Phase transitions and Bose-Einstein condensation in alpha-nucleon matter

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Recent results -> Phys. Rev. C **99**, 024909 (2019)

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- Equation of state (EoS) of iso-symmetric α-N matter with mean-field interaction $U = U(n_N, n_\alpha)$ $\rightarrow$ Skyrme-like attraction + repulsion
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  - pure nucleon matter $(n_\alpha=0)$ with interaction $\rightarrow$ fixing NN interaction terms
  - pure alpha matter $\rightarrow$ fixing αα interaction terms, using [1,2]
  - cold α-N matter $\rightarrow$ ground state (GS) at T=0 $\rightarrow$ upper limit for αN attraction term
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- Stable (N-like) and metastable (α-like) LGPT
- Conclusions and outlook

[1] J. Clark and T.-W. Wang, Ann. Phys. 40 (1966) 127 $\rightarrow$ microscopic calculations for cold α-matter
[2] L. M. Satarov et al., J. Phys. G44 (2017) 125102 $\rightarrow$ phase diagram of α-matter with Skyrme interaction

Previous studies of strongly interacting matter with clusters

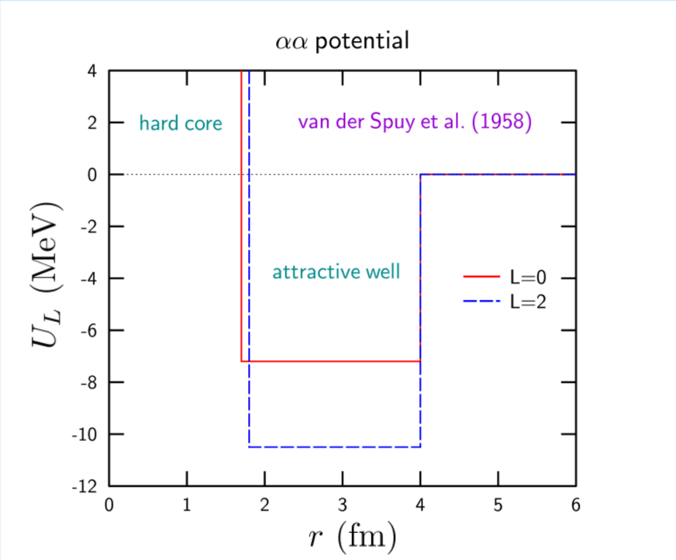
- generalized liquid-drop model → J. Lattimer and F. Swesty, Nucl. Phys. A535 (1991)
- statistical models → A. Botvina and I. Mishustin, Nucl. Phys. A843 (2010);
  A. Buyukcizmeci et al., Nucl. Phys. A907 (2013);
  S. Furusawa and I. Mishustin, Phys. Rev. C97 (2018)
- RMF models → M. Hempel and J. Schaffner-Bielich, Nucl. Phys. A837 (2010);
  J. Pais et al., Phys. Rev. C97 (2018);
  S. Typel, J. Phys. G45 (2018)
- **virial EoS** → C. Horowitz and A. Schwenk, Nucl. Phys. A776 (2006)
  (use information on observed phase shifts of NN, Nα and αα scatterings,
  the model applicable only at small particle densities)
- multi-component van der Waals model → V. Vovchenko et al., Phys. Rev. C96 (2017)
  (Nα and αα attractive interactions are disregarded)
- quasi-particle model → X.-H. Wu et al., J. Low Temp. Phys. 189 (2017) [*]
  (only small densities are considered)

