Multi-particle fields on the subset of simultaneity

Dmytro Ptashynskyi

Odessa National Polytechnic University

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- The major problem of existing field theories is not only the complexity of the interaction description, but the single-particle nature of these theories.

The tensor product of Minkowski spaces for two particles is an eight-dimensional linear space. Its elements can be considered as columns:

$$z^a = \begin{pmatrix} x^0_{(1)} \\ x^1_{(1)} \\ x^2_{(1)} \\ x^3_{(1)} \\ x^0_{(2)} \\ x^1_{(2)} \\ x^2_{(2)} \\ x^3_{(2)} \end{pmatrix}. \tag{1}$$

$\left(x^0_{(1)}, x^1_{(1)}, x^2_{(1)}, x^3_{(1)}\right)$ - for the quark, $\left(x^0_{(2)}, x^1_{(2)}, x^2_{(2)}, x^3_{(2)}\right)$ - for the antiquark

We introduce a scalar product in this eight-dimensional space by the following expression:

$$\langle z|z\rangle = \frac{1}{2}\left(g_{ab}^{Minc} x_{(1)}^{a} x_{(1)}^{b} + g_{ab}^{Minc} x_{(2)}^{a} x_{(2)}^{b}\right). \tag{2}$$

Using the Jacobi coordinates, expression (2) takes the form:

$$\langle z|z\rangle = g_{ab}^{Minc}\left(X^{a} X^{b} + \frac{1}{4} y^{a} y^{b}\right) \tag{3}$$

A condition for the subset of simultaneity is:

$$y^{0} = 0. \tag{4}$$

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The coordinates of a point on a subset of simultaneity are denoted by a seven-component column q^a. And the metric tensor of this space g^{ab} is:

$$q^a = \begin{pmatrix} X^0 \\ X^1 \\ X^2 \\ X^3 \\ y^1 \\ y^2 \\ y^3 \end{pmatrix}, g^{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4 \end{pmatrix} \tag{5}$$

Next we define the scalar product on a subset of simultaneity so that it coincides with the product (3), taking into account the condition (4):

$$\langle q|q\rangle = g_{ab}q^a q^b, \tag{6}$$

The following group of matrices acts on a subset of simultaneity:

$$\hat{G} = \begin{pmatrix} \Lambda^0_0 & \Lambda^0_1 & \Lambda^0_2 & \Lambda^0_3 & 0 & 0 & 0 \\ \Lambda^1_0 & \Lambda^1_1 & \Lambda^1_2 & \Lambda^1_3 & 0 & 0 & 0 \\ \Lambda^2_0 & \Lambda^2_1 & \Lambda^2_2 & \Lambda^2_3 & 0 & 0 & 0 \\ \Lambda^3_0 & \Lambda^3_1 & \Lambda^3_2 & \Lambda^3_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R^1_1 & R^1_2 & R^1_3 \\ 0 & 0 & 0 & 0 & R^2_1 & R^2_2 & R^2_3 \\ 0 & 0 & 0 & 0 & R^3_1 & R^3_2 & R^3_3 \end{pmatrix}. \qquad (7)$$

The indices of the G^a_b matrix take the values from 0 to 6. $\Lambda^a_b(a, b = 0, 1, 2, 3)$ are the elements of the Lorentz transformation matrix, and $R^a_b(a, b = 1, 2, 3)$ are the elements of the rotation matrix.

We use the notation $\psi_{c_1,c_2;f_1,f_2}(q)$ for the two-particle meson field, which after quantization describes the processes of creation and annihilation of bound states of quark and antiquark. The field $\psi_{c_1,c_2;f_1,f_2}(q)$ takes the value on which the mixed tensor representations of the $SU_c(3)$ and $SU_f(3)$ groups are realized:

$$\psi'_{c_1,c_2;f_1,f_2}(q) = u^{(c)\dagger}_{c_1 c_3} u^{(c)}_{c_2 c_4} u^{(f)\dagger}_{f_1 f_3} u^{(f)}_{f_2 f_4} \psi_{c_3,c_4;f_3,f_4}(q) . \quad (8)$$

