Equation of state for hot QCD and compact stars from a mean field approach

based on 1905.00866

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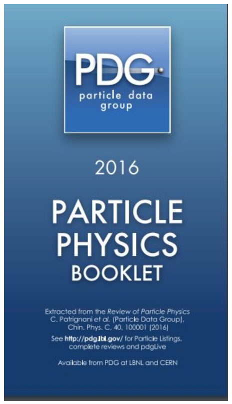
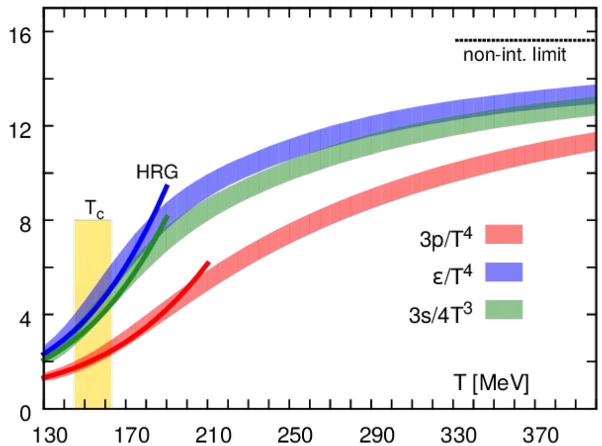
Known QCD phenomenology

hadron-resonance gas at low T

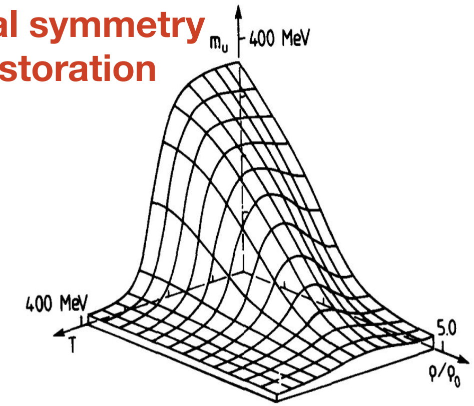


Stefan-Boltzmann limit for massless quarks



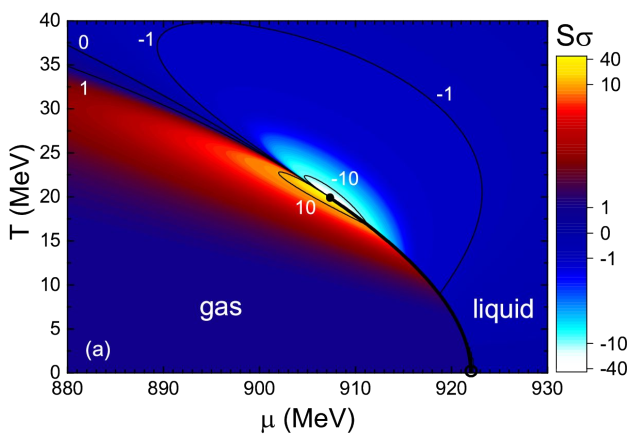
HotQCD, 1407.6387

Chiral symmetry restoration



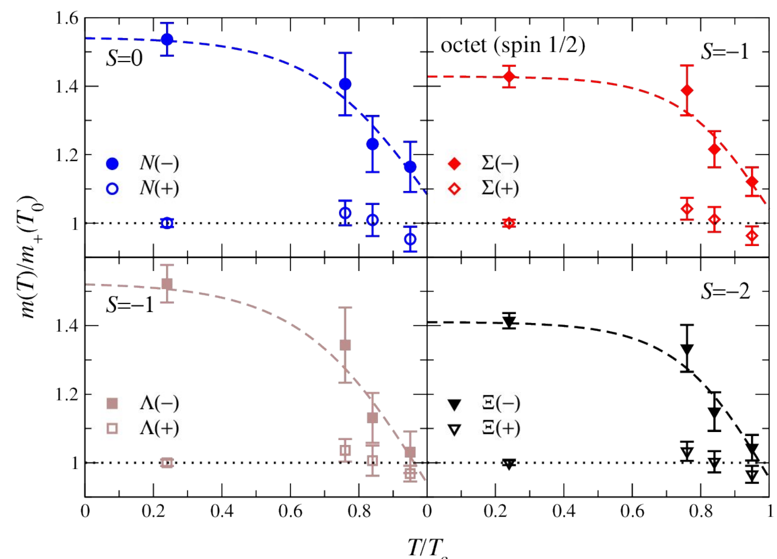
U. Vogl, W. Weise, 1991

Nuclear first order liquid-vapor phase transition



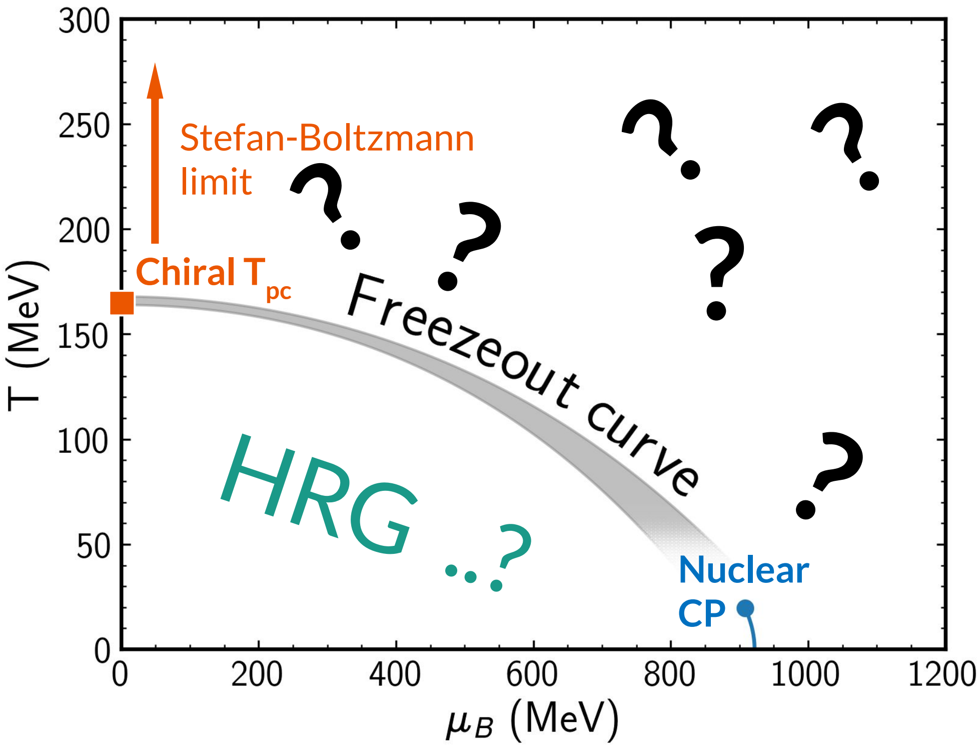
V. Vovchenko et al., 1506.05763

Parity doubling among baryons



G. Aarts et al., 1710.08294

Unknown QCD phenomenology



How one can map known phenomenology to the **QCD phase diagram**?

We build a **unified approach to equation of state** that incorporates most features of QCD phenomenology.

Chiral SU(3)$_f$ parity-doublet Polyakov-loop quark-hadron model

- **SU(3)** — 3-flavor (u, d, s) quark model: respective baryon octet interacting through mesonic fields.
  Realization of σ model.
  P. Papazoglou et al., nucl-th/9706024
- **parity-doublet** — parity doubling among the baryon octet
  C. E. Detar and T. Kunihiro, Phys.Rev. D39 (1989)
  T. Hatsuda and M. Prakash, Phys.Lett. B224 (1989)
  G. Aarts et al., 1703.09246 and 1812.07393
- **quark-hadron** — realization of the deconfinement, PNJL-like
  K. Fukushima, hep-ph/0310121
  C. Ratti, M.A. Thaler, W. Weise, hep-ph/0506234
  J. Steinheimer, S. Schramm, H. Stoecker, 1009.5239
- **chiral** — chiral symmetry restoration among parity partners and in the quark sector, chiral field is a proxy interaction between quarks and hadrons

A **single framework** to QCD thermodynamics, **simultaneously** satisfies constraints from **lattice QCD** and known **nuclear matter properties**, as well as **neutron star** observations.

