Scattering in high energy QCD:

particle production from high p_t to low p_t and back

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OUTLINE

QCD at high transverse momentum:

asymptotic freedom
parton model
hard scattering: collinear factorization (twist expansion)

QCD at high energy (CGC):

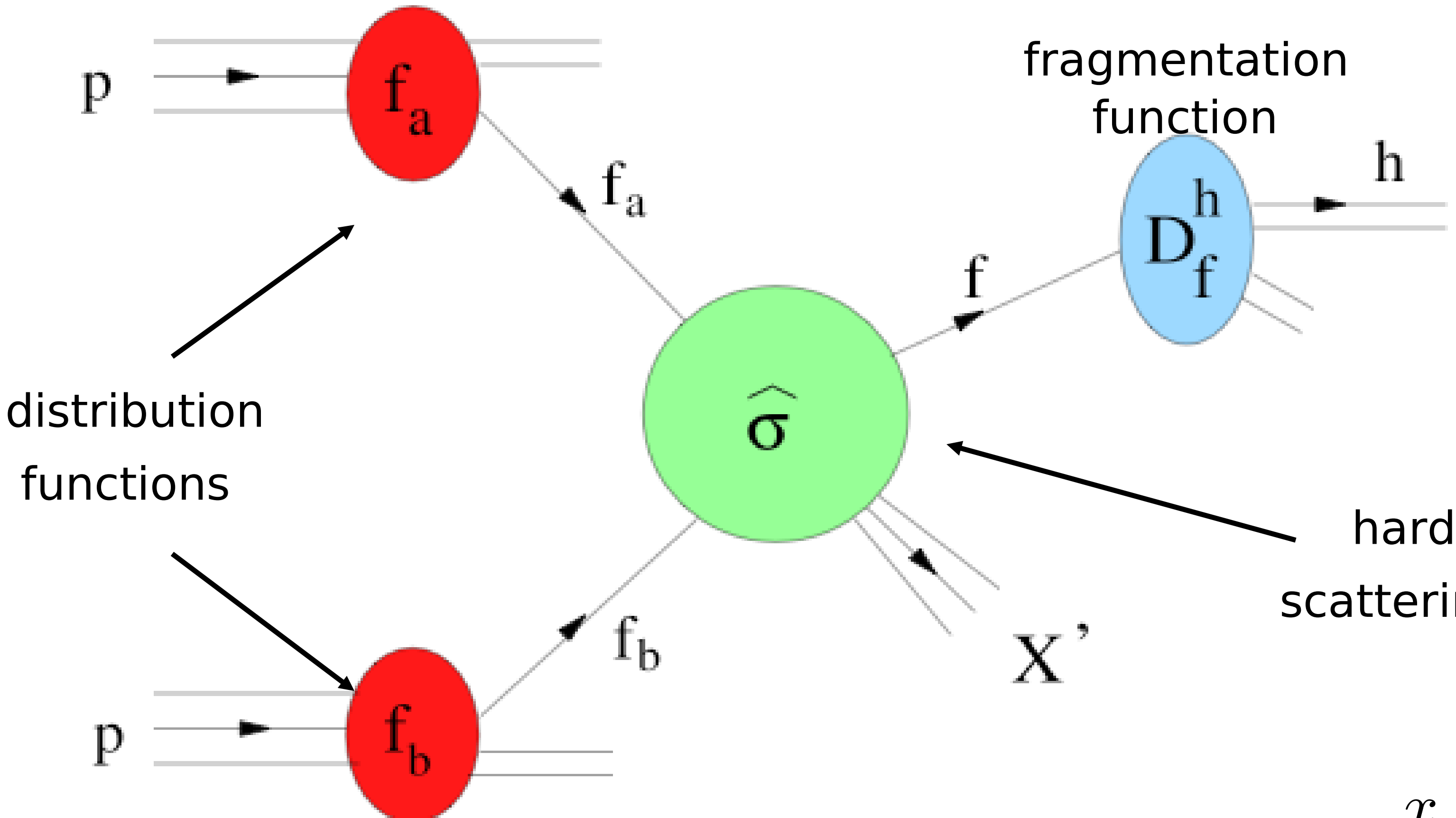
high gluon density effects: multiple soft scatterings
high energy effects: quantum corrections

Toward a unified formalism:

hard + multiple soft scatterings

High p_t particle production: pp collisions

collinear factorization: separation of soft (long distance) and hard (short distance)



