Scattering in high energy QCD:
particle production from high $p_t$ to low $p_t$ and back

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OUTLINE

QCD at high transverse momentum:

- asymptotic freedom
- parton model
- hard scattering: collinear factorization (twist expansion)

QCD at high energy (CGC):

- high gluon density effects: multiple soft scatterings
- high energy effects: quantum corrections

Toward a unified formalism:

- hard + multiple soft scatterings
**High $p_t$ particle production:** $pp$ collisions

collinear factorization: separation of soft (long distance) and hard (short distance)

\[
\frac{d\sigma^{pp\rightarrow hX}}{d^2p_t\, dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots \cdots
\]

\[x \equiv \frac{p^+}{P^+}\]

**power corrections**
pQCD: the standard paradigm

$$E \frac{d\sigma}{d^3p} \sim f_1(x, p_t^2) \otimes f_2(x, p_t^2) \otimes \frac{d\sigma}{dt} \otimes D(z, p_t^2)$$

bulk of QCD phenomena happens at low $p_t$ (small $x$)

$x \sim \frac{p_t}{\sqrt{s}} e^{-y}$

smaller $x$

high $x$
Deep Inelastic Scattering

\[ x = \frac{p^+}{P^+} \]

\( x \) is the fraction of hadron energy carried by a parton.
A hadron/nucleus at high energy: *gluon saturation*

- High gluon density: multiple scattering via Wilson lines
- $p_t$ broadening
- Energy dependence: $x$-evolution via JIMWLK
- Suppression of spectra/away side peaks

\[
Q_s^2(x, b_t, A) \sim A^{1/3} \left( \frac{1}{x} \right)^{0.3}
\]

\[
Q_s^2(x = 3 \times 10^{-4}) \sim 1 \text{ GeV}^2
\]

For a proton target (quarks)

A framework for multi-particle production in QCD at small $x$/low $p_t$

*Initial conditions for hydro*
*Thermalization?*
*Long range rapidity correlations*
*Azimuthal angular correlations*
*Nuclear modification factor*

\[ x \leq 0.01 \]
unifying saturation with high $p_t$ (large $x$) physics?

kinematics of saturation: where is saturation applicable?

jet physics, high $p_t$ (polar and azimuthal) angular correlations
cold matter energy loss, spin physics, ultra-high energy neutrinos
Classical CGC: multiple scattering in eikonal approximation

\[ i \mathcal{M}_n = 2\pi \delta(p^+ - q^+) \bar{u}(q) \frac{\hbar}{\Lambda} \int d^2 x_t \ e^{-i(q_t - p_t) \cdot x_t} \]

\[ \left\{ (i g)^n (-i)^n (i)^n \int dx_1^+ dx_2^+ \cdots dx_n^+ \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \right\} u(p) \]

Sum over all scatterings \[ i \mathcal{M} = \sum_{n=1}^{\infty} i \mathcal{M}_n \]

\[ i \mathcal{M}(p, q) = 2\pi \delta(p^+ - q^+) \bar{u}(q) \frac{\hbar}{\Lambda} \int d^2 x_t \ e^{-i(q_t - p_t) \cdot x_t} [V(x_t) - 1] u(p) \]

With \[ V(x_t) \equiv \hat{P} \exp \left\{ i g \int_{-\infty}^{+\infty} dx^+ n^- S_a(x^+, x_t) t_a \right\} \]

\[ \frac{d\sigma}{d^2 p_t dy} \sim |i \mathcal{M}|^2 \sim F.T. \quad < Tr V(x_t) V^\dagger(y_t) > \]
hard scattering: large deflection
scattered quark travels in the new “z” direction: \( \vec{z} \) 
\[
\begin{pmatrix}
\vec{x} \\
y \\
\vec{z}
\end{pmatrix} = \mathcal{O} 
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

\[
i\mathcal{M}_1 = (i g) \int d^4x \ e^{i(\vec{q} - \vec{p})x} \ \bar{u}(\vec{q}) \ [\hat{A}(x)] \ u(p)
\]

\[
i\mathcal{M}_2 = (i g)^2 \int d^4x \ d^4x_1 \ \int \frac{d^4p_1}{(2\pi)^4} \ e^{i(p_1 - p)x_1} \ e^{i(\vec{q} - \vec{p}_1)x}
\]
\[
\bar{u}(\vec{q}) \ \left[ \hat{A}(x) \ \frac{i\vec{p}_1}{\vec{p}_1^2 + i\epsilon} \ \hat{\sigma} \ S(x_1) \right] \ u(p)
\]

\[
i\mathcal{M}_2 = (i g)^2 \int d^4x \ d^4\vec{x}_1 \ \int \frac{d^4\vec{p}_1}{(2\pi)^4} \ e^{i(\vec{p}_1 - \vec{p})x} \ e^{i(\vec{q} - \vec{p}_1)\vec{x}_1}
\]
\[
\bar{u}(\vec{q}) \ \left[ \hat{\sigma} \ S(\vec{x}_1) \ \frac{i\vec{p}_1}{\vec{p}_1^2 + i\epsilon} \ \hat{A}(x) \right] \ u(p)
\]