SU(3)$_f$ baryon octet and parity doubling

We include all states of the **SU(3)$_f$** baryon octet:

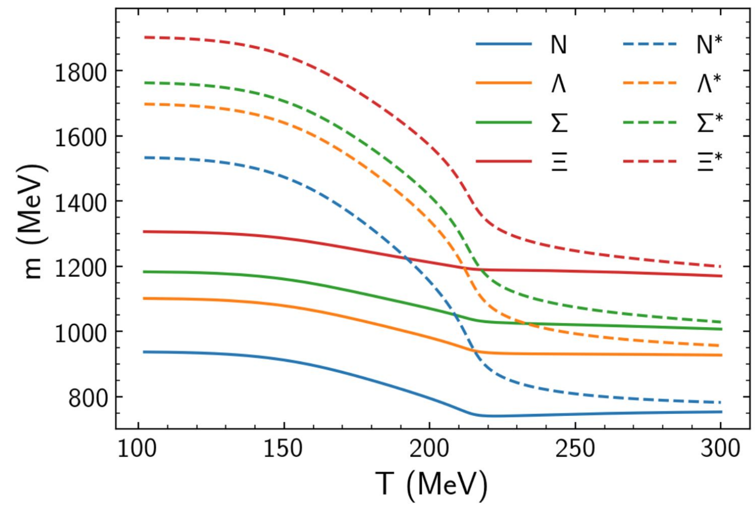
$$\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda}{\sqrt{6}} \end{pmatrix}$$

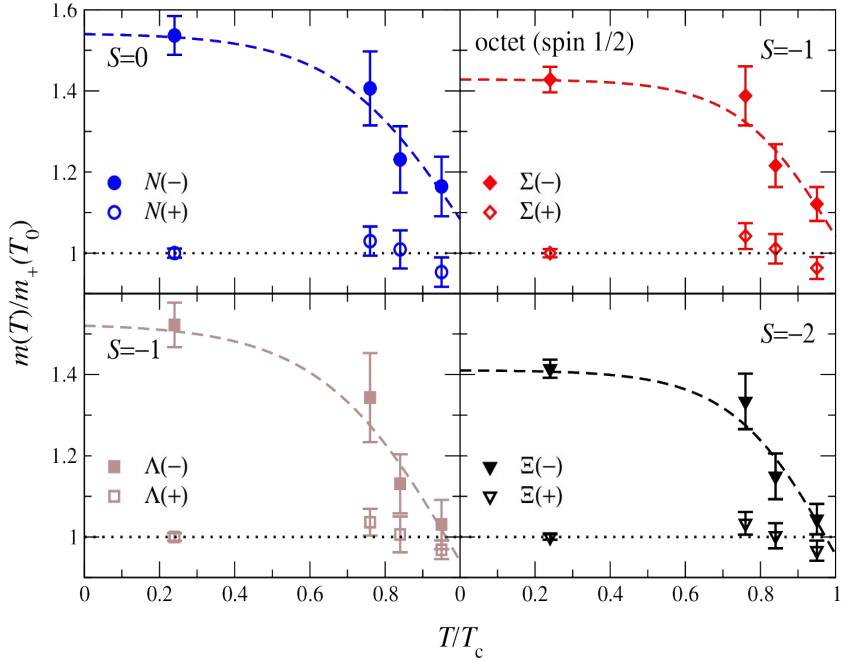
together with their **parity partners**, i.e. states with the same quantum numbers but **opposite parity**. Those interact within SU(3)$_f$ sigma model:

$$\mathcal{L}_B = \sum_i (\bar{B}_i i \partial\!\!\!/ B_i) + \sum_i (\bar{B}_i m_i^* B_i) + \sum_i (\bar{B}_i \gamma_\mu (g_{\omega i}\omega^\mu + g_{\rho i}\rho^\mu + g_{\phi i}\phi^\mu) B_i)$$

with effective masses generated by chiral fields **σ** and **ζ**:

$$m_{i\pm}^* = \sqrt{\left[(g_{\sigma i}^{(1)}\sigma + g_{\zeta i}^{(1)}\zeta)^2 + (m_0 + n_s m_s)^2\right]} \pm g_{\sigma i}^{(2)}\sigma \pm g_{\zeta i}^{(2)}\zeta$$





G. Aarts et al., 1710.08294

Chiral SU(3)$_f$ parity-doublet Polyakov-loop quark-hadron model

Mesonic fields:

$$\mathcal{L}_{meson} = -\frac{1}{2}(m_\omega^2\omega^2 + m_\phi^2\phi^2 + m_\rho^2\rho^2)$$
$$- g_4\left(\omega^4 + \frac{\phi^4}{4} + \frac{\rho^4}{2} + 3\omega^2\phi^2 + 3\omega^2\rho^2 + \frac{4\omega^3\phi}{\sqrt{2}} + \frac{2\omega\phi^3}{\sqrt{2}} + \frac{3\rho^2\phi^2}{2}\right)$$
$$+ \frac{1}{2}k_0(\sigma^2+\zeta^2) - k_1(\sigma^2+\zeta^2)^2$$
$$- k_2\left(\frac{\sigma^4}{2} + \zeta^4\right) - k_3\sigma^2\zeta + k_6(\sigma^6 + 4\zeta^6)$$
$$+ m_\pi^2 f_\pi \sigma + \left(\sqrt{2}m_k^2 f_k - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta - k_4 \ \ln\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0}$$

σ and ζ drive chiral symmetry breaking of non-strange and strange sector respectively.

Baryon octet + partners:

$$\mathcal{L}_B = \sum_i(\bar{B}_i i\partial\!\!\!/ B_i) + \sum_i(\bar{B}_i m_i^* B_i)$$
$$+ \sum(\bar{B}_i\gamma_\mu(g_{\omega i}\omega^\mu + g_{\rho i}\rho^\mu + g_{\phi i}\phi^\mu)B_i) \ ,$$

$$m_{i\pm}^* = \sqrt{\left[(g_{\sigma i}^{(1)}\sigma + g_{\zeta i}^{(1)}\zeta)^2 + (m_0 + n_s m_s)^2\right]} \pm g_{\sigma i}^{(2)}\sigma \pm g_{\zeta i}^{(2)}\zeta \ , B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda}{\sqrt{6}} \end{pmatrix}$$

Quarks in PNJL-like approach:

$$\Omega_q = -T\sum_{i\in Q}\frac{d_i}{(2\pi)^3}\int d^3k \ln\left(1 + \Phi \exp\frac{-(E_i^* - \mu_i)}{T}\right)$$
$$m_q^* = -g_{q\sigma}\sigma + \delta m_q + m_{0q},$$
$$m_s^* = -g_{s\zeta}\zeta + \delta m_s + m_{0q}$$

where Polyakov loop Φ controls deconfinement with the following potential U(Φ):

Excluded volume corrections for hadrons:

$$\rho_i = \frac{\rho_i^{id}(T, \mu_i^* - v_i p)}{1 + \sum_i v_j \rho_j^{id}(T, \mu_j^* - v_j p)}$$

$$U = -\frac{1}{2}(a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2)\Phi\Phi^* + b_3 T_0^4 \log[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2]$$

Ratti, Thaler, Weise, hep-ph/0506234

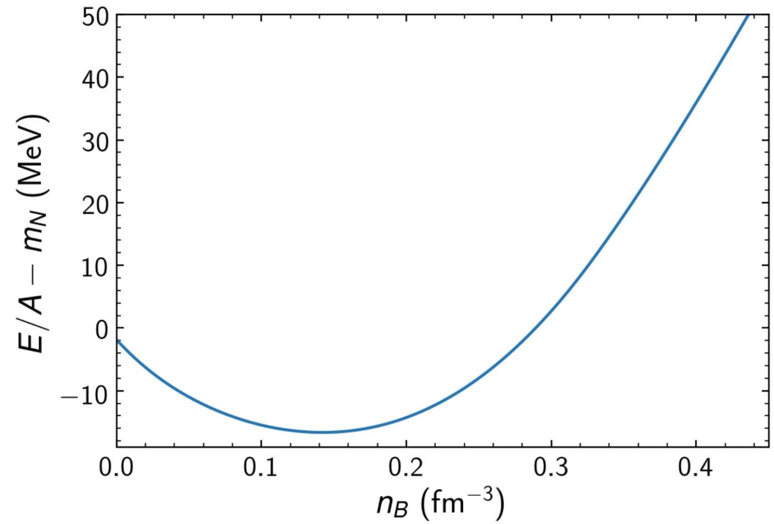
All calculations are done in the mean field approximation.