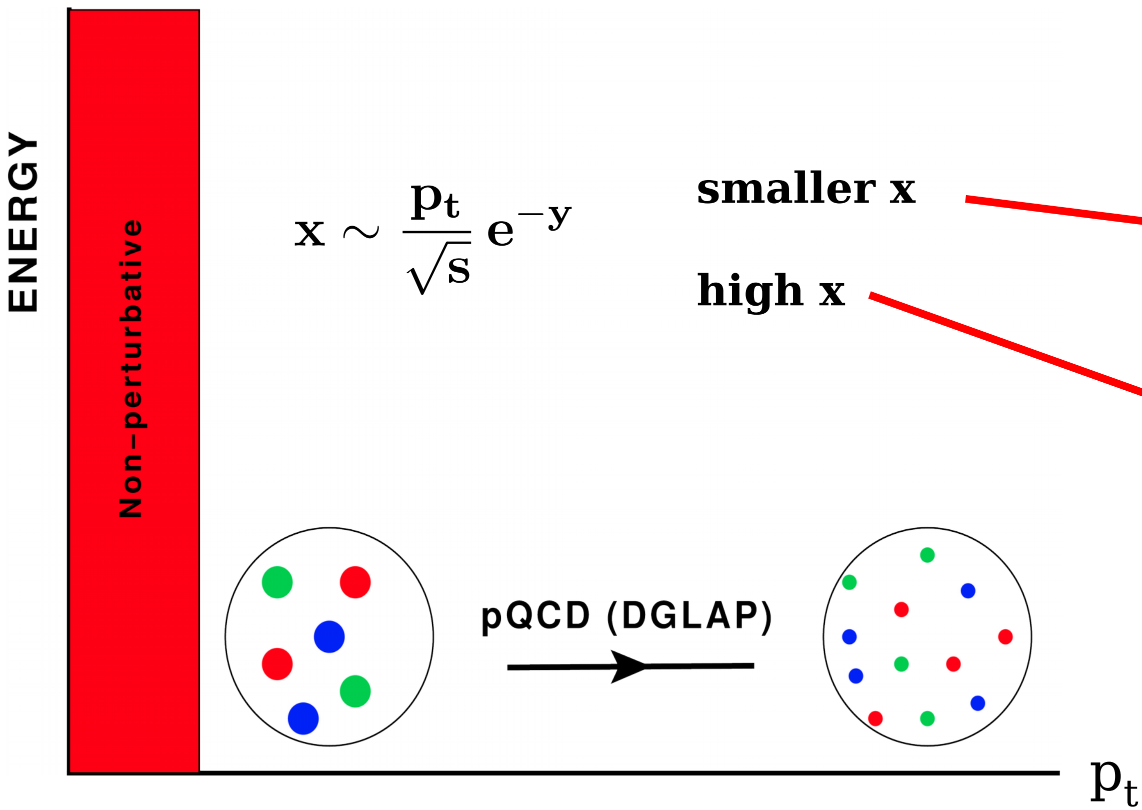
$$x \equiv \frac{p^+}{P^+}$$

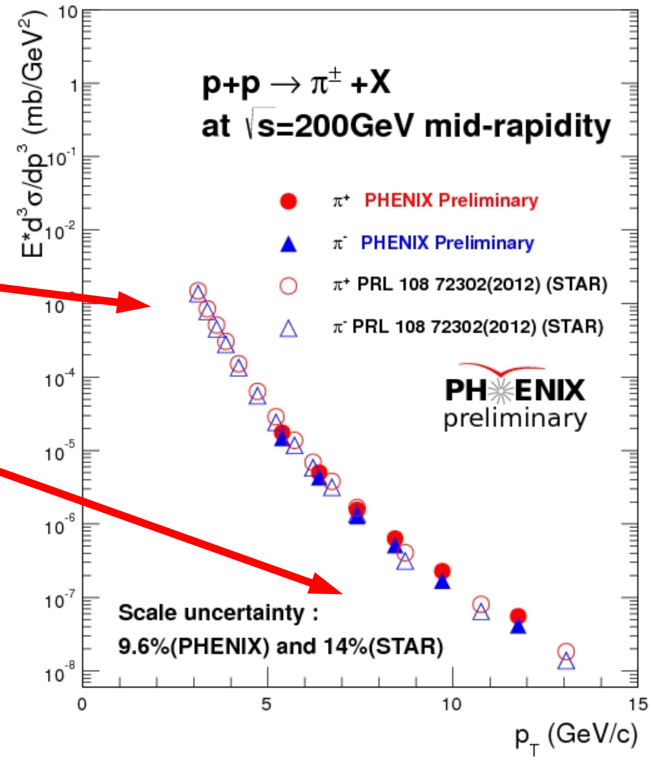
$$\frac{d\sigma^{pp\to h\,X}}{d^2p_t\,dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections

pQCD: the standard paradigm

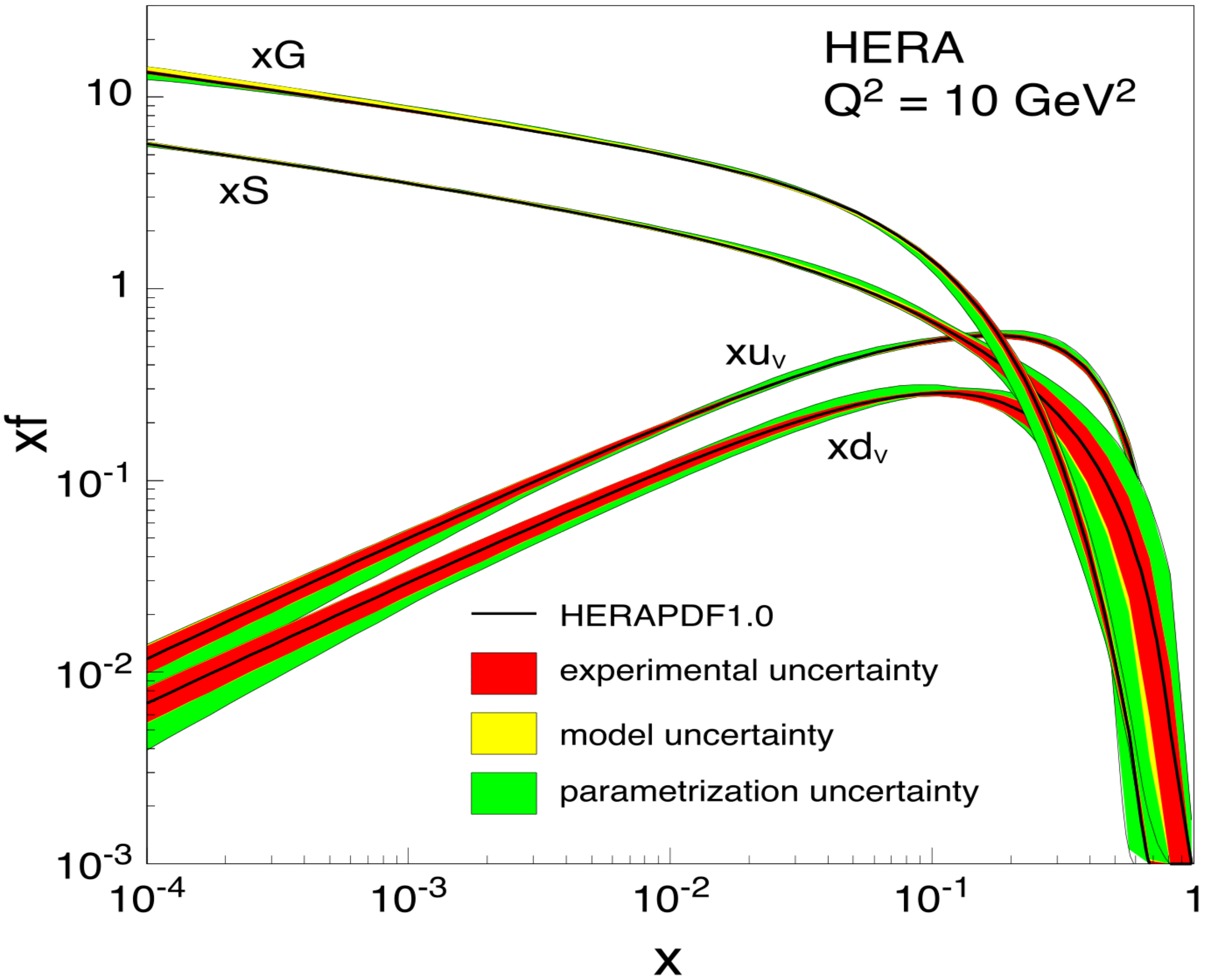
$$E\,\frac{d\sigma}{d^3p} \sim f_1(x, p_t^2) \otimes f_2(x, p_t^2) \otimes \frac{d\sigma}{dt} \otimes D(z, p_t^2)$$





bulk of QCD phenomena happens at low p_t (small x)

