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Investigation of soft processes within the QCD color dipole picture

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Motivations

- Goal is to investigate if the color dipole (CD) model and QCD saturation phenomenon can be useful to describe soft processes (at least in the transition region).
- Starting point is the description of the DIS by QCD approaches.
- In the Regge limit (high energy, $Q^2 = \text{cte}$, $W^2 \longrightarrow \infty$) one has strong rise of F_2 on the x-Bjorken variable.
- Power-like dependence: $F_2 \sim x^{-4\bar{\alpha}_S\, ln(2)}$ - unitarity violation for $x \longrightarrow 0$.
- Therefore, unitarization is needed: nonlinear evolution equations of QCD – gluons recombine.
- Significant progress in recent years – high density QCD.

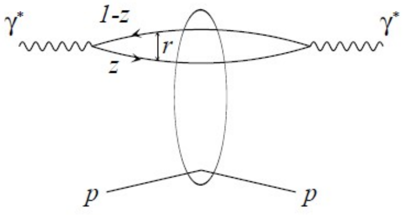
+ Guided by the idea of parton saturation:

- Gluons form high density system in which they easily recombine.
- Nonlinear evolution equations: GLR, BK, JIMWLK,...
- Unitarity restored (Froissart bound: $\sigma_{tot} < C \ln^2 s$, $|T(b)| < 1$).

+ Two basic features:

- Saturation scale $Q_s(x)$ - intrinsic scale of a dense gluonic system.
- Geometric scaling - quantities like $\gamma^* p$ cross section scale:

$$\sigma^{\gamma^* p}(x, Q^2) \sim \frac{F_2}{Q^2} = \sigma^{\gamma^* p}\left(\tau = \frac{Q^2}{Q_s^2(x)}\right).$$



r - transverse size

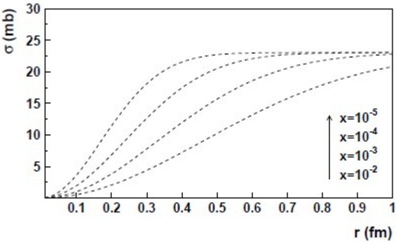
z - photon momentum fraction

- Color dipole formation + color dipole interaction.
- Dipole cross section $\sigma_{dip} \longrightarrow$ unitarized interaction.

$$\sigma^{\gamma^* p}(x,Q^2) = \int d^2r \int_0^1 dz \left|\Psi^{\gamma}_{T+L}(z,r,Q^2)\right|^2 \sigma_{dip}(x,r),$$
$$= 2\int d^2b \left[\int d^2r \int_0^1 dz \left|\Psi^{\gamma}_{T+L}(z,r,Q^2)\right|^2 N(x,r;b)\right].$$

- Unitarity condition: N never exceeds a bound when $x \to 0$.

Example: the celebrated GBW model



- $\hat{\sigma}(r,x) = \sigma_0 \, \{1 - \mathrm{e}^{-r^2 Q_s^2(x)/4}\}$
- Saturation scale: $Q_s^2 = Q_0^2 \; x^{-\lambda}$
- Geometric scaling: $\hat{\sigma}(rQ_s(x))$

- Unitarity condition: $\hat{\sigma}$ never exceeds σ_0 when $x \to 0$

Photon wavefunction and dipole x-section

- Expression for the transverse photon wavefunction:

$$|\psi^{\gamma}_{T,q}(z,r,Q^2)|^2 = e_q^2 N_c \frac{\alpha_{em}}{2\pi^2}\left[(z^2+(1-z)^2)\epsilon^2 K_1^2(\epsilon r) + m_q^2 K_0^2(\epsilon r)\right],$$
$$\epsilon^2 = z(1-z)Q^2 + m_q^2.$$

- Here, e_q is the quark charge, N_c the number of colors and $K_{0,1}$ are the modified Bessel functions. Light quark mass, $m_q \simeq 0.14$ GeV.
- Dipole cross section, $\sigma_{dip}(x,r)$, or dipole amplitude, $N(x,r,b)$, are extracted from the deep inelastic structure function $F_2^{\gamma^* p}(x,Q^2)$.
- Several phenomenological models in literature, including or not impact-parameter dependence.