Model parameterization

Model has numerous couplings that got to be determined.

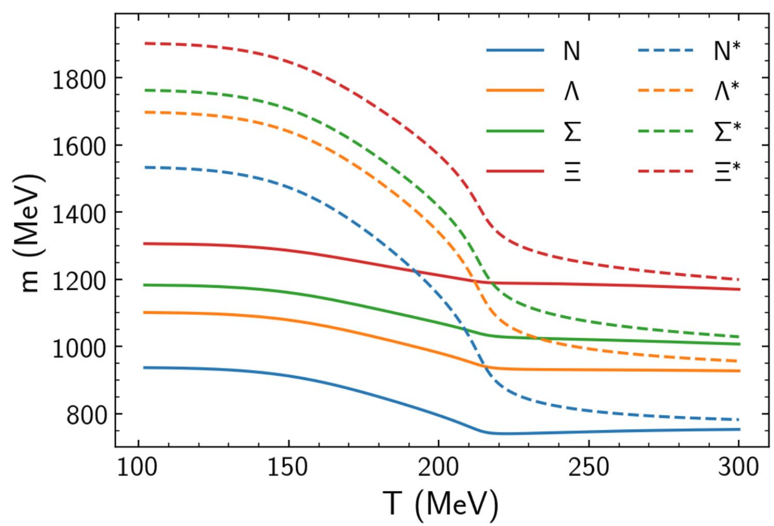
Nuclear physics comes to provide constraints on parameters:

- Nuclear matter ground state:
  ➡ $E/A(n_0) = -16$ MeV, $n_0 = 0.16$ fm^{-3};
- Compressibility ➡ $K(n_0) = 267$ MeV;
- Asymmetry energy ➡ $S(n_0) = 31.9$ MeV;
- Vacuum masses of octet baryons



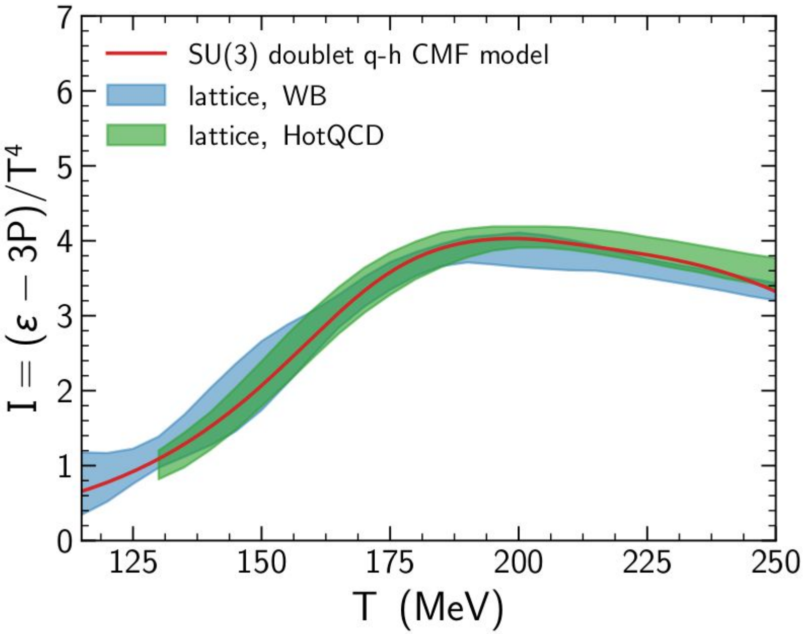
+

Model reproduces binding energies for nuclei.
S. Schramm, *Phys.Rev. C66 (2002) 064310*



➡ **Parameters of quark sector still to be fixed.**

Fitting the quark sector to the lattice QCD data



Wuppertal-Budapest collab., 1112.4416, 1309.5258, 1507.04627
HotQCD collab., 1203.0784, 1407.6387, 1701.04325

Standard parameters of PNJL don't work because of the hadron degrees of freedom at lower T.

How to fit the lattice data?

We reproduce the trace anomaly ***I*** by fitting the parameters of the quark sector:

- quark couplings to meson fields: $g_{q\sigma}$, $g_{q\zeta}$

$$m_q^* = -g_{q\sigma}\sigma + \delta m_q + m_{0q},$$
$$m_s^* = -g_{s\zeta}\zeta + \delta m_s + m_{0q}$$

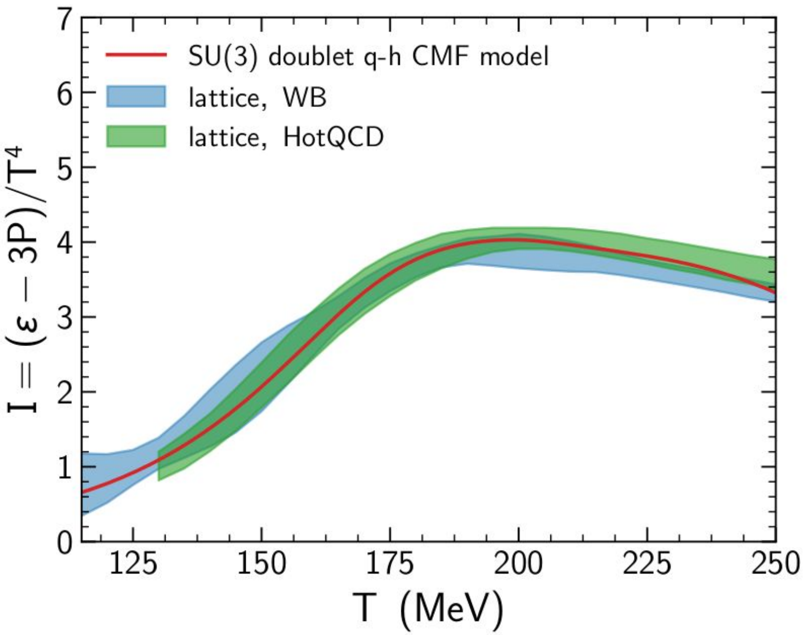
- parameters of the Polyakov loop potential *U(Φ)*: T_0, a_1, a_2, b_3

$$U = -\frac{1}{2}a(T)\Phi\Phi^* + b(T)\log[1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2],$$
$$a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2, \; b(T) = b_3 T_0^4$$

This controls quark thermodynamics:

$$P_q = \frac{1}{3}\frac{d_i}{(2\pi)^3}\int d^3k \frac{k^2}{E^*}\frac{1}{\frac{1}{\Phi}exp(\frac{E^* - mu^*}{T}) + 1}$$

Fitting the quark sector to the lattice QCD data



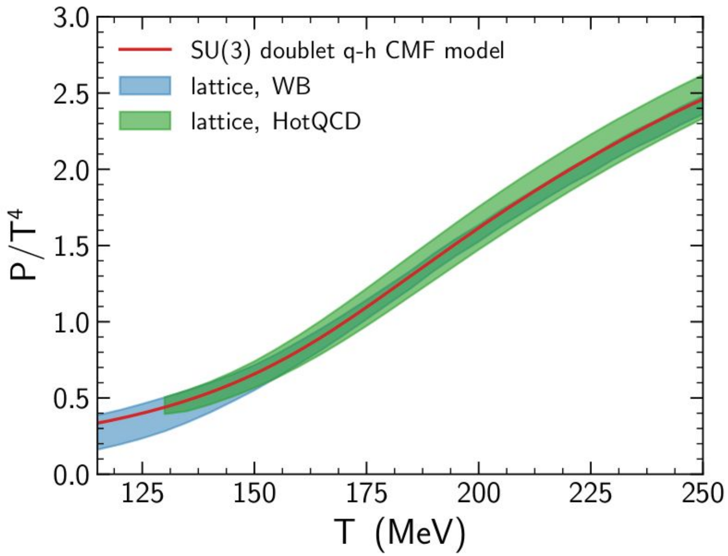
Wuppertal-Budapest collab., 1112.4416, 1309.5258, 1507.04627
HotQCD collab., 1203.0784, 1407.6387, 1701.04325