with \( \vec{\nu} = \mathcal{O} \ \vec{\nu} \)
summing all the soft re-scatterings gives:

\[
\begin{align*}
L_1 &= \int d^4x \, d^2z_t \, d^2\bar{z}_t \, \int \frac{d^2k_t}{(2\pi)^2} \, \frac{d^2\bar{k}_t}{(2\pi)^2} \, e^{i(\bar{k} - k) \cdot x} \, e^{-i(\bar{q}_t - \bar{k}_t) \cdot \bar{z}_t} \, e^{-i(k_t - p_t) \cdot z_t} \\
\bar{u}(q) \left[ \overline{V}_{AP}(x^+, \bar{z}_t) \, \frac{k}{2k^+} \, \left[ igA(x) \right] \, \frac{\bar{k}}{2\bar{k}^+} \, \delta V_{AP}(z_t, x^+) \right] \, u(p)
\end{align*}
\]

with

\[
\overline{V}_{AP}(x^+, \bar{z}_t) \equiv \hat{P} \exp \left\{ ig \int_{x^+}^{+\infty} d\bar{z}^+ \, \bar{S}^-_{a} (\bar{z}_t, \bar{z}^+) \, t_a \right\}
\]

\[
V_{AP}(z_t, x^+) \equiv \hat{P} \exp \left\{ ig \int_{-\infty}^{x^+} d\bar{z}^+ \, S^-_{a} (z_t, \bar{z}^+) \, t_a \right\}
\]

how about multiple scatterings of hard gluon?
interactions of large and small x gluons
multiple scatterings of hard gluon can be resummed

\[
i \mathcal{M}_2 = \frac{2i}{(p - \bar{q})^2} \int d^4x \, e^{i(q-p)x} \, \bar{u}(\bar{q}) \left[ (igt^a) \left[ \partial_x^+ U_{AP}^+(x_t, x^+) \right]^{ab} \right. \\
\left. n \cdot (p - \bar{q}) A_b(x) - (p - \bar{q}) \cdot A_b(x) \, \eta \right] u(p)
\]

with \( U_{AP}(x_t, x^+) \equiv \hat{P} \exp \left\{ ig \int^{x^+}_{-\infty} dz^+ \, S_a^{-}(z^+, x_t) T_a \right\} \)

but there is more!
how about the final state quark interactions?

these contributions resum to

\[ iM_3 = -2i \int d^4x \, d^2\bar{x}_t \, d\bar{x} + \frac{d^2\bar{p}_{1t}}{(2\pi)^2} \, e^{i(q^+-p^+)x_-} \, e^{-i(\bar{p}_{1t}-p_t)x_t} \, e^{-i(\bar{q}_t-\bar{p}_{1t})\cdot \bar{x}_t} \]

\[ \bar{u}(q) \left[ \left[ \partial_{\bar{x}^+} + \bar{V}_{AP}(\bar{x}^+, x_t) \right] \not{\bar{p}_1} \left(igt^a\right) \left[ \partial_x + U_{AP}^\dagger(x_t, x^+) \right] \right]^{ab} \]

\[ \frac{\left[ n \cdot (p-q) A^b(x) - (p-\bar{p}_1) \cdot A^b(x) \not{\gamma} \right]}{[2n \cdot \bar{q} \, 2n \cdot (p-\bar{q}) p^- - 2n \cdot (p-\bar{q}) \bar{p}_{1t}^2 - 2n \cdot \bar{q} (\bar{p}_{1t} - p_t)^2]} u(p) \]
full amplitude: \[ iM = iM_{\text{eik}} + iM_1 + iM_2 + iM_3 \]

soft (eikonal) limit:

\[ A^\mu(x) \rightarrow n^- S(x^+, x_t) \]
\[ n \cdot \bar{q} \rightarrow n \cdot p \]
\[ iM \rightarrow iM_{\text{eik}} \]
cross section: \[ |i\mathcal{M}|^2 = |i\mathcal{M}_{eik} + i\mathcal{M}_1 + i\mathcal{M}_2 + i\mathcal{M}_3|^2 \]

\[
|i\mathcal{M}_2|^2 = \frac{8g^2}{(p - \bar{q})^4} \int d^4x \, d^4y \, e^{i(\bar{q}^+ - p^+)(x^- - y^-)} \, e^{-i(\bar{q}_t - p_t) \cdot (x_t - y_t)} \left\{ p^+ q^- (p^+ - \bar{q}^+)^2 A^b_\perp(x) \cdot A^c_\perp(y) + 2 (p^+)^2 q_\perp \cdot A^b_\perp(x) \, q_\perp \cdot A^c_\perp(y) \right\} \\
\left[ \partial_{y^+} U_{AP}(y_t, y^+) \right]^{ca} \left[ \partial_{x^+} U_{AP}^\dagger(x_t, x^+) \right]^{ab}
\]

other terms are more complicated: *spinor helicity formalism* for Dirac Algebra

**DIS: structure functions, di-jet production**

**PA: single inclusive particle production**

**PP: spin asymmetries (A_{LL, ...})**
particle production in high energy collisions

pQCD and collinear factorization at high $p_t$

breaks down at low $p_t$ (small $x$)