- C.A. Arguelles *et. al.*, Phys. Rev. D 92, 074040 (2015).
- Stating point is CD model for heavy quark production in *pp* collisions (good description of accelerator data):

$$\sigma_{pp\to q\bar{q}+X} = 2\int_0^{-\ln(2m_q/\sqrt{s})} dy\, x_1 g(x_1,\mu)\,\sigma_{gN\to q\bar{q}+X}(x_2;Q^2).$$

- Here, $g(x_1,\mu)$ is gluon PDF at the scale μ, and $x_{1,2}$ are given by:

$$x_1 \simeq \frac{2m_q}{\sqrt{s}}\exp(+y)$$
$$x_2 \simeq \frac{2m_q}{\sqrt{s}}\exp(-y);$$

- Quantity $\sigma_{gN\to q\bar{q}+X}$ is the partonic cross section:

$$\sigma_{gN\to q\bar{q}+X}(x_2;Q^2) = \int dz\, d^2\mathbf{r}\, |\psi^g_{T,q}(z,\mathbf{r};Q^2)|^2\sigma_{gq\bar{q}}(x_2,z,\mathbf{r})$$

CD asymptotic cross section

- Partonic cross section is directly related to the dipole cross section (here, $\bar{z} = (1 - z)$):

$$\hat{\sigma}_{gq\bar{q}}(x_2, z, \mathbf{r}) \quad = \quad \frac{9}{8}\left[\sigma_{dip}(x_2, z\mathbf{r}) + \sigma_{dip}(x_2, \bar{z}\mathbf{r})\right] - \frac{1}{8}\sigma_{dip}(x_2, \mathbf{r}).$$

- Gluon wave function is related to the photon wave function in following way:

$$|\psi^{g}_{T,q}(z, \mathbf{r}; Q^2)|^2 = \frac{\alpha_s}{N_c \alpha_{em}} |\psi^{\gamma}_{T,q}(z, \mathbf{r}; Q^2)|^2.$$

- Asymptotic cross section is obtained with prescription: $\hat{\sigma}_{gq\bar{q}} \rightarrow \hat{\sigma}_{ggg}$ and $|\psi^{g}_{T,q}(z, \mathbf{r}; Q^2)|^2 \rightarrow |\psi^{g}_{T,gg}(z, \mathbf{r}; Q^2)|^2$.
- Namely, $|\psi^{g}_{T,gg}|^2 = 2(N_c - 1)|\psi^{g}_{T,q}|^2$.
- In addition, replace $m_q \rightarrow m_g$, an effective gluon mass, in evaluating x_1 and x_2.
- Problem: to evaluate gluon PDF at very small scale, $\mu^2 = (m_g)^2$.

- Asymptotic expression is given by:

$$\sigma_{tot}(pp) \approx 2\int_0^{-\ln(2m_g/\sqrt{s})} dy\, x_1 g(x_1,\mu)\, \sigma_{gN\to gg+X}(x_2;Q^2).$$

$$\sigma_{gN\to gg+X}(x_2;Q^2) = \int dz\, d^2\mathbf{r}\, |\psi^g_{T,gg}(z,\mathbf{r};Q^2)|^2 \sigma_{ggg}(x_2,z,\mathbf{r}),$$

$$\sigma_{ggg} = \frac{1}{2}\left[\sigma_{dip}(x_2,z\mathbf{r}) + \sigma_{dip}(x_2,\bar{z}\mathbf{r}) + \sigma_{dip}(x_2,\mathbf{r})\right].$$

- In numerical calculations we use $m_g = 400$ MeV and light quarks. The πp cross section is also obtained by quark counting rules.
- The gluon PDF at very low Q^2 takes the form:

$$x\,G(x,Q^2) = \int_0^{Q^2} dl_t^2\, \mathcal{F}(x,l_t^2) = \frac{3\,\sigma_0 Q_{sat}^2}{4\pi^2\alpha_s}\left[1 - \left(1+\frac{Q^2}{Q_{sat}^2}\right) e^{-\frac{Q^2}{Q_{sat}^2}}\right].$$

- K. Golec-Biernat, M. Wusthoff, Phys. Rev. D60, 114023 (1999).

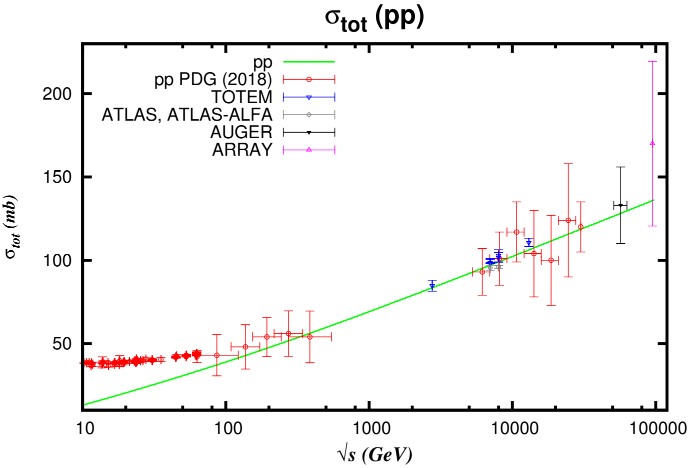


Figure: The *pp* total cross section including LHC data and cosmic rays measurements. Low energies require reggeon contribution.