The dynamic equations for the field $\psi_{c_1,c_2;f_1,f_2}(q)$ must be symmetric relative to the transformations (7) and (8). So it may take the following form:

$$L^{(0)} = g^{ab} \frac{\partial \psi^*_{c_1,c_2;f_1,f_2}(q)}{\partial q^a} \frac{\partial \psi_{c_1,c_2;f_1,f_2}(q)}{\partial q^b} - \\ - M^2_\mu \psi^*_{c_1,c_2;f_1,f_2}(q) \psi_{c_1,c_2;f_1,f_2}(q) . \quad (9)$$

Introducing a covariant derivative, we get:

$$\begin{aligned}
L_\mu = g^{ab} \Big(\partial\psi^*_{c_1,c_2;f_1,f_2}(q)/\partial q^a - igA^{(1)}_{a,g_1}(q)\,\psi^*_{c_{21},c_2;f_1,f_2}(q)\,\lambda^{g_1}_{c_1 c_{21}} + \\
+igA^{(2)}_{a,g_1}(q)\,\lambda^{g_1}_{c_{22} c_2}\psi^*_{c_1,c_{22};f_1,f_2}(q)\Big) \times \\
\times \Big(\partial\psi_{c_1,c_2;f_1,f_2}(q)/\partial q^b \ + igA^{(1)}_{b,g_{11}}(q)\,\psi_{c_{31},c_2;f_1,f_2}(q)\,\lambda^{g_{11}}_{c_{31} c_1} - \\
-igA^{(2)}_{b,g_{11}}(q)\,\lambda^{g_{11}}_{c_2 c_{32}}\psi_{c_1,c_{32};f_1,f_2}(q)\Big) - \\
- M^2_\mu \psi^*_{c_1,c_2;f_1,f_2}(q)\,\psi_{c_1,c_2;f_1,f_2}(q)\,.
\end{aligned} \qquad (10)$$

$A^{(1)}_{a,g_1}(q)$ and $A^{(2)}_{a,g_1}(q)$ are the compensating fields. Further, instead of these fields, it would be convenient to consider their linear combinations, similar to Jacobi variables:

$$\begin{aligned}
A^{(+)}_{a,g_1}(q) = \frac{1}{2}\Big(A^{(1)}_{a,g_1}(q) + A^{(2)}_{a,g_1}(q)\Big)\,, \\
A^{(-)}_{a,g_1}(q) = A^{(2)}_{a,g_1}(q) - A^{(1)}_{a,g_1}(q)\,.
\end{aligned} \qquad (11)$$

A local $SU_c(3)$ group representation is given for the values domain of the field functions $\psi_{c_1,c_2;f_1,f_2}(q)$. So this domain may be decomposed into a direct sum of subspaces which are invariant relative to transformations of this representation. And the field $\psi_{c_1,c_2;f_1,f_2}(q)$ can be given as:

$$\psi_{c_1,c_2;f_1,f_2}\left(q\right) = \delta_{c_1 c_2}\psi_{f_1,f_2}\left(q\right), \tag{12}$$

where $\psi_{f_1,f_2}(q)$ are the new field functions for the further dynamical equations, which after quantization should describe the processes of creation and annihilation of mesons. These dynamic equations can be obtained from the Lagrangian that is formed if one substitutes (12) into (10) and takes into account the notation (11). After these transformations, the Lagrangian (10) takes the form:

$$\begin{gathered} L_\mu = 3g^{ab}\left(\partial\psi^*_{f_1,f_2}\left(q\right)/\partial q^a\right)\left(\partial\psi_{f_1,f_2}\left(q\right)/\partial q^b\right) + \\ + 2g^2 g^{ab} A^{(-)}_{a,g_1}\left(q\right) A^{(-)}_{b,g_1}\left(q\right)\psi^*_{f_1,f_2}\left(q\right)\psi_{f_1,f_2}\left(q\right) - 3M^2_\mu\psi^*_{f_1,f_2}\left(q\right)\psi_{f_1,f_2}\left(q\right). \end{gathered} \tag{13}$$