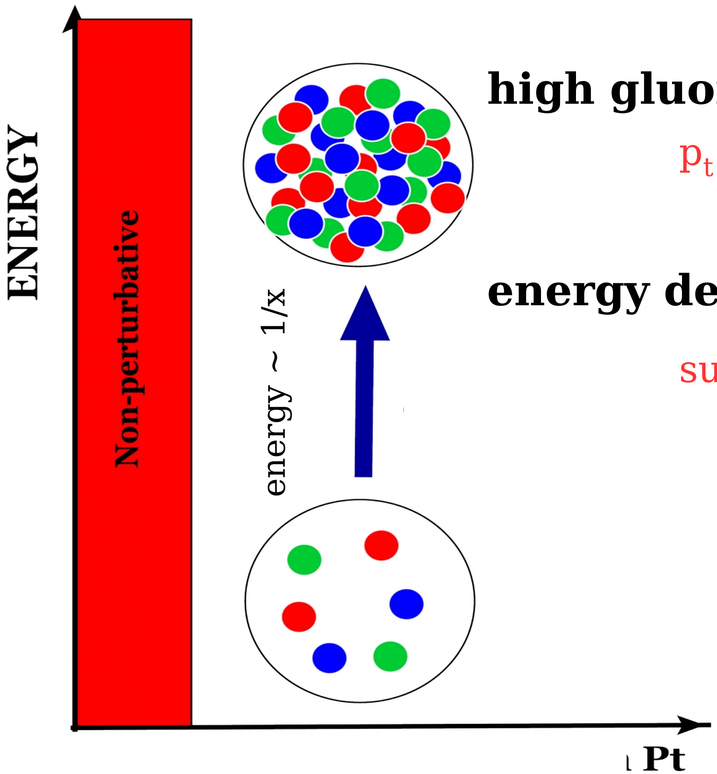
Deep Inelastic Scattering



$$x = \frac{p^+}{P^+}$$

x *is the fraction of hadron energy carried by a parton*

A hadron/nucleus at high energy: ***gluon saturation***



high gluon density: multiple scattering via Wilson lines

p_t broadening

energy dependence: x-evolution via JIMWLK

suppression of spectra/away side peaks

$$\mathbf{Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}}$$

$$Q_s^2(x = 3 \times 10^{-4}) \sim 1\, GeV^2$$

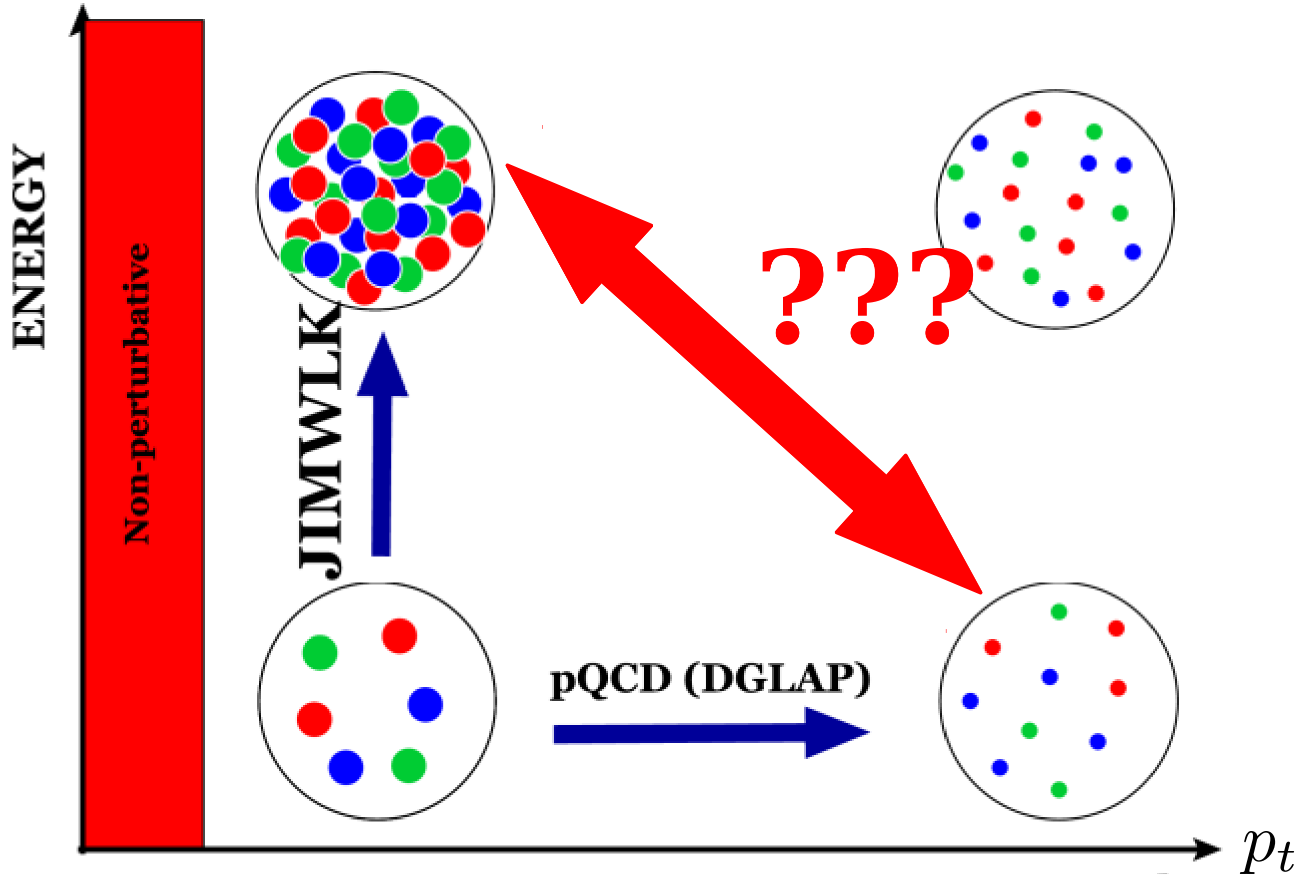
for a proton target (quarks)

a framework for multi-particle production in QCD at small x/low p_t

Initial conditions for hydro
Thermalization ?
Long range rapidity correlations
Azimuthal angular correlations
Nuclear modification factor

$$\mathbf{x \leq 0.01}$$

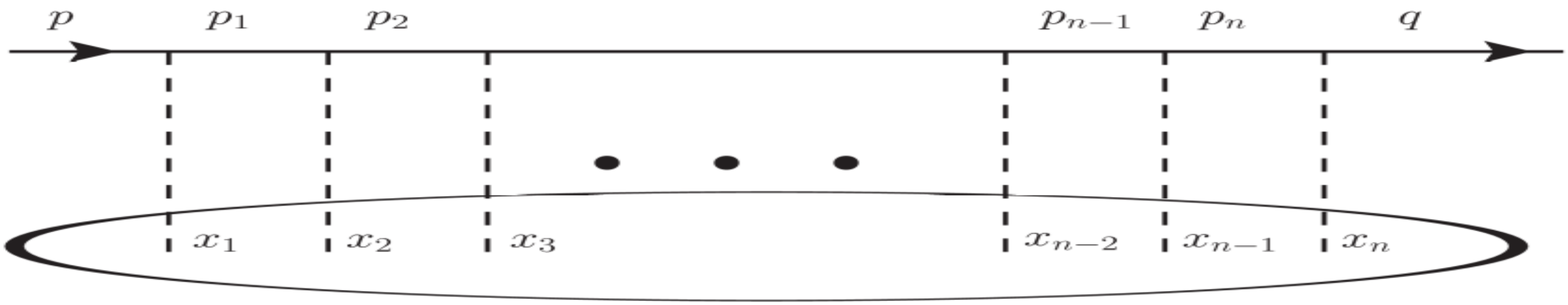
QCD kinematic phase space



unifying saturation with high p_t (large x) physics?