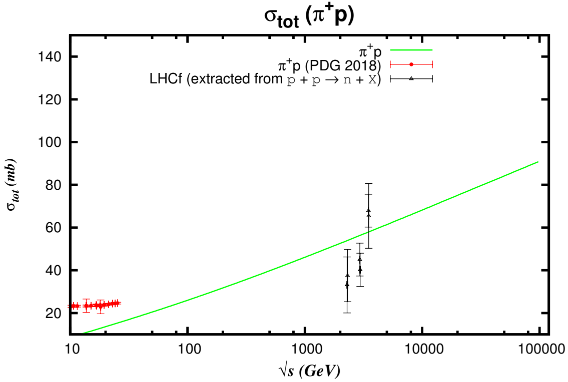


Figure: The $\pi^+ p$ total cross section extracted from LHCf measurements on leading neutrons spectra.

- R.A. Ryutin, Eur. Phys. J. C 77, 114 (2017); V.A. Khoze, A.D. Martin, M.G. Ryskin, Phys. Rev. D 96, 034018 (2017).

- J. Bartels, E. Gotsman, E. Levin, M. Lublinsky and U. Maor, Phys. Lett. B556, 114 (2003).
- The hadron-proton cross section is obtained based on the DIS cross section within the CD picture:

$$\sigma_{hp}(\sqrt{s}) = 2\int d^2b \int_0^1 dz \int d^2r\, |\Psi_h(r,z)|^2\; N(s,r,b).$$

- Ψ_h represents the wave function of the hadron which scatters off the target proton. N is the dipole-proton scattering amplitude.
- For meson-proton scattering, meson is treated as a $q\bar{q}$ pair: calculation follows that of DIS, i.e. the interaction of a CD with a proton target, with the replacement $\Psi_{\gamma^*}(r,z) \to \Psi_h(r,z)$.
- Saturation physics can be embedded in CD amplitude, $N(s,r,b)$.

- To characterize mesons and baryons, we use the phenomenological Gaussian Wirbel-Stech-Bauer (WSB) ansatz:

$$\psi_h(z_i,\vec{r}_i) = \sqrt{\frac{z_i(1-z_i)}{2\pi S_h^2 N_h}}\, e^{-(z_i-\frac{1}{2})^2/(4\Delta z_h^2)}\, e^{-|\vec{r}_i|^2/(4S_h^2)}.$$

- The hadron wave function normalization to unity, $\int dz_i d^2r_i\, |\psi_i(z_i,\vec{r}_i)|^2 = 1$, requires the normalization constant:

$$N_h = \int_0^1 dz_i\; z_i(1-z_i)\, e^{-(z_i-\frac{1}{2})^2/(2\Delta z_h^2)}.$$

- Mesons and baryons are assumed to have a $q\bar{q}$ and quark-diquark valence structure. As quark-diquark systems are equivalent to $q\bar{q}$ systems, this allows us to model not only mesons but also baryons as color-dipoles.

Using saturation models: impact parameter

- b-CGC model: G. Watt and H. Kowalski, Phys. Rev. D78, 014016 (2008).
- In the b-CGC, the color dipole-proton amplitude is given by,

$$N(x,r,b) = \begin{cases} N_0 \left(\frac{rQ_s}{2}\right)^{2\gamma_{eff}} & rQ_s \leq 2\,, \\ \\ 1 - \exp\left(-\mathcal{A}\ln^2\left(\mathcal{B}rQ_s\right)\right) & rQ_s > 2\,, \end{cases}$$

- The impact-parameter dependent effective anomalous dimension and the saturation scale Q_s are defined as:

$$\gamma_{eff} = \gamma_s + \frac{1}{\kappa\lambda Y}\ln\left(\frac{2}{rQ_s}\right), \qquad Q_s = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \exp\left\{-\frac{b^2}{4\gamma_s B_{CGC}}\right\} \text{GeV}.$$

- Here, $Y = \ln(1/x)$ and $\kappa = \chi''(\gamma_s)/\chi'(\gamma_s) = 9.9$, with χ being the LO BFKL characteristic function.