In order to obtain the dynamic equations for a two-gluon field, we consider the simplest tensor that can be formed from single-gluon fields:

$$A_{ab,g_1g_2}(q) = g^2 \left(A^{(-)}_{a,g_1}(q)\, A^{(-)}_{b,g_2}(q) \right), \qquad a, b = 4, 5, 6. \tag{14}$$

Expanding the linear space of tensors $A_{ab,g_1g_2}(q)$ relative to the group (7) into the direct sum of invariant subspaces, we pick a term corresponding to a projection on a scalar subspace:

$$A_{ab,g_1g_2}(q) = -A_{g_1g_2}(q)\, g_{ab} + \ldots \tag{15}$$

Convolving both sides of the equality (15) with the metric tensor g^{ab} we obtain:

$$A_{g_1g_2}(q) = \frac{4}{7} g^2 \sum_{b=4}^{6} \left(A^{(-)}_{b,g_1}(q)\, A^{(-)}_{b,g_2}(q) \right) \tag{16}$$

Next we apply a similar procedure for internal indices:

$$\begin{aligned} A_{g_1 g_2}(q) &= A(q)\,\delta_{g_1 g_2} + \dots, \\ A(q) &= \frac{1}{14} g^2 \sum_{b=4}^{6} \left(A_{b,g_1}^{(-)}(q)\, A_{b,g_1}^{(-)}(q) \right) = \\ &= \frac{1}{14} V(q), \end{aligned} \tag{17}$$

where $V(q)$ is defined by the relation (18), and the summation is performed over the repeated index g_1:

$$V(q) = g^2 \left(\sum_{b=4}^{6} \left(A_{a,g_1}^{(-)}(q) \right)^2 \right), \tag{18}$$

The kinetic part of the Lagrangian for the $A_{g_1 g_2}(q)$ field can be given as:

$$L_G^{(0)} = \frac{1}{2} g^{ab} \frac{\partial A_{g_1 g_2}(q)}{\partial q^a} \frac{\partial A_{g_1 g_2}(q)}{\partial q^b} - \frac{1}{2} M_G^2 A_{g_1 g_2}(q) A_{g_1 g_2}(q) . \qquad (19)$$

Replacing ordinary derivatives by covariant ones, and performing some calculations, we obtain:

$$L_V = \frac{1}{2} g^{ab} \frac{\partial V(q)}{\partial q^a} \frac{\partial V(q)}{\partial q^b} - \frac{3}{2} (V(q))^3 - \frac{1}{2} M_G^2 (V(q))^2 . \qquad (20)$$

Having a Lagrangian for the field $V(q)$, we can obtain a dynamic equation for this field as the Euler-Lagrange equation:

$$-g^{ca}\frac{\partial^2 V(q)}{\partial q^c \partial q^a} - M_G^2 V(q) - \frac{9}{2}(V(q))^2 = 0. \tag{21}$$

We introduce the function $V(X, \vec{y})$ in the form

$$\begin{aligned} V(X,\vec{y}) &= V_0(\vec{y}) + V_1(X,\vec{y}), \\ V_1(X,\vec{y}) &\equiv V(X,\vec{y}) - V_0(\vec{y}), \end{aligned} \tag{22}$$

Then the function $V_0(\vec{y})$ will enter the complete Lagrangian as the potential energy of the interaction of nonrelativistic constituent quarks. At the same time, it will satisfy the equation:

$$4\Delta_{\vec{y}} V_0(\vec{y}) - M_G^2 V_0(\vec{y}) - \frac{9}{2}(V_0(\vec{y}))^2 = 0. \tag{23}$$

Analyzing the properties of the solutions of equation (23), we can obtain information on the potential of the quarks interaction.