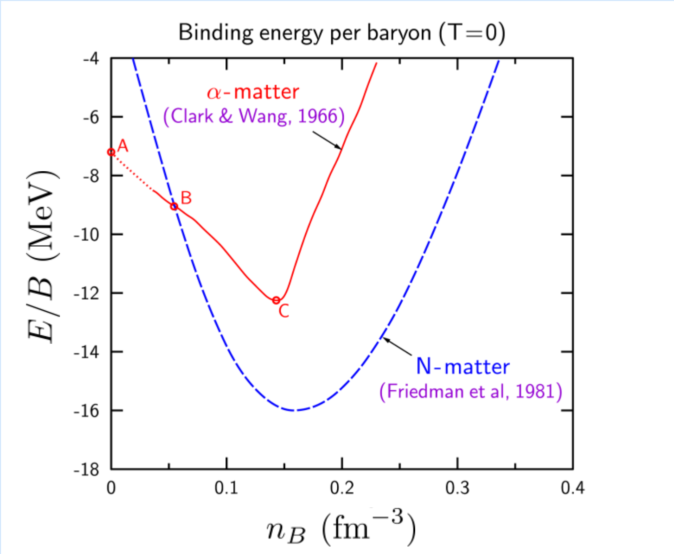
All these models disregard the BEC effects (except [*])

Energy per baryon in cold α-matter

Clark & Wang (1966): variational calculation with phenomenological αα potential, comparison with isospin-symmetric nucleon matter

extracted from αα scattering data





baryon density:

$$n_B = 4n_\alpha = n_N$$

dilute α-matter:

$$\frac{E}{B} = \frac{m_\alpha}{4} - m_N$$

$$\equiv -B_\alpha \simeq -7.1 \text{ MeV}$$

B_α= binding energy of single α-particle

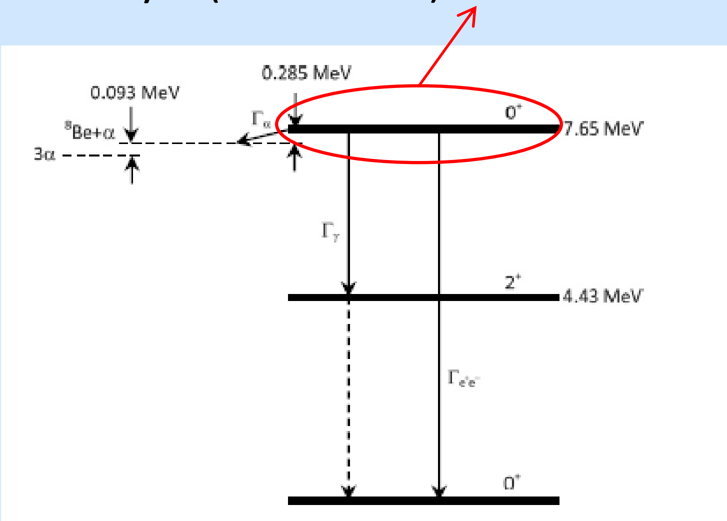
+ Coulomb potential ($4e^2/r$)

- α-matter is energetically favorable at low baryon densities $n_\alpha \lesssim 0.01 \text{ fm}^{-3}$ (section AB)
- ground state (GS) of pure α-matter at $n_\alpha \simeq 0.036 \text{ fm}^{-3}$ (point C)
  It has smaller binding energy $E/B \simeq -12 \text{ MeV}$ as compared to nuclear matter

Systems/processes with enhanced formation of α's

- low-density excited states of light nuclei (^{12}C, ^{16}O, ^{20}Ne, … ← large-size isomers)
- periphery of heavy nuclei
- multifragmentation reactions in heavy-ion collisions
- outer regions of compact stars
- neutron star mergers, supernovae matter (dilute and warm matter)

The Hoyle (2^{nd} excited) state of ^{12}C → key role in stellar nucleosynthesis



two-step process:

1) $\alpha + \alpha \rightarrow {}^{8}Be$
2) $\alpha + {}^{8}Be \rightarrow {}^{12}C^{*} \rightarrow {}^{12}C + \gamma$

Enhanced rate of ^{12}C formation at $T \gtrsim 200$ keV

Röpke et al. (1998): Hoyle state ≈ BEC state of 3α

Our assumptions

- isospin symmetry $(N_p = N_n)$
- homogeneous matter (no surface terms)
- no Coulomb interactions
- no clusters, except α (we neglect d, t, ^{3}He, ^{5}He … and their excited states)
- moderate temperatures $(T \lesssim 30 \text{ MeV})$