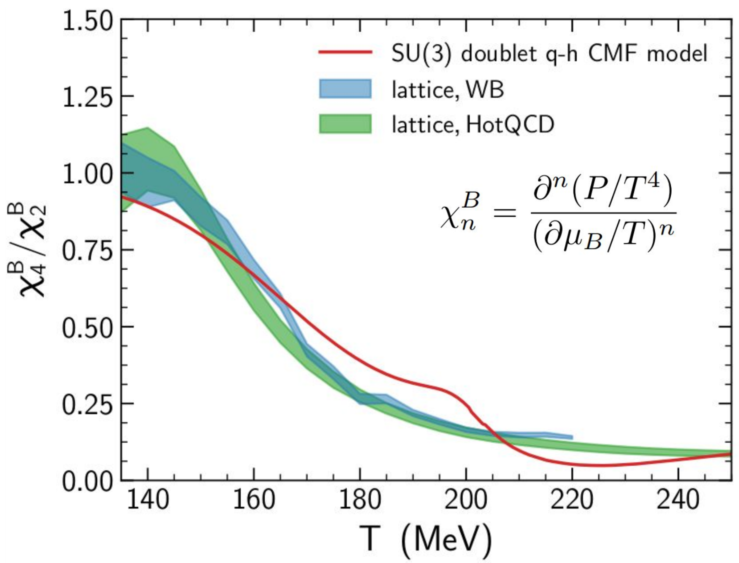
Our result:

T_0 (MeV)	a_1	a_2	b_3	$g_{q\sigma} = g_{s\zeta}$
180.0	-11.67	9.33	-0.53	-1.0

- T_0 is smaller than for pure gauge (270 MeV), approximately correspond to the location of the maximum of ***I***;
- Quark couplings to chiral field are 3 times smaller than for baryons;

Properties at μ_B=0





Description of pressure is good, general feature of PNJL models (however we have **very modified PNJL by hadrons**);

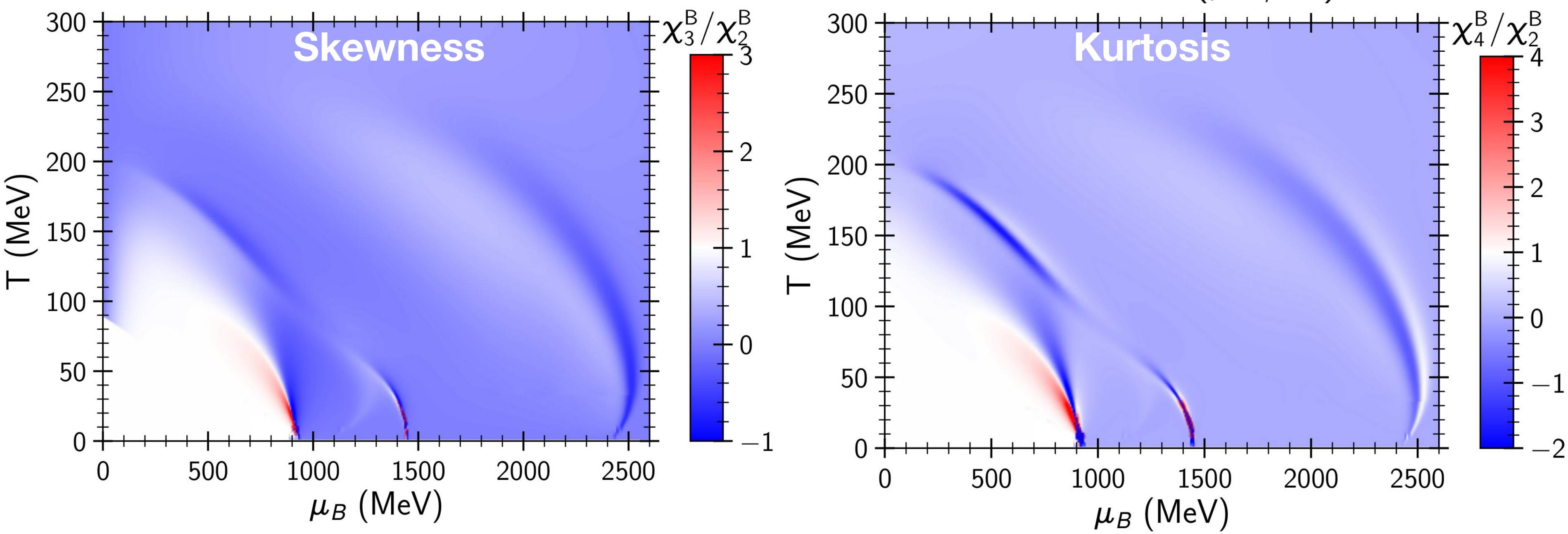
Kurtosis is similar to the lattice data, except the bump at 200 MeV — remnant of the chiral transition;

Still a problem to solve — **how take into account the contribution of PDG hadrons to chiral field?**

Fluctuations in T-μ_B plane

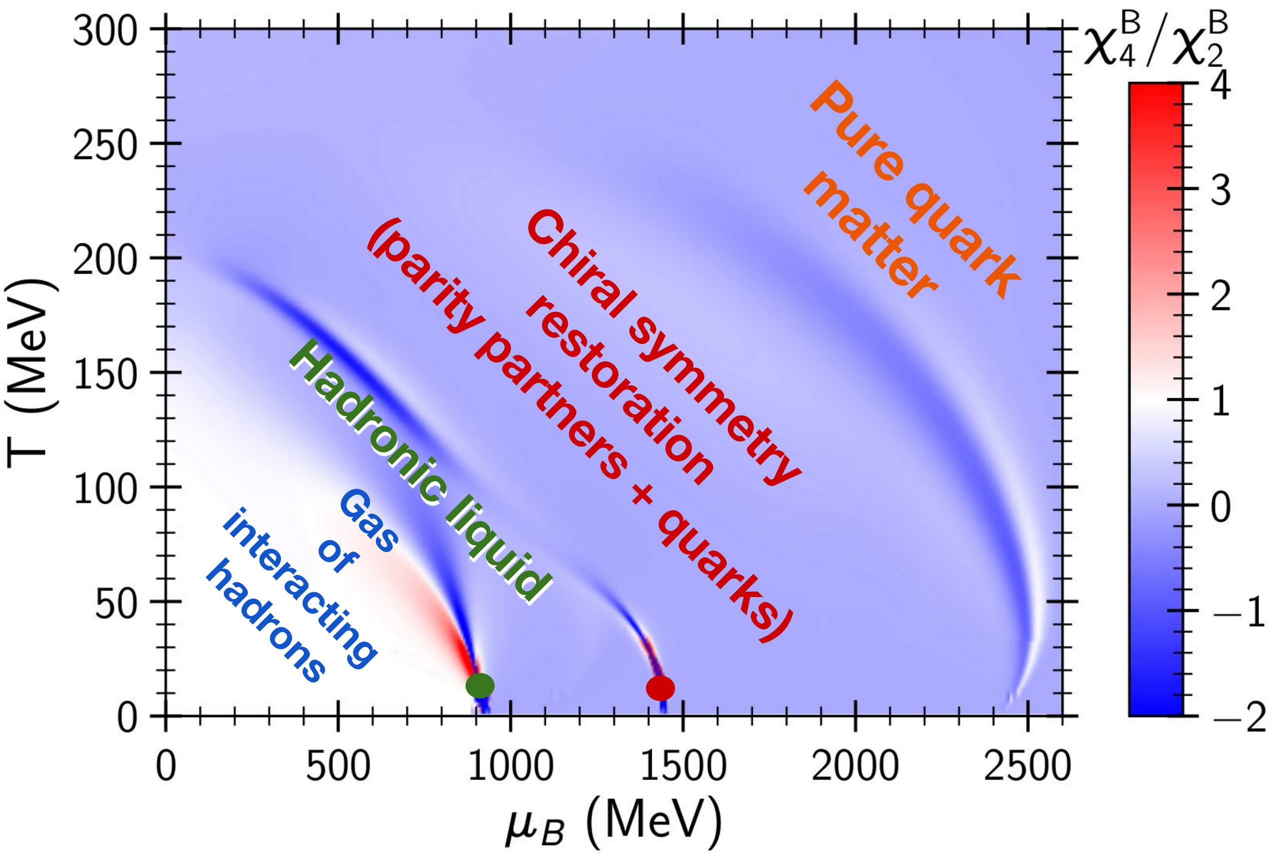
Skewness and Kurtosis — higher order measures of baryon number **fluctuations**. Allow to probe critical regions in phase diagram, non-monotonic behavior = criticality.