Parameters for numerical calculations

Table: Hadron wave function parameters.

Hadron	Δz_h	S_h [fm]
$p, \bar{p}$	0.3	0.86
$\pi^{\pm}$	2	0.607

Table: Parameters of b-CGC model (fit in the range $x \leq 0.01$ and $Q^2 \in [0.75, 650]\,\text{GeV}^2$, with $m_c = 1.4$ GeV, using high precision combined HERA data).

B_{CGC}/GeV^{-2}	γ_s	N_0	x_0	λ	χ^2/d.o.f.
5.5	0.6492	0.3658	0.00069	0.2023	1.249

- A. H. Rezaeian and I. Schmidt, Phys. Rev. D88, 074016 (2013).

Results using b-CGC

- We need rewrite the energy dependence from photon-hadron scattering in terms of the appropriate Bjorken scaling variable-x. The following ansatz has been considered:

$$\frac{1}{x} = \frac{sr^2}{(s_0\, R_c^2)}, \qquad \text{with } s_0 \sim m_h^2 \text{ and } R_c = 0.22 \text{ fm.}$$

- See, for instance: A. Donnachie and H.G. Dosch, Phys. Rev. D65, 014019 (2002).
- We consider Tevatron and LHC data for pp cross sections and indirect determination of $\pi^+ p$ (total) cross section.
- Ansatz is numerically equivalent to the proposal $\frac{1}{x} = \frac{s}{Q_0^2}$, with $Q_0^2 \sim (2m_q)^2$, in J. Bartels et al., Phys. Lett. B556, 114 (2003).

Proton-proton cross section: σ_{tot} and σ_{el}

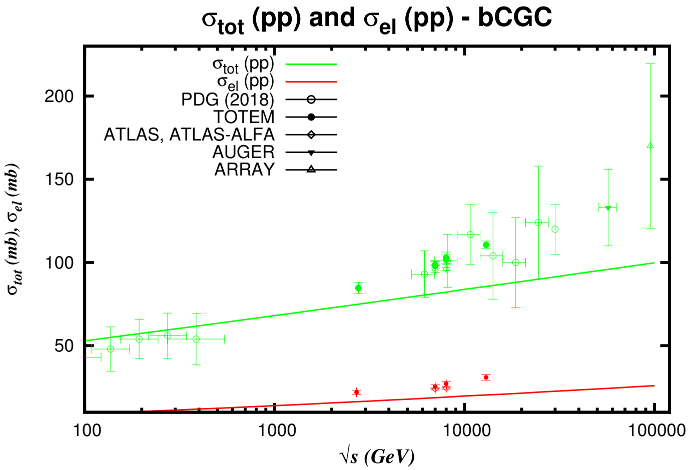


Figure: The *pp* total and elastic cross sections including LHC data and cosmic rays measurements.

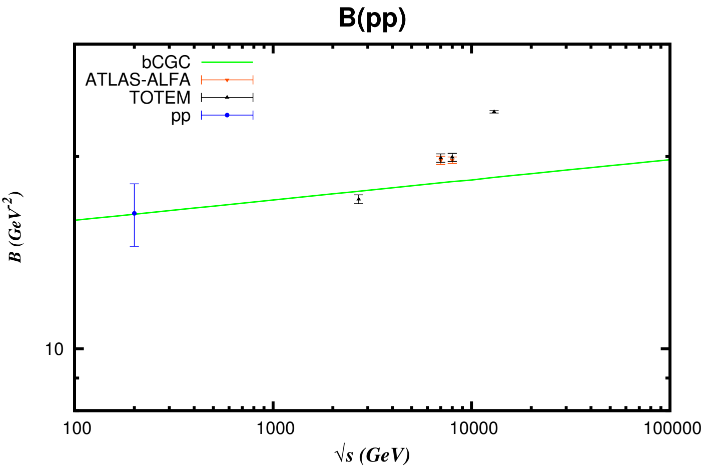


Figure: The slope parameter B_{el} in elastic proton-proton scattering.