  $\rightarrow$ neglect contributions of mesons (π, ρ, K …), other baryons (Δ, N*, Λ …) and antibaryons
  $\rightarrow$ nonrelativistic limit is accurate, since $T \ll m_N \simeq 0.939 \text{ GeV}, m_\alpha \simeq 3.727 \text{ GeV}$
- chemical equilibrium with respect to reactions $\alpha \leftrightarrow 4N$
- mean-field approximation for particle interactions
- no in-medium modification of particles (vacuum masses etc.)

Thermodynamic functions (α-N matter)

- free energy density $F/V = f(T, n_N, n_\alpha)$ → thermodynamic potential in canonical ensemble
- chemical potentials $\mu_i(T, n_N, n_\alpha) = (\partial f / \partial n_i)_T$ $\quad (i = N, \alpha)$ → (1)
- pressure $p = \mu_N n_N + \mu_\alpha n_\alpha - f$ entropy density $s = -(\partial f/\partial T)_{\{n_i\}}$
- energy density $\varepsilon = Ts + f$ baryon density $n_B = n_N + 4n_\alpha = B/V$
- 'mass' fraction of alphas $\chi = 4n_\alpha/n_B$ → one can use (n_B, χ) instead of (n_N,n_α) $\quad (\chi \leqslant 1)$

condition of chemical equilibrium: $\mu_N = \mu_\alpha/4 \equiv \mu_B$ → (2) (μ_B- baryon chem. potential)

substituting (1) into (2) → isotherms of chem. equilibrium in $(n_N, n_\alpha), (n_B, \chi)$ or (μ_B, p) planes

- $p = p(T, \mu_B),\ n_B = (\partial p/\partial \mu_B) \ldots$ (grand canonical ensemble)

stability with respect to fluctuations of partial densities (necessary condition):

$$\det||\partial^2 f/\partial n_i \partial n_j|| = (\partial \mu_N/\partial n_N)_{\{n_\alpha, T\}} (\partial \mu_\alpha/\partial n_\alpha)_{\{n_N, T\}} - (\partial \mu_N/\partial n_\alpha)^2_{\{n_N, T\}} > 0$$

Müller & Serot (1995)

Bose-Einstein condensation (BEC) in ideal boson gas

condition of BEC: $\mu = m \to T < T_{\rm BEC}(n)$ (T_{BEC} = threshold temperature of BEC)

$$n = \begin{cases} n_{\rm id}(T,\mu), & T > T_{\rm BEC}(n) \\ n_{\rm id}(T,m) + n_{\rm bc}, & T < T_{\rm BEC}(n) \end{cases}$$

→ equivalent to $\mu < \mu_{max} = m$ (boson mass)

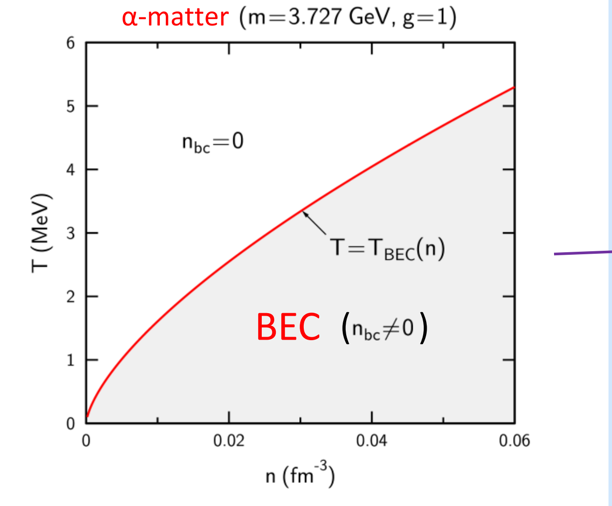
(n_{bc} – density of Bose-condensed particles with zero momenta)