CGC at low $p_t$

breaks down at large $x$ (high $p_t$)

**Toward a unified formalism:**

parton scattering from small and large $x$ fields

*gluon radiation, 1-loop corrections*

*particle production in pp, pA in both small and large $p_t$ regions*
both initial state quark and hard gluon interacting:

integration over $p_1^-$

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)} \left[ p_1^2 + i\epsilon \right]\left[ (p_1 - \bar{q})^2 + i\epsilon \right]}{\left[ p_1^2 - \bar{q} - \frac{(p_1t - \bar{q}_t)^2}{2(p_+ - \bar{q}^+)} \right] + \left[ \bar{q} - \frac{(p_1t - \bar{q}_t)^2}{2(p_+ - \bar{q}^+)} - \frac{p_1^2}{2p^+} \right]}$$

both poles are below the real axis, we get

$$e^{i\frac{p_1^2}{2p^+} (x_1^+ - x^+)} \left[ e^{i\bar{q} - \frac{(p_1t - \bar{q}_t)^2}{2(p_+ - \bar{q}^+)}} (x_1^+ - x^+) \right]$$

ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!
toward unifying small and large x
(multiple scattering)

scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$p^\mu = (p^+ \sim \sqrt{s}, p^- = 0, p_t = 0)$$

$$S = S(p^+ \sim 0, p^- / P^- \ll 1, p_t)$$

allow hard scattering by including one hard field $A_{a^\mu}(x^+, x^-, x_t)$ during which there is large momenta exchanged and quark can get deflected by a large angle.

include eikonal multiple scattering before and after (along a different direction) the hard scattering
eliminate/minimize medium effects \textit{(proton-nucleus)}

Eikonal approximation

\[ J^\mu_a \simeq \delta^{\mu -} \rho_a \]

\[ D_\mu J^\mu = D_- J^- = 0 \]

\[ \partial_- J^- = 0 \quad \text{(in } A^+ = 0 \text{ gauge)} \]

does not depend on \( x^- \)

solution to EOM:

\[ A_a^- (x^+, x_t) \equiv n^- S_a (x^+, x_t) \]

with

\[ n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0) \]

\[ n^2 = 2 n^+ n^- - n_\perp^2 = 0 \]

recall (eikonal limit):

\[ \bar{u}(q) \gamma^\mu u(p) \rightarrow \bar{u}(p) \gamma^\mu u(p) \sim p^\mu \]

\[ \bar{u}(q) A^- u(p) \rightarrow p \cdot A \sim p^+ A^- \]

multiple scattering of a quark from background color field

\[ A_a^- (x^+, x_t) \]
\[ iM_1 = (ig) \int d^4 x_1 \, e^{i(q-p)x_1} \, \bar{u}(q) \, [\not_h S(x_1)] \, u(p) \]

\[ = (ig)(2\pi)\delta(p^+ - q^+) \int d^2 x_{1t} \, dx_1^+ \, e^{i(q^- - p^-)x_1^+} \, e^{-i(q_t - p_t)x_{1t}} \, \bar{u}(q) \, [\not_h S(x_1^+, x_{1t})] \, u(p) \]

\[ iM_2 = (ig)^2 \int d^4 x_1 \, d^4 x_2 \, \int \frac{d^4 p_1}{(2\pi)^4} \, e^{i(p_1 - p)x_1} \, e^{i(q - p_1)x_2} \, \bar{u}(q) \, \left[ \frac{i\not p_1}{p_1^2 + i\epsilon} \not_h S(x_1) \right] \, u(p) \]

\[ \int \frac{dp_1^-}{(2\pi)^2} \frac{e^{ip_1(x_1^+ - x_2^+)}}{2p^+ \left[p_1^- - \frac{p_1^- - i\epsilon}{2p^+}\right]} = -\frac{i}{2p^+} \theta(x_2^+ - x_1^+) \, e^{i\frac{p_1^2}{2p^+}(x_1^+ - x_2^+)} \]

Contour integration over the pole leads to path ordering of scattering

Ignore all terms: \( O\left(\frac{p_t}{p^+}, \frac{q_t}{q^+}\right) \) and use \( \not_h \frac{p_1}{2n \cdot p} \not_h = \not_h \)

\[ iM_2 = (ig)^2 (-i)(i) \, 2\pi \delta(p^+ - q^+) \int dx_1^+ \, dx_2^+ \, \theta(x_2^+ - x_1^+) \int d^2 x_{1t} \, e^{-i(q_t - p_t) \cdot x_{1t}} \, \bar{u}(q) \, [S(x_2^+, x_{1t}) \not_h S(x_1^+, x_{1t})] \, u(p) \]
Pion production at RHIC: **kinematics**
collinear factorization

CGC

GSV, PLB603 (2004) 173-183

DHJ, NPA765 (2006) 57-70

\[ p_T = 1.5 \text{ GeV} \]
\[ \eta = 3.2 \]

\[ \int_{x_{min}}^{1} dx \, xG(x, Q^2) \rightarrow x_{min}G(x_{min}, Q^2) \]

this is an extreme approximation with potentially severe consequences!