$$\textbf{Skewness}: \chi_3^B/\chi_2^B, \quad \textbf{Kurtosis}: \chi_4^B/\chi_2^B, \quad \chi_n^B = \frac{\partial^n p/T^4}{\partial(\mu_B/T)^n}$$



- Skewness and kurtosis suggest a separation in four different phases.
- Transition from HRG to dense liquid is reflected even at μ_B=0.
- Signals in crossover at μ_B=0 are remnants from nuclear liquid-vapor transition
- Chiral symmetry restoration and pure quark phase are at very high μ_B and/or T.

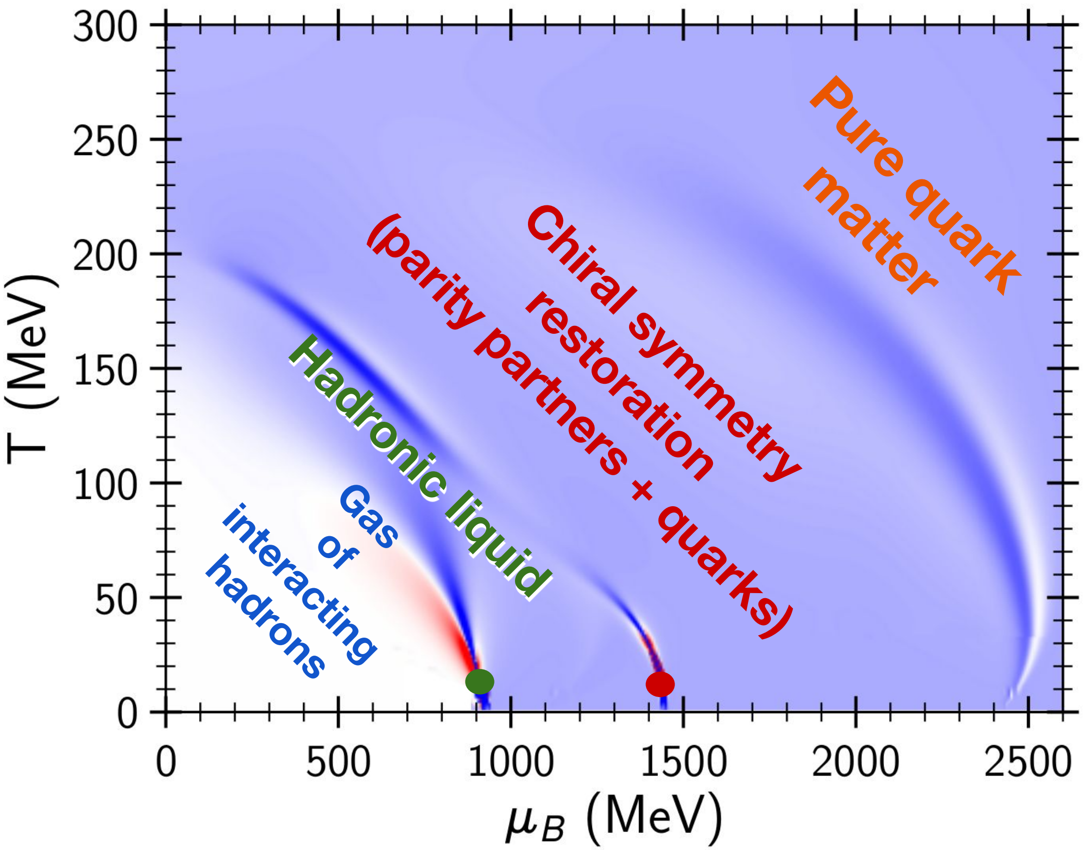
Phase diagram



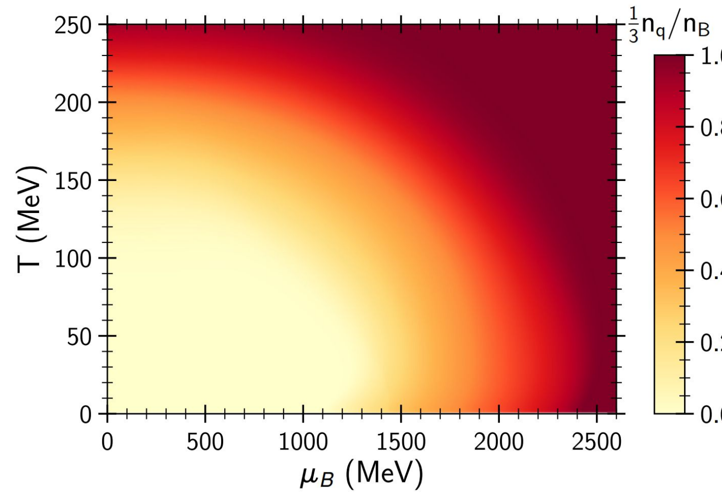
Two critical points:
Nuclear liquid-vapor phase transition;
Chiral symmetry restoration;
both at low T ≲ 20 MeV.

Transition to pure quark matter:
⅓ n_q/n_b=1, is always of second order.