$$n_{\rm id}(T,\mu) = \frac{g}{(2\pi)^3}\int d^3p \left[\exp\left(\frac{\sqrt{m^2+p^2}-\mu}{T}\right)-1\right]^{-1}$$

$$\to n_*(T) \equiv n_{\rm id}(T,m) \simeq g\left(\frac{mT}{2\pi}\right)^{3/2}\zeta(3/2) \quad (T \ll m)$$

at $\mu \to m$ (g – degeneracy factor, $\zeta(3/2) = \sum_{k=1}^{\infty} k^{-3/2} \simeq 2.612$)

$$T_{\rm BEC}(n) \simeq \frac{2\pi}{m}\left[\frac{n}{\zeta(3/2)g}\right]^{2/3} \propto n^{2/3}$$



BEC boundary (line): $n=n_*(T) \to T=T_{BEC}(n)$

BEC occurs at low T or at large n

interacting α-matter in mean-field approximation: $T_{BEC}(n_\alpha)$ is the same as in ideal α-gas with density $n=n_\alpha$

Ideal α-N matter in chemical equilibrium

pressure: $p = p_N^{\rm id}(T,\mu_N) + p_\alpha^{\rm id}(T,\mu_\alpha)$ partial densities: $n_i = \partial p_i^{\rm id}/\partial\mu_i \; (i = N, \alpha)$

(+ n_{bc} term for i=α at $T<T_{BEC}$)

$$p_i^{\rm id}(T,\mu_i) = \frac{g_i}{(2\pi)^3}\int d^3k\,\frac{k^2}{3E_i}\left[\exp\left(\frac{E_i-\mu_i}{T}\right)+\eta_i\right]^{-1}$$

$$\left(E_i = \sqrt{m_i + k^2},\; g_N = 4,\; g_\alpha = 1,\; \eta_N = 1,\; \eta_\alpha = -1\right)$$

chemical equilibrium: $\mu_N = \mu_\alpha/4$ → isotherms $n_\alpha = n_\alpha(T, n_N)$

region of BEC states: $\mu_\alpha = m_\alpha \rightarrow \mu_N = m_\alpha/4 \simeq m_N - B_\alpha$

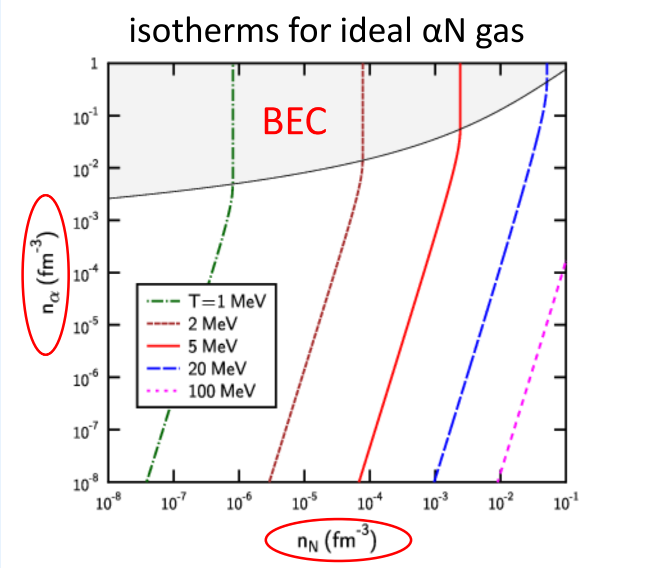
$$T < T_{\rm BEC} \simeq \frac{2\pi}{m_\alpha}\left[\frac{n_\alpha}{\zeta(3/2)g_\alpha}\right]^{2/3}$$

binding energy (per baryon) of single α: $B_\alpha \simeq 7.1$ MeV

$$n_N|_{T<T_{\rm BEC}} = g_N\left(\frac{m_N T}{2\pi}\right)^{3/2}\sum_{k=1}^{\infty}(-1)^{k+1}k^{-3/2}e^{-B_\alpha k/T}$$

