Multidimensional scalar-tensor gravity: theory and experiment

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Contents

The Post-Newtonian Limit

$$ds^2 = \left(1 - 2\frac{\varphi}{c^2} + 2\beta\frac{\varphi^2}{c^4}\right)c^2dt^2 - \left(1 + 2\gamma\frac{\varphi}{c^2}\right)\sum_{\tilde{\mu}=1}^{3} dx^{\tilde{\mu}} \qquad (1)$$

Perihelion Shift

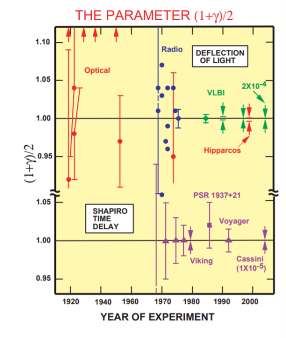
$$\delta\Psi = \frac{6\pi G_N m}{a(1-e^2)c^2} \qquad \delta\Psi = \frac{6\pi G_N m}{a(1-e^2)c^2}\frac{1}{3}(2 + 2\gamma - \beta)$$

Deflection of Light

$$\delta\Psi = (1+\gamma)\frac{2G_N m}{c^2\rho}$$

Shapiro Time-Delay Effect

$$\delta t = (1+\gamma)\frac{2G_N m}{c^3} ln\left(\frac{r_{Earth} r_{planet}}{R^2_{Sun}}\right)$$



*Will, C.M. Living Rev. Relativ. (2014) 17:4

$$\gamma - 1 = 2.1 \pm 2.3 \times 10^{-5*}$$

The Metric

$$\hat{g}_{MN} dX^M \otimes dX^N = \hat{g}_{\mu\nu} dx^\mu \otimes dx^\nu + \hat{g}_{mn} dx^m \otimes dx^n$$
$$M, N = 0, ..., D, \quad \mu, \nu = 0, 1, 2, 3; \qquad (2)$$
$$m, n = 4, ..., D.$$

$$\hat{g}_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$\hat{R}_{mn}[\hat{g}^{(d)}] = \lambda g_{mn}, \quad \lambda = 0. \qquad (3)$$

$\mathcal{D}$-dimensional Gravitational Action

$$S_g = \frac{1}{2\kappa_{\mathcal{D}}^2} \int d^{\mathcal{D}}x \sqrt{|g|} \times \left[f(\Phi)R + h(\Phi)\, \nabla_M \Phi \nabla^M \Phi - U(\Phi) \right] \quad (4)$$

MF	PF
$\frac{\delta S}{\delta g^{MN}} = 0$	$\frac{\delta S}{\delta g^{MN}} = 0$
	$\frac{\delta S}{\delta \Gamma^K_{LM}} = 0$
$\frac{\delta S}{\delta \Phi} = 0$	$\frac{\delta S}{\delta \Phi} = 0$

$$\kappa_{\mathcal{D}}^2 \equiv 2S_{D-1}\tilde{G}_{\mathcal{D}}/c^4$$

Metric vs Palatini Formulation

$$
\begin{aligned}
&fG_{MN} + \left[f'' - \frac{h}{2}\right] g_{MN}(\nabla\Phi)^2 \\
&+ g_{MN} f' \Delta_D \Phi + (h - f'')\nabla_M\Phi\nabla_N\Phi \\
&- f'\nabla_M\nabla_N\Phi + \frac{1}{2} g_{MN} U = \kappa_{\mathcal{D}}^2 T_{MN}, \\
&f'R - h'(\nabla\Phi)^2 - 2h\Delta_D\Phi - U' = 0
\end{aligned}
$$

$$
\begin{aligned}
&fG_{MN} + \left[f'' - \frac{\chi}{2}\right] g_{MN}(\nabla\Phi)^2 \\
&+ g_{MN} f' \Delta_D \Phi + (\chi - f'')\nabla_M\Phi\nabla_N\Phi \\
&- f'\nabla_M\nabla_N\Phi + \frac{1}{2} g_{MN} U = \kappa_{\mathcal{D}}^2 T_{MN}, \\
&f'R - \chi'(\nabla\Phi)^2 - 2\chi\Delta_D\Phi - U' = 0
\end{aligned}
$$

$$h \leftrightarrow \chi$$

$$\chi \equiv \frac{D-1}{D-2}\frac{(f')^2}{f} + h$$

Metric vs Palatini Formulation

Is it possible to obtain physical equivalence in both frames?
Without loss of generality, let's try $f(\Phi) \equiv \Phi$ and
$h(\Phi) \equiv -\omega(\Phi)/\Phi$:

$$\chi \equiv -\left[\omega(\Phi) - \frac{D-1}{D-2}\right]\frac{1}{\Phi} \equiv -\frac{\widetilde{\omega}(\Phi)}{\Phi} \qquad (5)$$

$$\omega(\Phi) = \omega = \text{const.}$$

A Note on Background $\hat{\Phi}$

Stable vacuum for Φ:

$$U(\Phi) = \frac{\mu^2}{2}(\Phi - \hat{\Phi})^2 + o\left((\Phi - \hat{\Phi})^2\right) \tag{6}$$

μ : mass scale for the scalar Φ

Background value $\hat{\Phi}$ cannot be zero; Φ determines the strength of gravitational coupling!