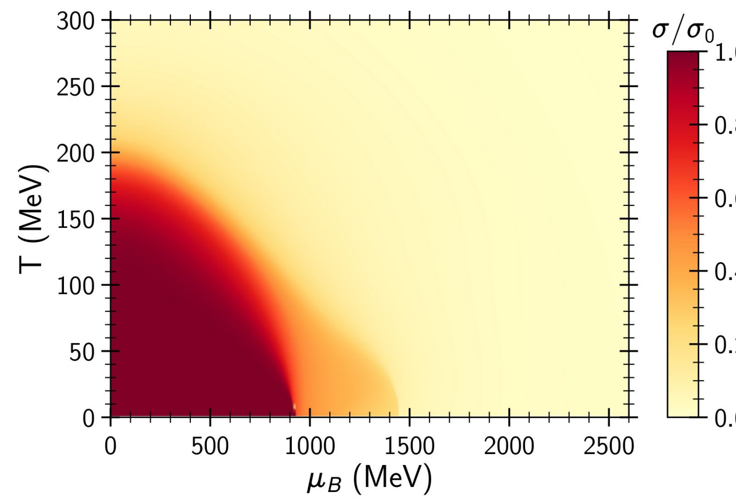
Phase diagram



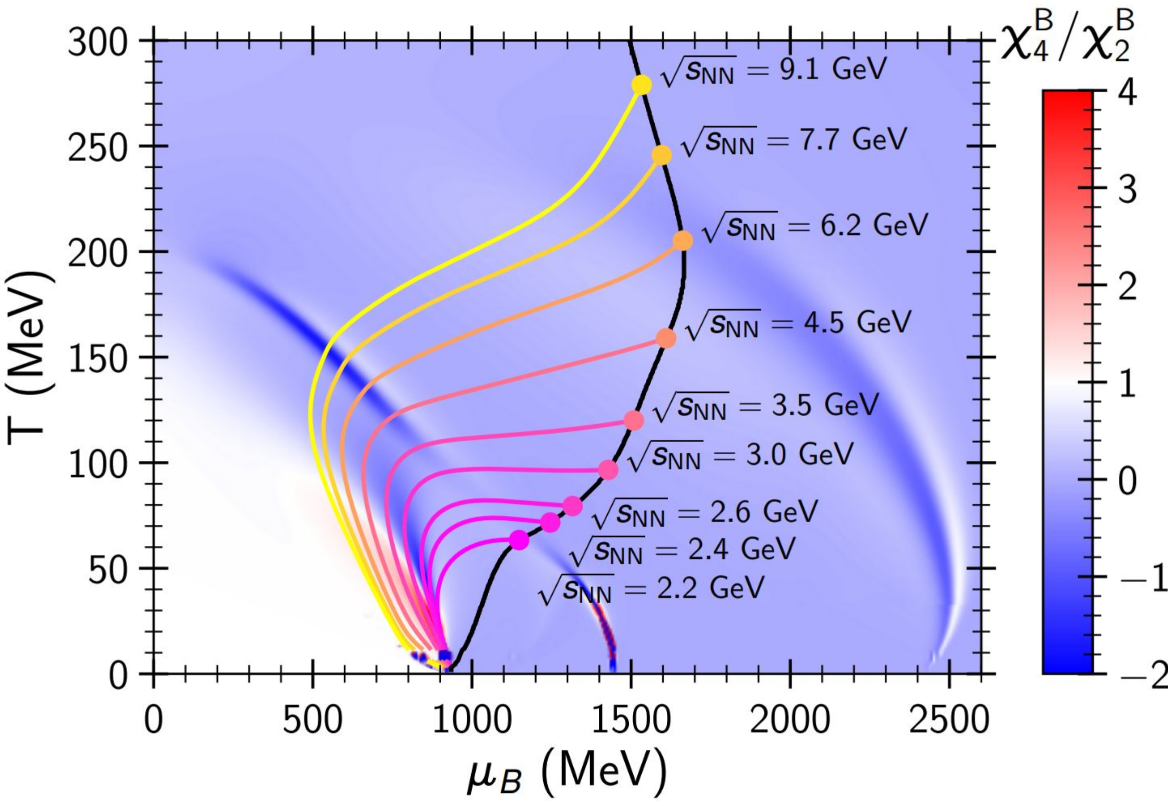
Quark fraction:



Chiral condensate:



Probing phase diagram by heavy ions collisions



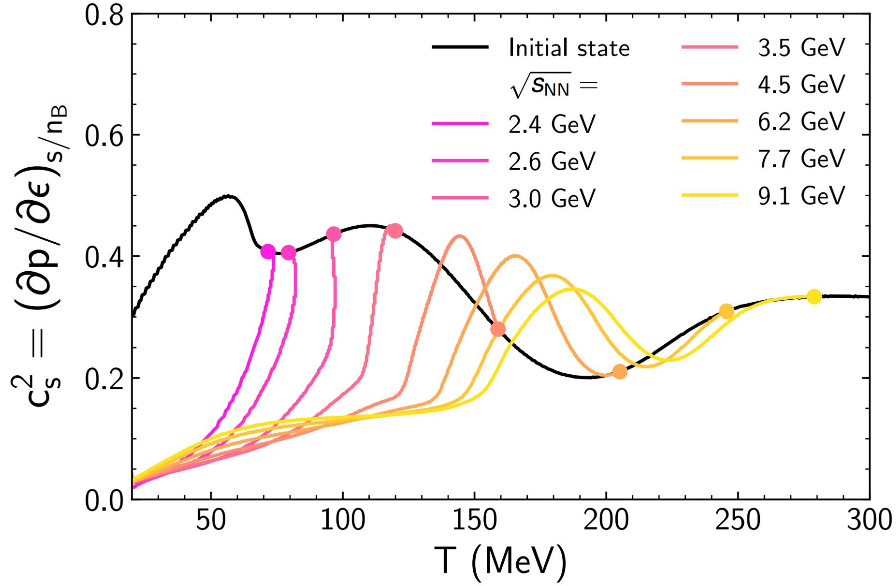
Isentropes (**S/A** = const) illustrate hydrodynamical evolution of the **central region in heavy ion collision**.

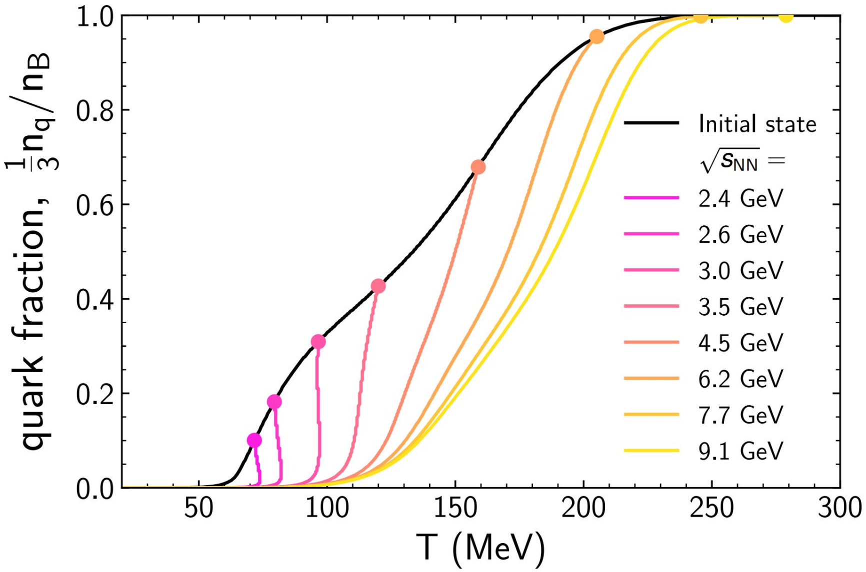
To estimate initial entropy per baryon S/A, **Taub adiabat** (shock wave solution) was used: (*K. S. Thorne, Astrophysical Journal 179, pp. 897-908 (1973)*)

$$\begin{cases} \frac{(P_0+\varepsilon_0)\,(P+\varepsilon_0)\,n^2}{(P_0+\varepsilon)\,(P+\varepsilon)n_0^2} = 1, \\ \gamma = \frac{\varepsilon n_0}{\varepsilon_0 n} \\ \gamma = \sqrt{\frac{1}{2}\left(1+\frac{E_{\rm lab}}{m_N}\right)} \end{cases}$$

P_0, ε_0, and n_0 correspond to the initial pressure, energy density, and baryon density in the local rest frame of each slab

Probing phase diagram by heavy ions collisions





speed of sound c_s^2 (**left**) and **quark fraction** (**right**) along the **isentropes** as functions of temperature T.

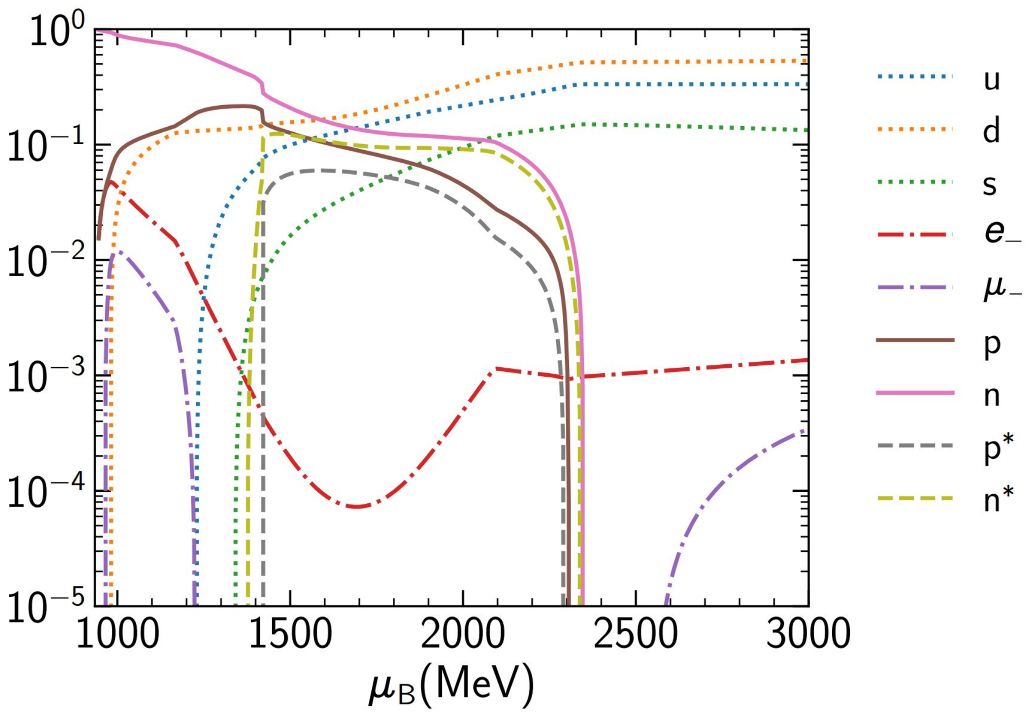
Colored lines = different collision energies (initial S/A), black solid line correspond to the initial state speed of sound and quark fraction respectively.