kinematics of saturation: *where is saturation applicable?*
jet physics, high p_t (polar and azimuthal) angular correlations
cold matter energy loss, spin physics, ultra-high energy neutrinos

Classical CGC: multiple scattering in eikonal approximation

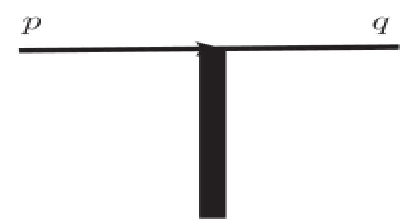


$$
\begin{aligned}
i\mathcal{M}_n \;\; = \;\; & 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t} \\
& \Big\{ (ig)^n\, (-i)^n (i)^n \int dx_1^+\, dx_2^+ \cdots dx_n^+\, \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \\
& \left[S(x_n^+, x_t)\, S(x_{n-1}^+, x_t) \cdots S(x_2^+, x_t) S(x_1^+, x_t) \right] \Big\}\, u(p)
\end{aligned}
$$

sum over all scatterings $\quad i\mathcal{M} = \sum_n i\,\mathcal{M}_n$

$$
i\mathcal{M}(p,q) = 2\pi\delta(p^+ - q^+)\, \bar{u}(q)\, \not{n} \int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\, \left[V(x_t) - 1 \right]\, u(p)
$$

with $V(x_t) \equiv \hat{P}\, \exp\left\{ ig \int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a \right\}$

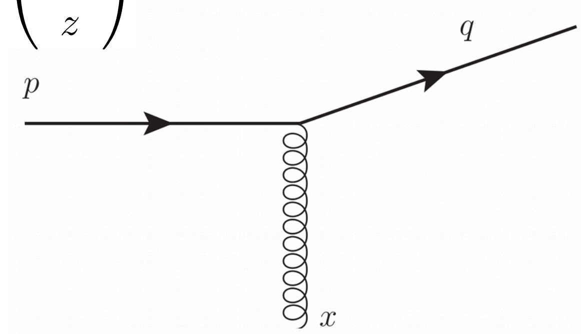


$$
\frac{d\,\sigma^{q\,T \to q\,X}}{d^2 p_t\, dy} \sim |i\mathcal{M}|^2 \sim \; F.T. \; < Tr\, V(x_t)\, V^\dagger(y_t) >
$$

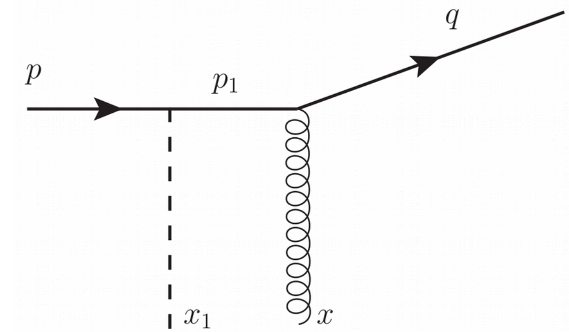
hard scattering: large deflection
scattered quark travels in the new "z" direction: $\bar{z}$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

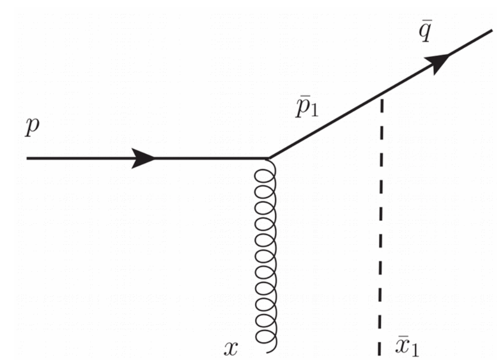
$$i\mathcal{M}_1 = (ig) \int d^4x\, e^{i(\bar{q}-p)x}\, \bar{u}(\bar{q}) \; \left[\not{A}(x)\right] \; u(p)$$



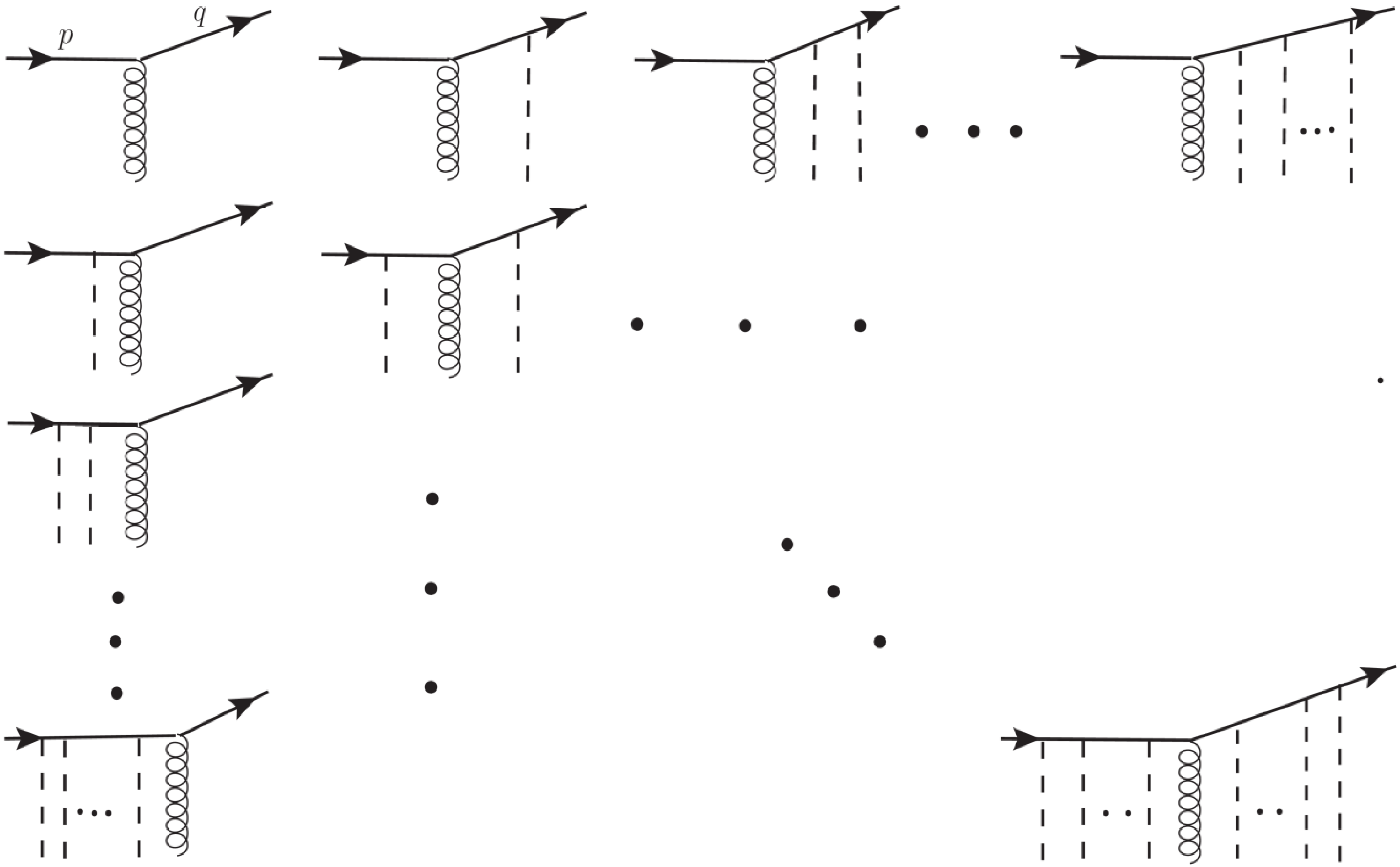
$$i\mathcal{M}_2 \quad = \quad (ig)^2 \int d^4x\, d^4x_1 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1-p)x_1}\, e^{i(\bar{q}-p_1)x}$$
$$\bar{u}(\bar{q}) \; \left[\not{A}(x)\, \frac{i\not{p}_1}{p_1^2+i\epsilon}\, \not{n}\, S(x_1)\right] \; u(p)$$