→ n_N does not depend on n_α for BEC states

$n_\alpha \gg n_N$ in the BEC region



One-component matter with mean-field interaction

(pure nucleon- or α- matter)

$U = U(n)$ - mean-field potential (depends only on density n, w/o explicit dependence on T)

shift of chemical potential $\mu = \widetilde{\mu} + U(n)$ with respect to the ideal gas

$\widetilde{\mu} = \widetilde{\mu}(T, n)$ - equivalent chemical potential of ideal gas (determined from $n = n_{\text{id}}(T, \widetilde{\mu})$)

pressure $p(T, \mu) = p_{\text{id}}(T, \widetilde{\mu}) + \Delta p(n)$ where $\Delta p(n) = nU(n) - \int_0^n dn_1 U(n_1)$

'excess' pressure

We use Skyrme-like parametrization:

$$U(n) = -2an + \frac{\gamma + 2}{\gamma + 1} b n^{\gamma + 1}$$

attraction short-range repulsion

parameters of interaction $a, b, \gamma > 0$
from fit of ground state (GS) at T=0

$$\Rightarrow \quad \Delta p(n) = -an^2 + bn^{\gamma+2}$$

we compare the results
for soft (γ=1/6) and hard (γ=1)
repulsive interactions

[1] Satarov, Dmitriev, Mishustin, Phys. At. Nucl. 72 (2009) 1390 (iso-symmetric nuclear matter)
[2] Satarov et al, J. Phys. G 44 (2017) 125102 (pure α-matter)

Iso-symmetric nucleon matter with Skyrme interaction

we choose Skyrme parameters a_N, b_N by fitting GS properties of such matter at T=0:

binding energy per baryon $W_N \equiv m_N - \min(\varepsilon/n) = 15.9$ MeV → equivalent to

equilibrium (saturation) density $n = n_0 = 0.15\ \mathrm{fm}^{-3}$ $p = 0,\ \mu = \mu_0 = 923$ MeV

equations for a_N, b_N : $E_F(n_0) + U(n_0) = \mu_0,\ p = p_{\mathrm{id}}(T=0, n_0) + \Delta p(n_0)$

$E_F(n_0)$ ← $\tilde{\mu}$

γ	a_N (GeV fm^3)	b_N (GeV fm$^{3+3\gamma}$)	K_N (MeV)	T_c (MeV)
1	0.40	2.05	372	21.3
1/6	1.17	1.48	198	15.3

T_c → temperature of critical point

$\gamma = 1$ → hard EoS

$\gamma = 1/6$ → soft EoS

⇒ reasonable values of compressibility $K_N = 9(dp/dn)_{\mathrm{GS}} = 200 - 240$ MeV for soft Skyrme repulsion (γ=1/6)

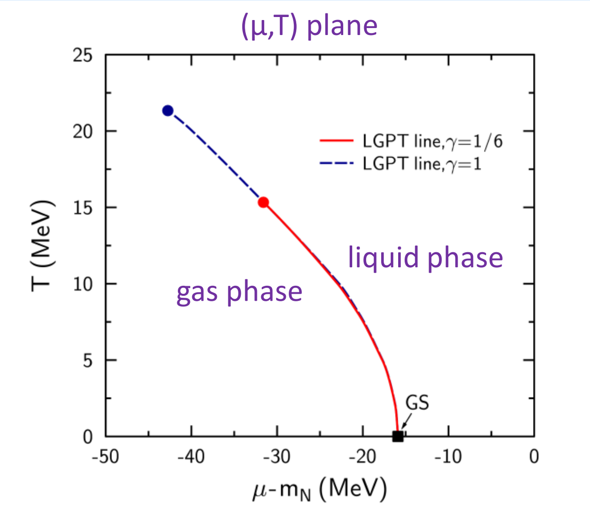
Phase diagram of iso-symmetric nucleon matter