Scenario for higher energy $\sqrt{s_{NN}} > 7$ GeV:

1. start at the quark phase
2. softest point of deconfinement
3. baryons rapidly appear providing repulsion and increase of c_s^2
4. transition to dilute hadronic phase and lowering of c_s^2

Equation of state at T=0: neutron stars

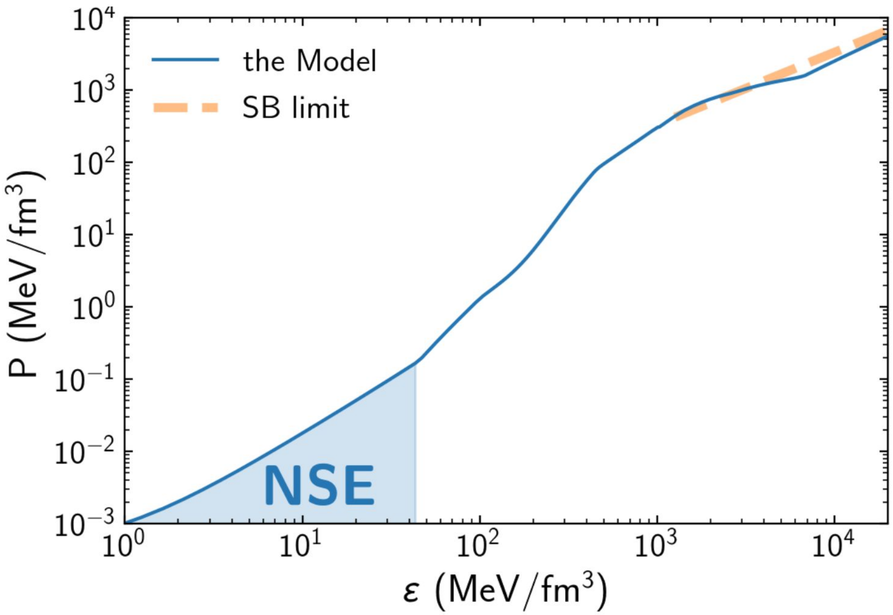
Particle densities normalized to baryon density:



Equation of state for **neutron stars**:

- **T=0**
- Electric charge is **zero**
- **Leptons** are included
- No nuclear ground state
- Small effects from chiral phase transition
- At T=0: polyakov loop **Φ = 1**
- **Hyperons** at T=0 are suppressed by hard-core repulsion (nevertheless are present at T ≠ 0)
- **Strange quarks** are included

Application to neutron stars



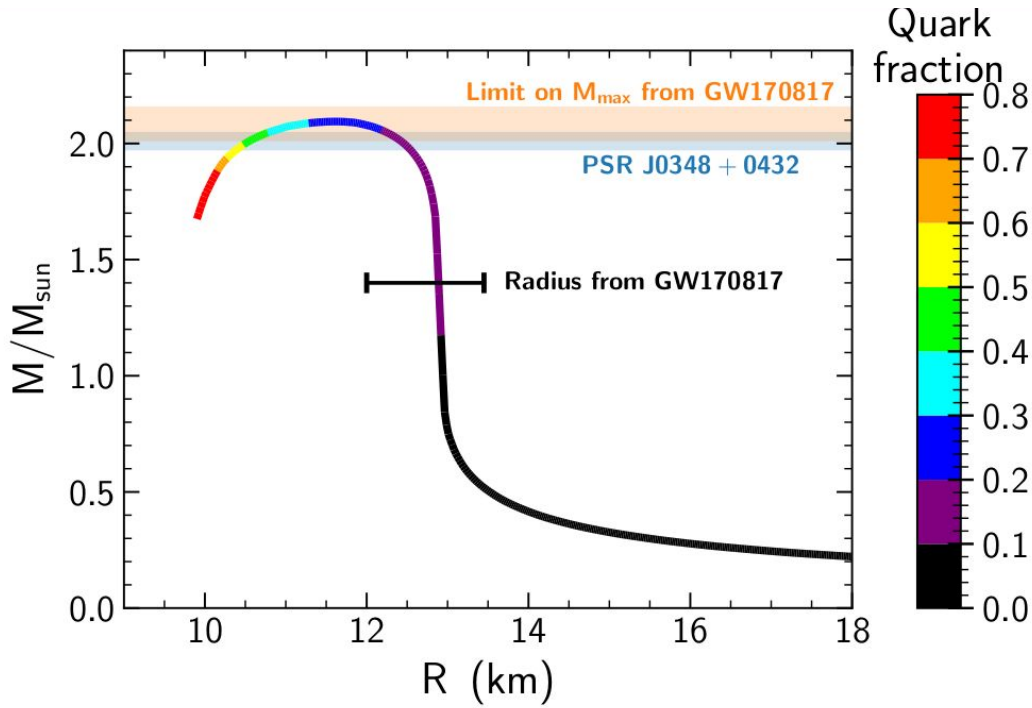
The model can be easily employed for the description of **neutron star matter** at **T=0 in beta-equilibrium** without any changes to the parameters. The EoS then can be used as an input to model neutron stars by solving Tolman–Oppenheimer–Volkoff equation:

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

Additional input is needed to model star's crust — Nuclear Statistical Equilibrium (*Baym, Pethick, Sutherland,1971, Astrophys. J. , 170,299.*)

- The model approaches **Stefan-Boltzmann** limit at high energy densities;
- Chiral phase transition is negligible

Mass-radius relations



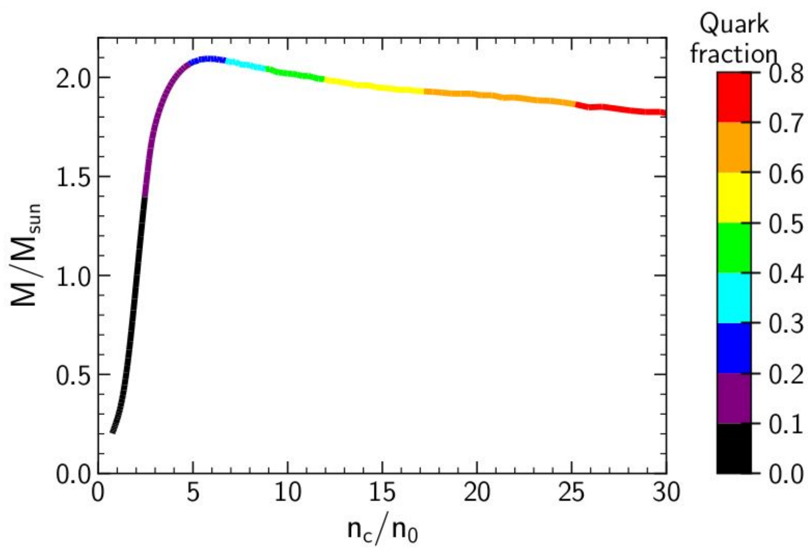
- EoS is stiff enough to provide $2M_{sun}$ neutron stars
- No phase transition — no second family
- quark fraction < 30% for stable stars
- Agreement with observations