Results for b-CGC model

Pion-proton cross section: $\sigma_{tot}(\pi^+ p)$

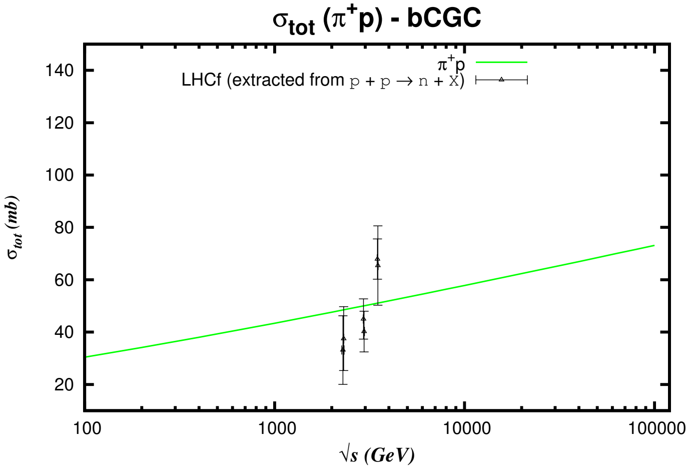


Figure: The total pion-proton cross section.

Results for a saturation model

- We have tried an eikonal-like expression for the dipole amplitude: different impact parameter dependence (dipole profile function).

$$
\begin{aligned}
N(x,r,b) &= 1 - \exp\left(-\frac{1}{2}\hat{\sigma}(x,r)S(b)\right), \\
\hat{\sigma}(x,r) &= \sigma_0 \frac{r^2 Q_s^2(x)}{4}, \quad S(b) = \frac{1}{\pi R^2}\frac{\sqrt{8}b}{R}K_1(\frac{\sqrt{8}b}{R}).
\end{aligned}
$$

- Parameters for (GBW) $\hat{\sigma}$ from K. Golec-Biernat and S.Sapeta, JHEP **1803**, 102 (2018).
- We have used $R^2 = 4.5\ \mathrm{GeV}^{-2}$.

Proton-proton cross section: σ_{tot}, σ_{el}

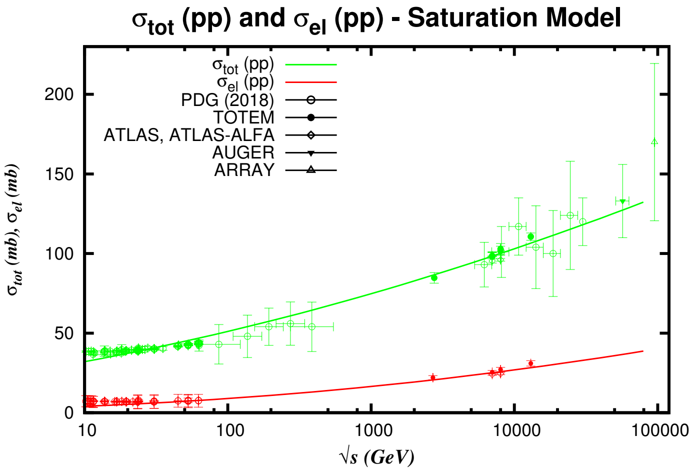


Figure: The total and elastic proton-proton cross section.

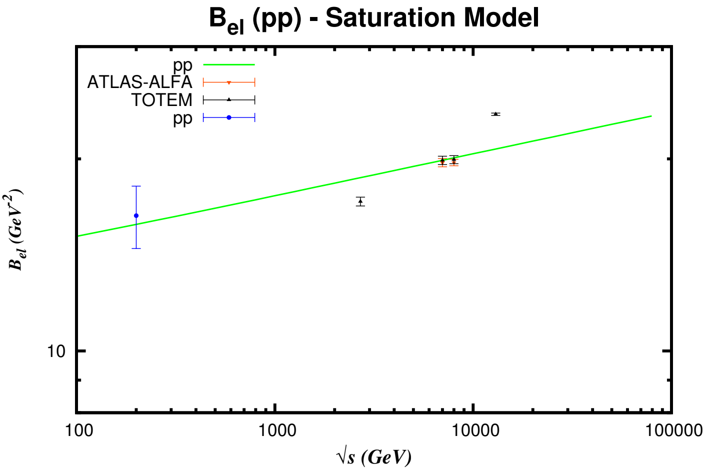


Figure: The elastic slope parameter B_{el} in pp scattering.

Conclusions and Comments

- In principle, CD approach can be used to describe some hadron soft observables, like σ_{tot}, σ_{el} and B_{el}.
- Main advantage: use dipole scattering amplitude, $N(x, r, b)$, found from inclusive/exclusive DIS.
- We have analysed two approaches: CD asymptotic model for *pp* collision and proposal by J. Bartels et al., PLB556 (2003).
- We considered saturation models: crucial role in parton saturation is played by the saturation scale which reduce the dependence of results on the soft region.
- Data description is reasonable. Fitting procedure is still needed (working in progress).
- Future step: investigate other impact parameter dependent dipole amplitudes (like IPSAT model, including DGLAP evolution).