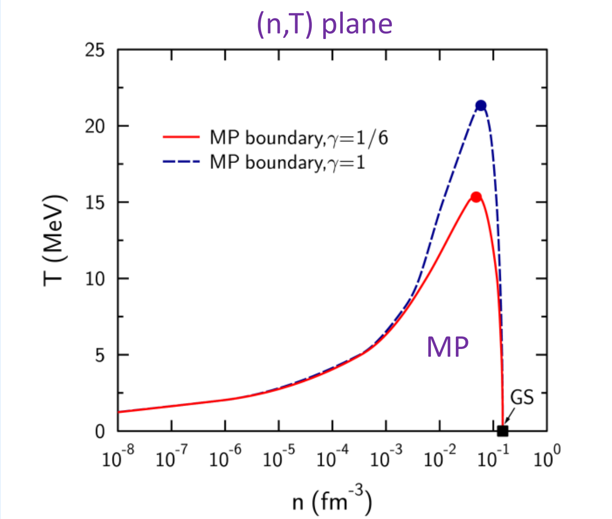
first-order liquid-gas phase transition (LGPT) $\rightarrow$ formation of mixed phase (MP) with coexisting gas ($n=n_g$) and liquid ($n=n_l$) domains ($n_g < n_l$)

exists for any a_N, $b_N > 0$ at $T \leqslant T_{\max} \equiv T_c$

Gibbs conditions of phase equilibrium: $p(T, n_g) = p(T, n_l),\ \ \mu(T, n_g) = \mu(T, n_l)$

$\rightarrow$ boundaries of MP in (n,T) plane ('binodals')





dots: critical points

$(\partial p/\partial n)_T = 0$

$(\partial^2 p/\partial n^2)_T = 0$

squares: ground state

$\varepsilon/n = \min$

$\Rightarrow$ temperature of critical point increases with γ

Pure α matter

Clark & Wong (1966) calculated characteristics of GS (T=0) using phenomenological αα-potentials:

binding energy per baryon $W_\alpha \simeq m_N - \min(\varepsilon_\alpha/n_B) \simeq 12\ \mathrm{MeV}$ ($n_B=4n_\alpha$)

density of GS $n_\alpha = n_{\alpha 0} \simeq 0.036\ \mathrm{fm}^{-3}$ all α's in GS are in BEC state with zero pressure

using $\widetilde{\mu}_\alpha = m_\alpha = 4(m_N - B_\alpha),\ p_\alpha = p_\alpha^{\mathrm{id}} = 0$ one gets

$$\mu_\alpha = m_\alpha + U(n_{\alpha 0}) = 4(m_N - W_\alpha),\ \Delta p_\alpha(n_{\alpha 0}) = 0$$

$$a_\alpha = b_\alpha n_{\alpha 0}^\gamma = \frac{4(\gamma+1)}{\gamma n_{\alpha 0}}(W_\alpha - B_\alpha)$$

(analytic relations for Skyrme parameters a_α, b_α)

γ	$a_\alpha\ (\mathrm{GeV\,fm}^3)$	$b_\alpha\ (\mathrm{GeV\,fm}^{3+3\gamma})$	$K_\alpha(\mathrm{MeV})$	$T_c(\mathrm{MeV})$
1	1.09	30.4	354	13.7
1/6	3.83	6.67	207	10.2

smaller critical temperatures as compared to nucleon matter (at the same γ)

Phase diagram of α matter

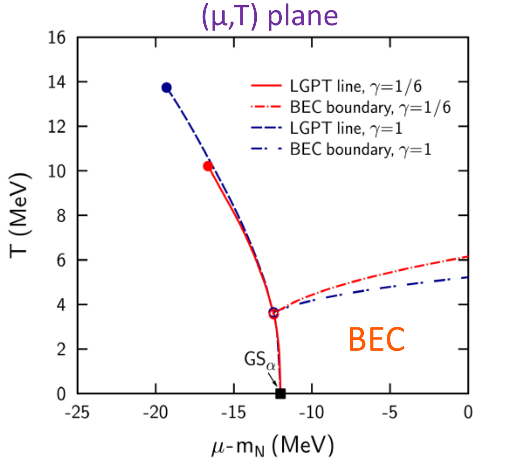
Satarov et al., J. Phys. G44 (2017) 125102 → simultaneous description of LGPT and BEC in pure α matter