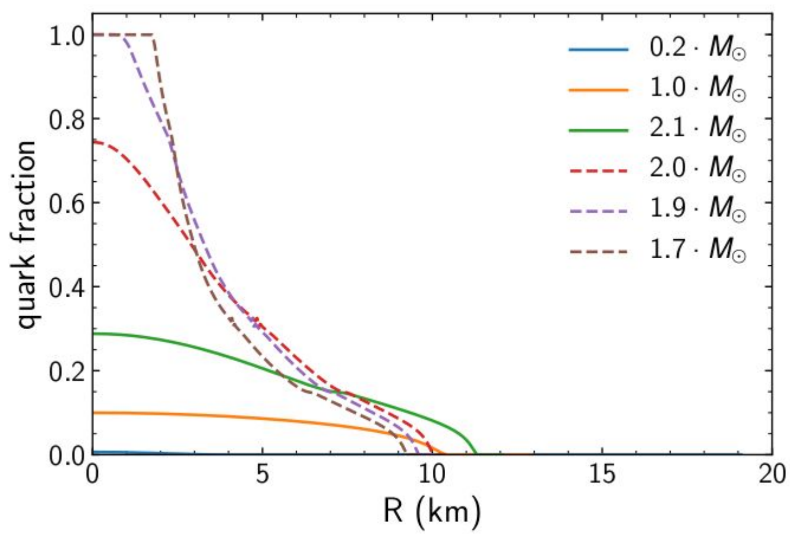
Maximal mass is in agreement with recent constraints from GW170817:

$$2.01^{+0.04}_{-0.04} \leq M_{\rm TOV}/M_{\odot} \lesssim 2.16^{+0.17}_{-0.15}$$

- *(Rezzolla, Most, Weih, Astrophys.J. 852 (2018) no.2, L25)*

Content of the star



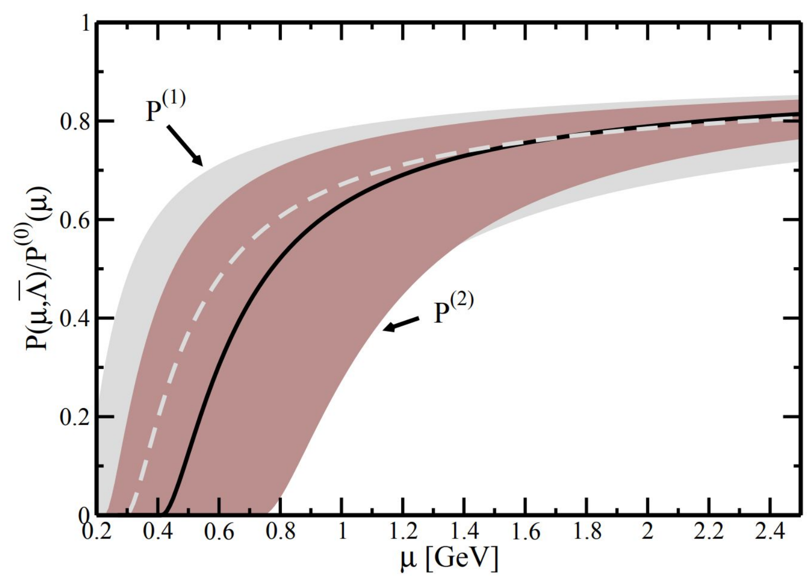


Local quark fraction (dashed — unstable stars):

Quarks appear smoothly — no separation between phases.

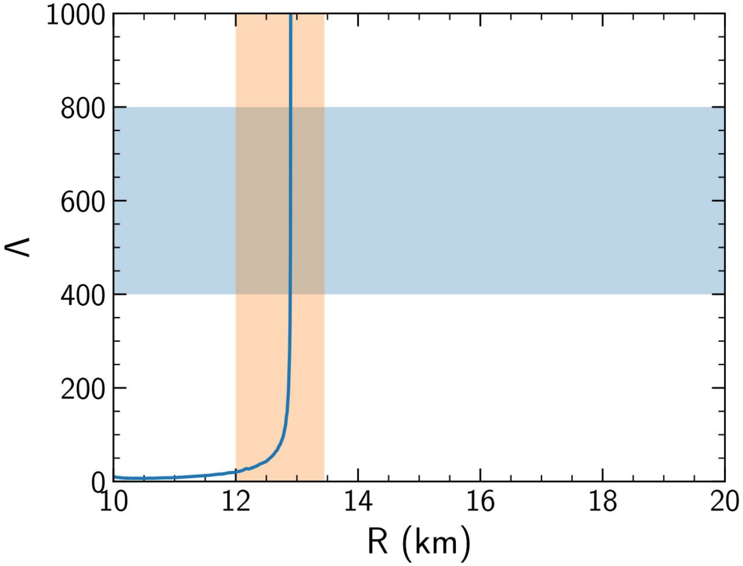
Strange quark fraction is <13%, produced by weak decays.

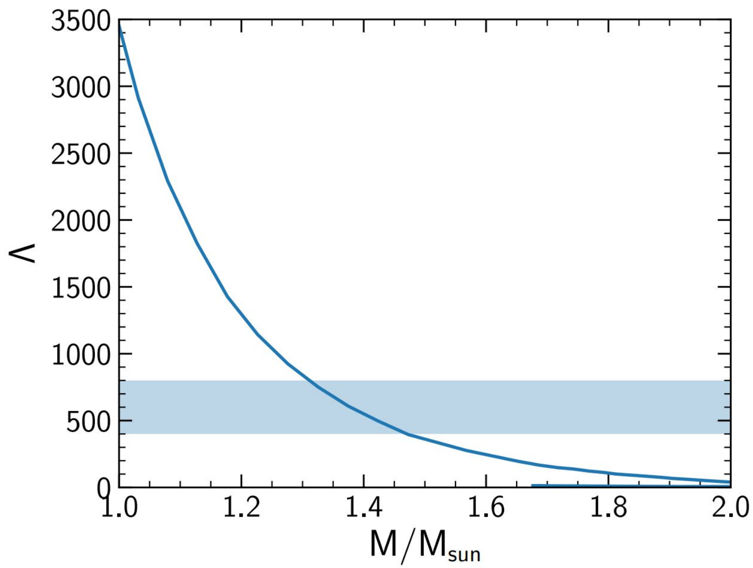
Quarks give significant contribution to stars with central density $n_c > 6n_0$, where only pQCD calculations are available:



A. Kurkela, P. Romatschke, A. Vuorinen, 0912.1856

Neutron star tidal deformabilities





Tidal deformability — measures stars' induced quadruple moment Q_{ij} as a response to the external tidal field $\mathcal{E}_{ij}$:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

important EoS-dependent quantity for inspiral phase of binary neutron star system. Related to second Love number k_2:

$$\lambda = \frac{2}{3} k_2 R^5$$

One presents the dimensionless tidal deformability Λ (mostly dependent on compactness M/R):

$$\Lambda = \frac{\lambda}{M^5} = \frac{2}{3} k_2 \left(\frac{R}{M}\right)^5$$

Bands — recent constraints for radius and tidal deformability of $1.4M_{sun}$ star.
Most, Weih, Rezzolla, Schaffner-Bielich., 1803.00549
Line — results on Λ using EoS obtained from the model.

Summary

- **Chiral SU(3) parity-doublet quark-hadron mean-field model** — is a unified phenomenological approach to model QCD thermodynamics at wide range of scales;
- μ_B=0 lattice QCD data is used to constrain parameters of model's quark sector;
- Nuclear liquid-vapor phase transition gives strong signals in fluctuations even at μ_B=0;
- Chiral symmetry restoration and transition to quark-dominated phase are at very high μ_B and/or T;
- Model produces neutron stars in agreement with today's constraints;
- Model's EoS can be used as input for both finite T and T=0 neutron star physics
- … as well as for hydro simulations of heavy ions collisions.

Thanks for your attention!