Metric vs Palatini Formulation

Within the metric formalism, dynamical equations read

$$\Phi G_{MN} + g_{MN}\frac{\omega}{2\Phi}(\nabla\Phi)^2 + g_{MN}\Delta_D\Phi - \frac{\omega}{\Phi}\nabla_M\Phi\nabla_N\Phi - \nabla_M\nabla_N\Phi + \frac{1}{2}g_{MN}U = \kappa_D^2 T_{MN}, \tag{7}$$

$$[(d+3)+\omega(d+2)]\ \Delta_D\Phi = \kappa_D^2 T + \Phi U' - \frac{d+4}{d+2}U\,.$$

No background matter $\rightarrow$ $T_{MN} = 0$

Linear Perturbations

Perturbed Quantities

$$g_{MN} \approx \hat{g}_{MN} + \delta g_{MN} \equiv \hat{g}_{MN} + h_{MN}, \;\; h^K_L \equiv \hat{g}^{KM} h_{ML} \; ;$$

$$\Phi \approx \hat{\Phi} + \delta\Phi = \hat{\Phi}(1+\phi), \quad \phi \equiv \delta\Phi/\hat{\Phi}.$$

Perturbations of the EMT:

$$\mathcal{T}^M_N = \delta\mathcal{T}^M_N = \mathcal{E}\delta^M_0\delta^0_N - \mathcal{P}_1\delta^M_l\delta^l_N, \tag{8}$$

$$\mathcal{E} = \rho c^2 = Mc^2\frac{\delta(\mathbf{r})}{\hat{V}_d}, \quad \mathcal{P}_1 = \Omega\mathcal{E}, \quad \Omega = \mathrm{const.} \tag{9}$$

$$h_{00} = \chi_1, \quad h_{\tilde{\mu}\tilde{\nu}} = \hat{g}_{\tilde{\mu}\tilde{\nu}}\chi_2 = -\delta_{\tilde{\mu}\tilde{\nu}}\chi_2, \quad h_{mn} = \hat{g}_{mn}\chi_3 \tag{10}$$

Linearized Field Equations

$$-\frac{1}{2}\mu^2\phi + (1+\omega)\Delta\phi - \frac{1}{2}\Delta\chi_1 = \kappa_{\mathcal{D}}^2\mathcal{E},$$

$$\frac{1}{2}\mu^2\phi - (1+\omega)\Delta\phi + \frac{1}{2}\Delta\chi_2 = 0,$$

$$-\frac{1}{2}\mu^2\phi + (1+\omega)\Delta\phi - \frac{1}{2}\Delta\chi_3 = -\kappa_{\mathcal{D}}^2\Omega\mathcal{E}$$

$$\left[(d+3) + \omega(d+2)\right]\,\Delta\phi - \mu^2\phi = \kappa_{\mathcal{D}}^2\mathcal{E}(1-\Omega d)$$

Solutions

The solutions of $\chi_1(\mathbf{r})$, $\chi_2(\mathbf{r})$, $\chi_3(\mathbf{r})$, and that of $\phi(\mathbf{r})$ may all be expressed as

$$f(r) = \frac{\alpha\kappa_{\mathcal{D}}^2 Mc^2}{4\pi\hat{V}_d}\frac{1}{r}\left[\mathbf{A} - \left(\mathbf{A} - \frac{\mathbf{B}}{C}\right)e^{-mr}\right] \tag{11}$$

$\mu = 0, \chi_1 = \frac{2\varphi}{c^2}$

$\lim_{r\to\infty}(\varphi/\varphi_N) = 1$

$$\frac{\kappa_{\mathcal{D}}^2}{\hat{V}_d}\frac{B_1}{C} = \frac{8\pi G_N}{c^4}$$

$\mu \neq 0$: Yukawa correction term

$lim_{r\to\infty}(\varphi/\varphi_N) = 1 \mid \mathrm{r} \gg 1/m$

$$\frac{\kappa_{\mathcal{D}}^2}{\hat{V}_d}A_1 = \frac{8\pi G_N}{c^4}$$

$$ds^2 = \left(1 - 2\frac{\varphi}{c^2}\right)c^2dt^2 - \left(1 + 2\gamma\frac{\varphi}{c^2}\right)\sum_{\tilde{\mu}=1}^{3} dx^{\tilde{\mu}} \tag{12}$$

Massless Scalar

- $d = 0, \gamma = 1$

$$\gamma = \frac{B_2}{B_1} = \frac{(1+\omega)(1-\Omega d)}{d(\omega+1)(\Omega+1)+\omega+2} \xrightarrow{d=0} \frac{\omega+1}{\omega+2} \tag{13}$$

- $\Omega = 0, \gamma = 1$

$$\frac{1+\omega}{d(\omega+1)+\omega+2} = 1 \quad \rightarrow \quad \omega = -1 - \frac{1}{d} < -1 \tag{14}$$

- $\omega \sim O(1), \;\; \omega > 0, \gamma = 1$

$$\Omega = -\frac{1}{2} - \frac{1}{2d(\omega+1)} < -\frac{1}{2} \tag{15}$$

Massive Fields

Provided that m is large enough, one gets

$$\gamma = -\lim_{r\to\infty} \frac{\chi_2(r)}{\chi_1(r)} = \frac{A_2}{A_1} = \frac{1-\Omega d}{1+\Omega d} \tag{16}$$

$$\gamma = 1 \tag{17}$$