda} (\vec{r} - \vec{r}_n) = \frac{1}{\lambda} (\vec{R} - \vec{R}_n)$$

1. Newtonian approximation:  $|\vec{R} - \vec{R}_n| \ll \lambda$  (and ignore velocities as a source of the grav. field)
2. Exponential suppression:  $|\vec{R} - \vec{R}_n| \gg \lambda$  (Great GRBs Wall)

the largest structures  $\leq \lambda$  ( $\lambda_0 \approx 3700 \text{ Mpc} > 3066 \text{ Mpc}$ );  
 the absence of the Neumann-Seeliger paradox.

# The Yukawa interaction range and the horizons

$$\lambda = \left[ \frac{3\kappa}{2} \bar{\varepsilon} \right]^{-1/2} = \sqrt{\frac{2a^3}{3\kappa\bar{\rho}_c c^2}} = \frac{1}{\sqrt{3(1+q)}} \frac{c}{H} \sim a^{3/2}$$

At the present time:  $\lambda_0 \approx 3.7 \times 10^3$  Mpc

Hubble horizon (radius):

(this horizon is not really a physical size)

$$\frac{c}{H_0} \approx 4.1 \times 10^3 \text{ Mpc} > \lambda_0 \quad \frac{c}{H_{\uparrow}} = \lambda \quad \text{at the deceleration parameter } q = -\frac{\ddot{a}}{(aH^2)} = -2/3$$

$$a = 1.16 a_0$$

Particle horizon:

(this is the farthest distance that any photon can freely stream from the Big Bang – the size of the observable Universe)

$$l_p(t_0) = a(t_0) \int_0^{t_0} \frac{cd\tilde{t}}{a(\tilde{t})} \approx 14.26 \times 10^3 \text{ Mpc} \quad - \quad \text{radius of the observable Universe}$$

# Universality of the Yukawa suppression

Additionally two sets of continuous perfect fluids:

$$\text{Linear EoS } p_I = \omega_I \varepsilon_I, \quad \omega_I = \text{const} \quad . \quad \text{Non linear EoS } p_J = f_J(\varepsilon_J)$$

$$\text{Dark energy: } (p \approx -\varepsilon), \quad \text{radiation: } (p = \varepsilon/3) \quad \text{etc}$$

Total  $\Phi$  can be split into individual contributions from inhomogeneities of each matter source:

$$\Phi = \Phi_{\text{CDM}} + \underbrace{\sum_I \Phi_I + \sum_J \Phi_J}_{\text{continuous perfect fluids}}$$

discrete CDM

Each of components  $\Phi$  satisfies the Helmholtz eq. with the **same**  $\lambda$

$$\left\{ \begin{array}{l} \Delta_c \Phi_{CDM} - \frac{a^2}{\lambda^2} \Phi_{CDM} = \frac{\kappa c^2}{2a} \delta\rho_c - \frac{3\kappa c^2 \tilde{H}}{2a} \Xi \\ \Delta_c \Phi_I - \frac{a^2}{\lambda^2} \Phi_I = \frac{\kappa}{2} \frac{\delta A_I}{a^{1+3\omega_I}} - \frac{3\kappa \tilde{H}}{2} \frac{1+\omega_I}{a^{1+3\omega_I}} \xi_I \\ \Delta_c \Phi_J - \frac{a^2}{\lambda^2} \Phi_J = \frac{\kappa a^2}{2} \bar{\varepsilon}_J \delta_J - \frac{3\kappa a^2 \tilde{H}}{2} (\bar{\varepsilon}_J + \bar{p}_J) \nu_J \end{array} \right.$$

$$\frac{1}{\lambda^2} = \frac{3\kappa}{2} \left[ \bar{\varepsilon}_{CDM} + \underbrace{\sum_I (\bar{\varepsilon}_I + \bar{p}_I) + \sum_J (\bar{\varepsilon}_J + \bar{p}_J)} \right] = \frac{3}{c^2} H^2 (1+q)$$

$$\bar{p} \approx -\bar{\varepsilon} \Rightarrow \approx 0$$

$$\longrightarrow \lambda \approx \lambda_0 \approx 3700 \text{ Mpc} \quad \text{at present time}$$

## Conclusions

1. Our approach works **at all cosmological scales** (i.e. sub-horizon and super-horizon) and incorporates **linear and nonlinear effects** with respect to the energy density fluctuations ( $\delta\varepsilon / \bar{\varepsilon}$  can be  $\gg 1$ ).
2. The gravitational potential can be split into individual contributions from inhomogeneities of each matter source (i.e. discrete CDM and continuous DE). Each of these contributions satisfies its own Helmholtz-type equation.

3. The gravitational potentials are characterized by a finite time-dependent Yukawa interaction range being the **same** for each individual contributions. At the present time

$$\lambda \approx \lambda_0 \approx 3700 \text{ Mpc}$$

The value  $\lambda_0$  is bigger than **the largest known structure** in the Universe (**Great GRBs Wall**):

$$\lambda_0 \approx 3700 \text{ Mpc} > 3066 \text{ Mpc}$$

4. At distances  $|\vec{R} - \vec{R}_n| > \lambda$  the gravitational potential of the  $n$ -th fluctuation is exponentially suppressed. This suppression is called the cosmological screening. The cosmological background is responsible for this effect (greetings to Ernst Mach!).

$$|\vec{R} - \vec{R}_n| \ll \lambda \longrightarrow \text{Newton}$$

$$\Phi(R) \sim \sum_n \frac{m_n}{|\vec{R} - \vec{R}_n|}$$

$$|\vec{R} - \vec{R}_n| \gg \lambda \longrightarrow \text{Yukawa}$$

$$\Phi \sim \sum_n \frac{m_n}{|\vec{R} - \vec{R}_n|} \exp(-q_n)$$

Newton is wrong at the cosmological distances  $|\vec{R} - \vec{R}_n| > \lambda$