$$i\mathcal{M}_2 \quad = \quad (ig)^2 \int d^4x\, d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4}\, e^{i(\bar{p}_1-p)x}\, e^{i(\bar{q}-\bar{p}_1)\bar{x}_1}$$
$$\bar{u}(\bar{q}) \; \left[\not{\bar{n}}\, \bar{S}(\bar{x}_1)\, \frac{i\not{\bar{p}}_1}{\bar{p}_1^2+i\epsilon}\not{A}(x)\right] \; u(p)$$



with $\quad \vec{\bar{v}} = \mathcal{O}\, \vec{v}$

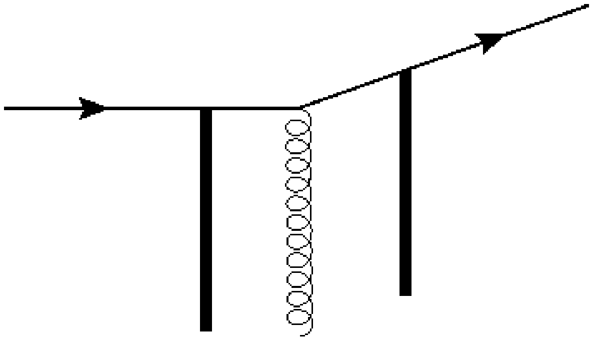

p
q

summing all the soft re-scatterings gives:

$$i\mathcal{M}_1 = \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\, \frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t}$$

$$\bar{u}(\bar{q}) \left[\overline{V}_{AP}(x^+,\bar{z}_t)\,\not{n}\,\frac{\not{\bar{k}}}{2\bar{k}^+}\,\left[ig\not{A}(x)\right]\,\frac{\not{k}}{2k^+}\,\not{n}\,V_{AP}(z_t,x^+)\right] u(p)$$

with

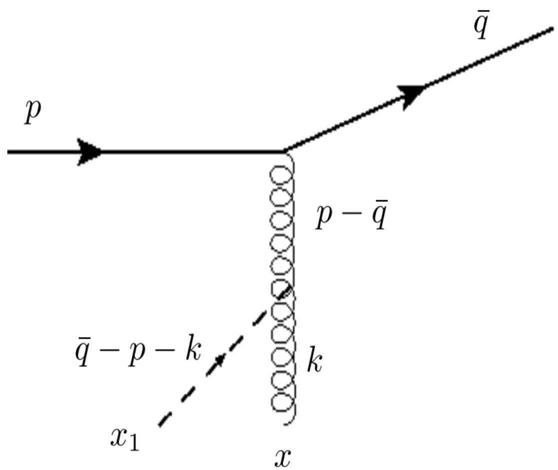


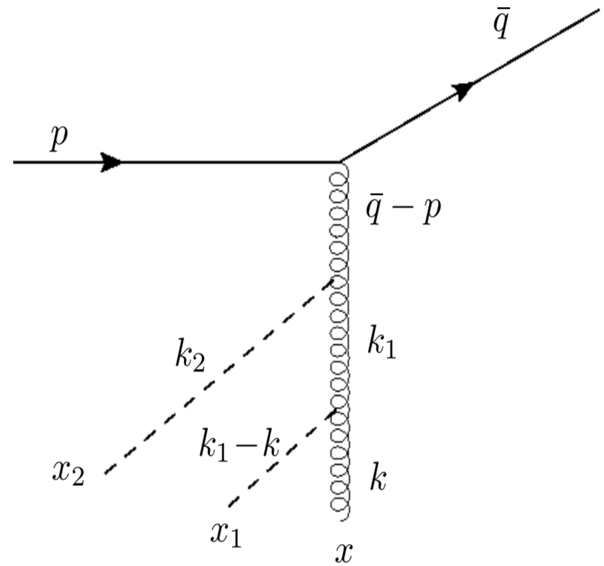
$$\overline{V}_{AP}(x^+,\bar{z}_t) \equiv \hat{P}\,\exp\left\{ig\int_{x^+}^{+\infty} d\bar{z}^+\,\bar{S}_a^-(\bar{z}_t,\bar{z}^+)\,t_a\right\}$$

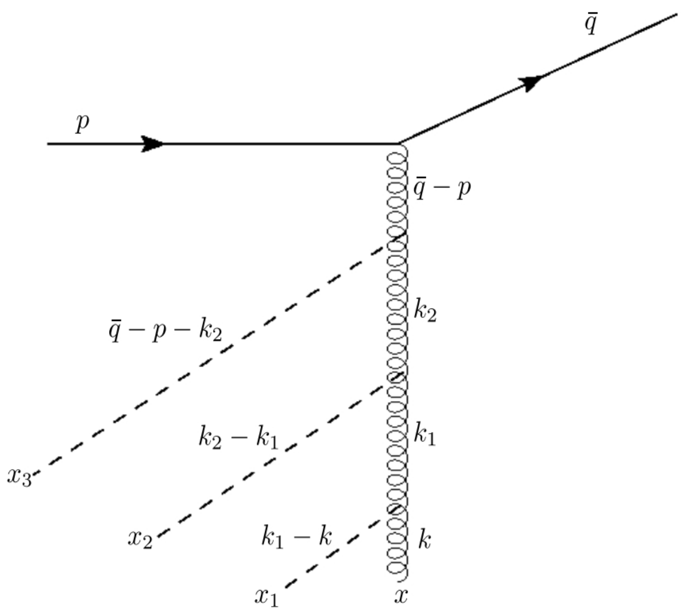
$$V_{AP}(z_t,x^+) \equiv \hat{P}\,\exp\left\{ig\int_{-\infty}^{x^+} dz^+\,S_a^-(z_t,z^+)\,t_a\right\}$$

how about multiple scatterings of hard gluon?