Introducing the dimensionless internal coordinates $\vec{r}$, dimensionless glueball mass m_G and dimensionless potential energy $u(\vec{r})$ as follows:

$$\begin{aligned} &\vec{y} = l\vec{r}, M_G = l^{-1} m_G, \\ &V_0(\vec{y}) = V_0(l\vec{r}) = l^{-2} u(\vec{r}). \end{aligned} \tag{24}$$

Then, instead of the equation (23) in the introduced dimensionless variables, we obtain:

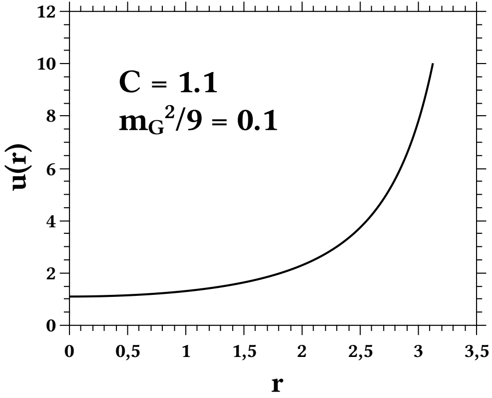
$$4\Delta_{\vec{r}} u(\vec{r}) - m_G^2 u(\vec{r}) - \frac{9}{2}(u(\vec{r}))^2 = 0. \tag{25}$$

Making a standard replacement and applying the boundary conditions:

$$u(r) = \frac{\chi(r)}{r}, \qquad \chi(r)|_{r=0} = 0, \quad \left.\frac{d\chi(r)}{dr}\right|_{r=0} = C, C \in \mathbb{R}, \tag{26}$$

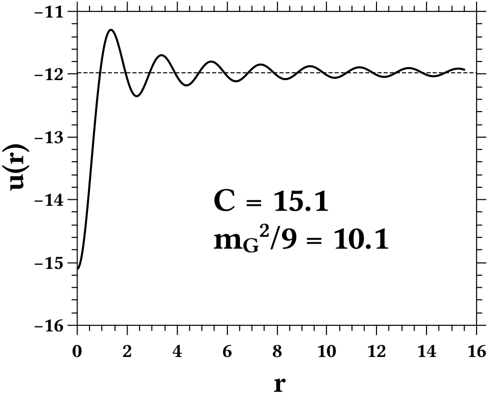
we finally obtain:

$$\frac{d^2\chi(r)}{dr^2} = \frac{9}{8}\frac{\chi(r)\left(\chi(r) + \left(m_G^2/9\right) r\right)}{r}. \tag{27}$$



When r increases, the potential $u(r)$ tends to infinity. I.e. the confinement.

Numerical calculation of the dimensionless inter-quark potential $u(r)$ dependence on dimensionless distance r for $C = 1.1, m_G^2/9 = 0.1$.



When r increases, the potential $u(r)$ tends to some negative constant value. Thus the eigenvalue of the square of internal Hamiltonian will definitely be negative. Since this eigenvalue is a coefficient at the squared field describing the bound state of two gauge bosons, this corresponds to the mechanism of spontaneous symmetry breaking.

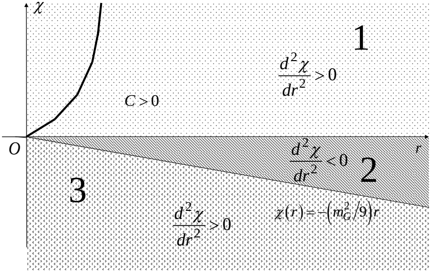
Numerical calculation of the dimensionless inter-quark potential $u(r)$ dependence on dimensionless distance r for $C = 15.1, m_G^2/9 = 10.1$.

Conclusions

- We propose the two-particle fields model. With this model we can describe the confiment of quarks in hadrons and the interaction of hadrons with each other.
- In this model, the energy-momentum conservation law holds true precisely for hadrons (as it is in the experiment), and not for the constituent particles.
- The model contains a dynamic equation which describes the confiment of quarks under certain boundary conditions, and the spontaneous symmetry breaking – under anothers. I.e. it reproduces the basic properties of strong and weak interactions.

Thank you for attention!

$$\hat{\phi}'_a(X') = \hat{U}^+(\Lambda)\,\hat{\phi}_a(X)\,\hat{U}(\Lambda) = \hat{D}_{ab}(\Lambda)\,\hat{\phi}_b\left(X = \hat{\Lambda}^{-1}X'\right) \tag{28}$$



χ
1
$\frac{d^2\chi}{dr^2} > 0$
C > 0
O
r
$\frac{d^2\chi}{dr^2} < 0$
2
3
$\frac{d^2\chi}{dr^2} > 0$
$\chi(r) = -(m_G^2/9)r$