interactions of large and small x gluons

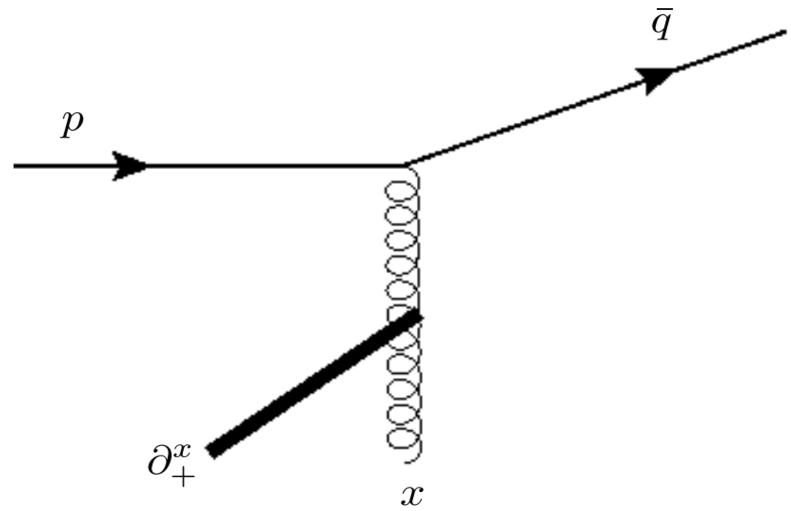






+ ....

multiple scatterings of hard gluon can be resummed



$$i\mathcal{M}_2 = \frac{2\,i}{(p-\bar{q})^2}\int d^4x\, e^{i(\bar{q}-p)x}\,\bar{u}(\bar{q})\Big[(ig\,t^a)\,\left[\partial_{x^+}U^{\dagger}_{AP}(x_t,x^+)\right]^{ab}$$

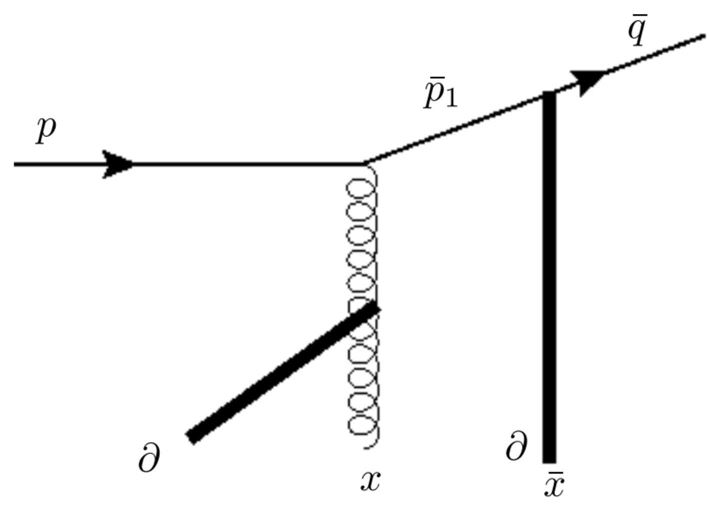
$$\left[n\cdot(p-\bar{q})\not{A}_b(x)-(p-\bar{q})\cdot A_b(x)\,\not{n}\right]\Big]u(p)$$

with $$U_{AP}(x_t,x^+)\equiv \hat{P}\,exp\left\{ig\int_{-\infty}^{x^+}dz^+\,S_a^-(z^+,x_t)\,T_a\right\}$$

but there is more!

how about the final state quark interactions?

these contributions resum to

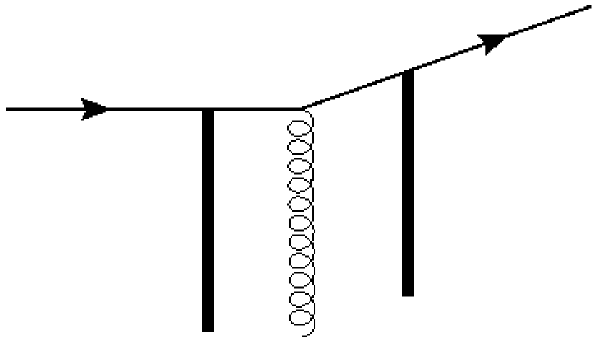


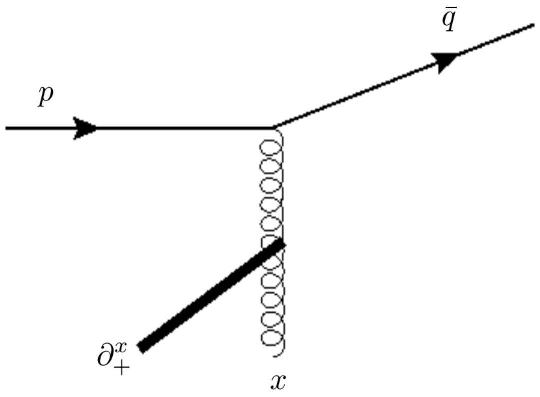
$$i\mathcal{M}_3 \;=\; -2\,i\int d^4x\,d^2\bar{x}_t\,d\bar{x}^+\,\frac{d^2\bar{p}_{1t}}{(2\pi)^2}\,e^{i(\bar{q}^+-p^+)x^-}\;e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\;e^{-i(\bar{q}_t-\bar{p}_{1t})\cdot\bar{x}_t}$$

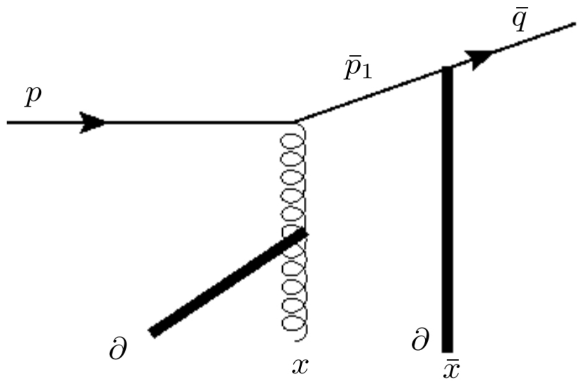
$$\bar{u}(\bar{q})\left[\left[\partial_{\bar{x}^+}\,\overline{V}_{AP}(\bar{x}^+,\bar{x}_t)\right]\,\not{n}\,\not{\bar{p}}_1\,(igt^a)\,\left[\partial_{x^+}\,U^\dagger_{AP}(x_t,x^+)\right]^{ab}\right.$$

$$\left.\frac{\left[n\cdot(p-\bar{q})\not{A}^b(x)-(p-\bar{p}_1)\cdot A^b(x)\,\not{n}\right]}{\left[2n\cdot\bar{q}\,2n\cdot(p-\bar{q})\,p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2 - 2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]}\right]u(p)$$

full amplitude: $\mathbf{i}\mathcal{M} = \mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}$







soft (eikonal) limit:

$$A^\mu(x) \rightarrow n^- S(x^+, x_t)$$
$$n\cdot\bar{q} \rightarrow n\cdot p$$

$$i\mathcal{M} \longrightarrow i\mathcal{M}_{eik}$$

cross section: $|\mathbf{i}\mathcal{M}|^2 = |\mathbf{i}\mathcal{M}_{\mathbf{eik}} + \mathbf{i}\mathcal{M}_{\mathbf{1}} + \mathbf{i}\mathcal{M}_{\mathbf{2}} + \mathbf{i}\mathcal{M}_{\mathbf{3}}|^2$

$$|i\mathcal{M}_2|^2 = \frac{8g^2}{(p-\bar{q})^4} \int d^4x\, d^4y\, e^{i(\bar{q}^+ - p^+)(x^- - y^-)}\, e^{-i(\bar{q}_t - p_t)\cdot(x_t - y_t)} \left\{ p^+ q^- (p^+ - \bar{q}^+)^2 A^b_\perp(x) \cdot A^c_\perp(y) + 2\,(p^+)^2 q_\perp \cdot A^b_\perp(x)\, q_\perp \cdot A^c_\perp(y) \right\} \left[\partial_{y^+} U_{AP}(y_t, y^+)\right]^{ca} \left[\partial_{x^+} U^\dagger_{AP}(x_t, x^+)\right]^{ab}$$

other terms are more complicated: *spinor helicity formalism* for Dirac Algebra

DIS: structure functions, di-jet production

PA: single inclusive particle production

PP: spin asymmetries (A_LL,...)

particle production in high energy collisions

pQCD and collinear factorization at high p_t

breaks down at low p_t (small x)

CGC at low p_t

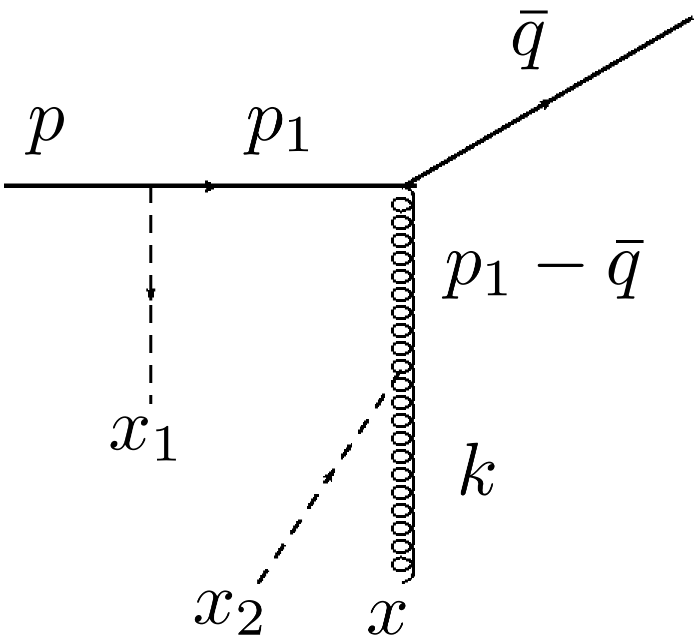
breaks down at large x (high p_t)

Toward a unified formalism:

parton scattering from small and large x fields

gluon radiation, 1-loop corrections

particle production in pp, pA in both small and large p_t regions



both initial state quark and hard gluon interacting:

integration over p_1^-

$$\int \frac{dp_1^-}{2\pi} \frac{e^{ip_1^-(x_1^+ - x^+)}}{[p_1^2 + i\epsilon]\ [(p_1 - \bar{q})^2 + i\epsilon]}$$

both poles are below the real axis, we get

$$\frac{e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x^+)}}{\left[\frac{p_{1t}^2}{2p^+} - \bar{q}^- - \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right]} + \frac{e^{i\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)}\right](x_1^+ - x^+)}}{\left[\bar{q}^- + \frac{(p_{1t} - \bar{q}_t)^2}{2(p^+ - \bar{q}^+)} - \frac{p_{1t}^2}{2p^+}\right]}$$

ignoring phases we get a cancellation!

this can be shown to hold to all orders whenever both initial state quark and hard gluon scatter from the soft fields!

toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field $A^- \equiv n^- S$ involves only small transverse momenta exchange (small angle deflection)

$$
\begin{aligned}
p^\mu &= (p^+ \sim \sqrt{s},\, p^- = 0, p_t = 0) \\
S &= S(p^+ \sim 0, p^-/P^- \ll 1, p_t)
\end{aligned}
$$

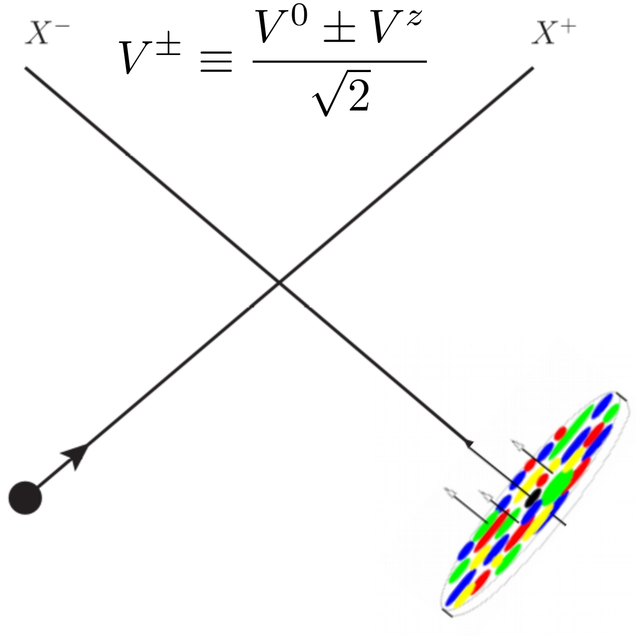
allow hard scattering by including one hard field $A_a^\mu(x^+, x^-, x_t)$ during which there is large momenta exchanged and quark can get deflected by a large angle.

include eikonal multiple scattering before and after (along a different direction) the hard scattering

eliminate/minimize medium effects (***proton-nucleus***)

<u>Eikonal approximation</u>

$$V^{\pm} \equiv \frac{V^0 \pm V^z}{\sqrt{2}}$$



$$J^\mu_a \simeq \delta^{\mu -} \rho_a$$

$$\begin{aligned} D_\mu \, J^\mu &= D_- \, J^- = 0 \\ \partial_- J^- &= 0 \end{aligned}$$

(in $A^+ = 0$ gauge)

does not depend on x^-

solution to EOM: $$A^-_a(x^+, x_t) \equiv n^- \, S_a(x^+, x_t)$$

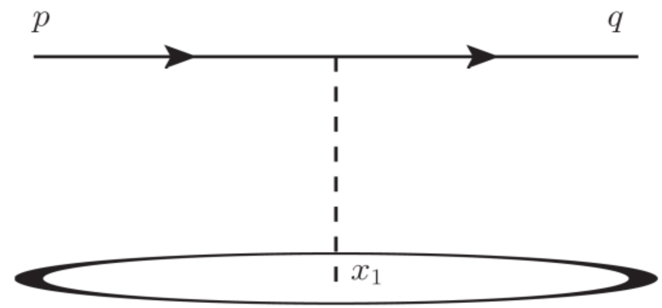
with $$\begin{aligned} n^\mu &= (n^+ = 0, n^- = 1, n_\perp = 0) \\ n^2 &= 2\, n^+ n^- - n^2_\perp = 0 \end{aligned}$$

recall (eikonal limit): $$\bar{u}(q)\gamma^\mu u(p) \rightarrow \bar{u}(p)\gamma^\mu \, u(p) \sim p^\mu$$

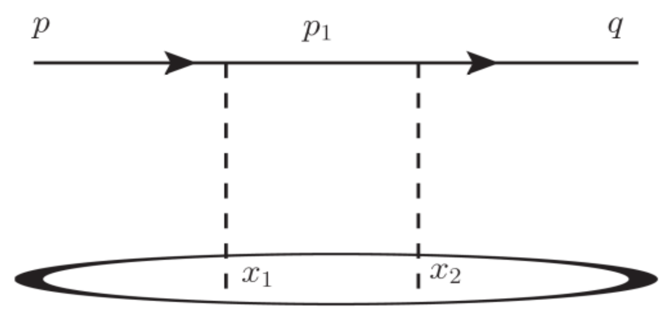
$$\bar{u}(q) \not{A} \, u(p) \rightarrow p \cdot A \sim p^+ \, A^-$$

multiple scattering of a quark from background color field $A^-_a(x^+, x_t)$

$$i\mathcal{M}_1 = (ig)\int d^4x_1\, e^{i(q-p)x_1}\, \bar{u}(q)\, \left[\not{n}\, S(x_1)\right]\, u(p)$$
$$= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t}\, dx_1^+\, e^{i(q^- - p^-)x_1^+}\, e^{-i(q_t - p_t)x_{1t}}$$
$$\bar{u}(q)\left[\not{n}\, S(x_1^+, x_{1t})\right] u(p)$$



$$i\mathcal{M}_2 = (ig)^2\int d^4x_1\, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4}\, e^{i(p_1 - p)x_1}\, e^{i(q-p_1)x_2}$$
$$\bar{u}(q)\left[\not{n}\, S(x_2)\, \frac{i\not{p}_1}{p_1^2 + i\epsilon}\, \not{n}\, S(x_1)\right] u(p)$$



$$\int \frac{dp_1^-}{(2\pi)}\frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+\left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+}\right]} = \frac{-i}{2p^+}\,\theta(x_2^+ - x_1^+)\, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}$$

contour integration over the pole leads to path ordering of scattering

ignore all terms: $O(\frac{p_t}{p^+}, \frac{q_t}{q^+})$ and use $\not{n}\,\frac{\not{p}_1}{2n\cdot p}\,\not{n} = \not{n}$

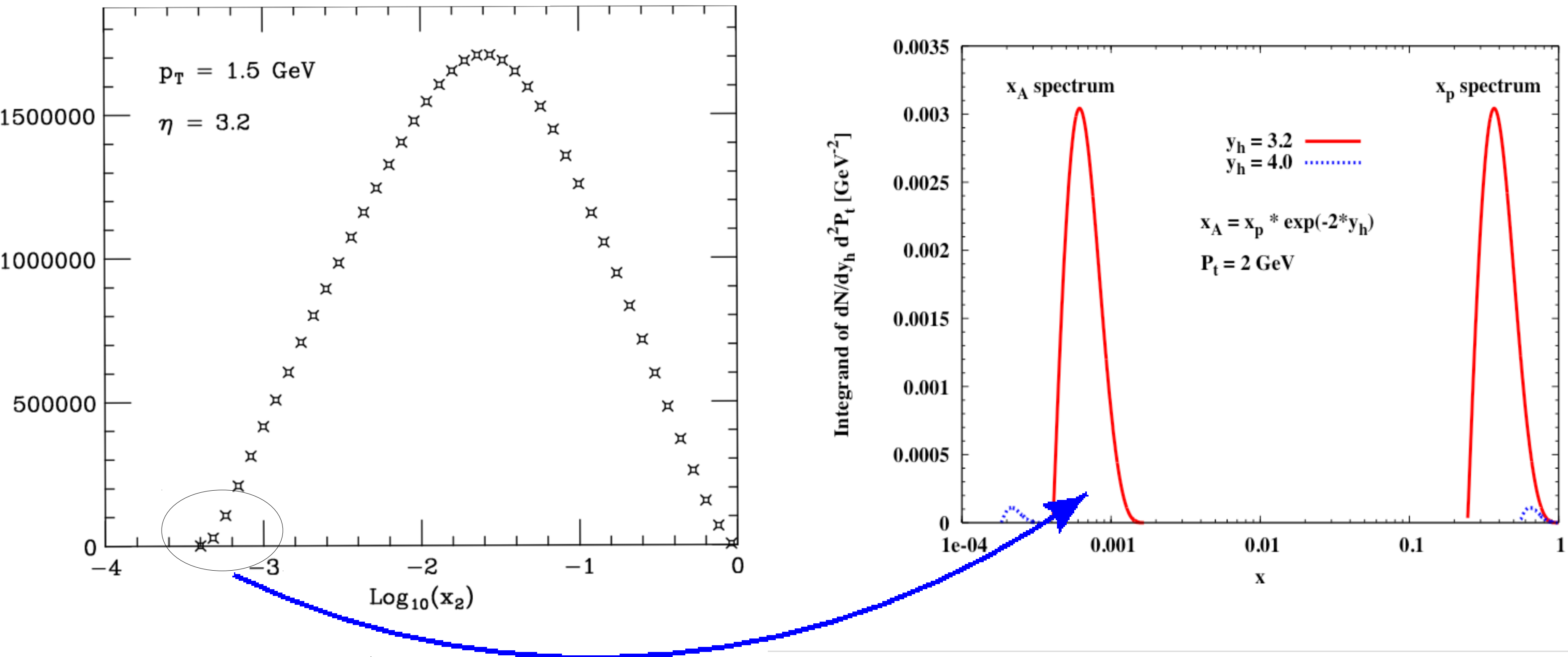
$$i\mathcal{M}_2 = (ig)^2\,(-i)(i)\, 2\pi\delta(p^+ - q^+)\int dx_1^+\, dx_2^+\, \theta(x_2^+ - x_1^+)\int d^2x_{1t}\, e^{-i(q_t - p_t)\cdot x_{1t}}$$
$$\bar{u}(q)\left[S(x_2^+, x_{1t})\,\not{n}\, S(x_1^+, x_{1t})\right] u(p)$$

Pion production at RHIC: kinematics

collinear factorization — CGC

GSV, PLB603 (2004) 173-183

DHJ, NPA765 (2006) 57-70



$$\int_{x_{min}}^{1} dx\, xG(x, Q^2) \cdots\cdots \longrightarrow x_{min} G(x_{min}, Q^2) \cdots$$

this is an extreme approximation with potentially severe consequences!