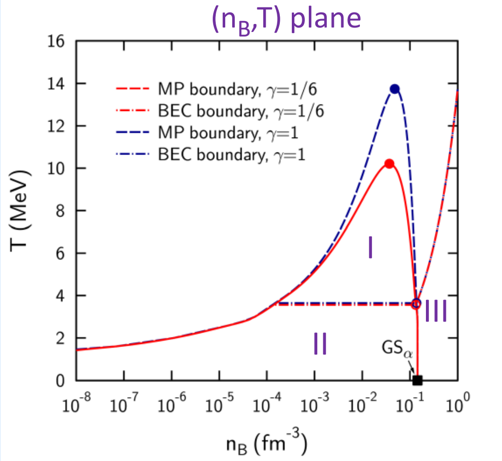
condition of BEC: $$\widetilde{\mu}_\alpha(T,n_\alpha) = m_\alpha \to T_{\rm BEC} \simeq \frac{2\pi}{m_\alpha}\left[\frac{n_\alpha}{\zeta(3/2)g_\alpha}\right]^{2/3}$$ (in the MP region $n_\alpha \to n_{\alpha l}$)

➡ BEC boundary in the (n_α,T) plane is not sensitive to interaction (in the mean-field appr.)

triple point (TP): crossing of BEC line with MP boundary $T_{\rm TP} \simeq 3.6$ MeV (for γ=1/6, 1)

➡ we obtain phase diagrams similar to those observed for atomic ^{4}He





full dots: critical points

open dots: triple points

squares: ground state

$$\mu = \mu_\alpha/4$$
$$n_B = 4n_\alpha$$

region II (MP states with $T<T_{\rm TP}$): gas domains w/o BEC + liquid domains with BEC

Skyrme-like interaction for α-N binary mixture

generalized Skyrme parametrization of excess pressure:

$$\Delta p(n_N,n_\alpha) = p - p_N^{\rm id}(T,n_N) - p_\alpha^{\rm id}(T,n_\alpha) = -\sum_{i,j} a_{ij} n_i n_j + \left(\sum_i B_i n_i\right)^{\gamma+2} \qquad (i,j=N,\alpha)$$

we assume $a_{ii}=a_i$, $B_i=b_i$ where a_i, b_i are Skyrme coefficients for one-component system of ith particles

$$\Delta p(n_N,n_\alpha) = -(a_N n_N^2 + 2a_{N\alpha} n_N n_\alpha + a_\alpha n_\alpha^2) + b_N (n_N + \xi n_\alpha)^{\gamma+2}$$

cross-term coefficient of attraction ($a_{N\alpha}$)

$$\xi = (b_\alpha/b_N)^{1/(\gamma+2)} \simeq 2.01\ (\gamma=1/6),\ 2.46\ (\gamma=1)$$

excess free energy:

$$\Delta f(n_N,n_\alpha) = f - f_N^{\rm id}(T,n_N) - f_\alpha^{\rm id}(T,n_\alpha) = \int_0^1 \frac{d\lambda}{\lambda^2}\Delta p(\lambda n_N,\lambda n_\alpha) \to U_i \equiv \mu_i - \widetilde{\mu}_i = \frac{\partial \Delta f}{\partial n_i}$$

chemical potentials:

$$\mu_N = \widetilde{\mu}_N(T,n_N) - 2(a_N n_N + a_{N\alpha} n_\alpha) + \frac{\gamma+2}{\gamma+1} b_N (n_N + \xi n_\alpha)^{\gamma+1}$$

$$\mu_\alpha = \widetilde{\mu}_\alpha(T,n_\alpha) - 2(a_{N\alpha} n_N + a_\alpha n_\alpha) + \frac{\gamma+2}{\gamma+1} b_N \xi (n_N + \xi n_\alpha)^{\gamma+1}$$

substituting into $\mu_N=\mu_\alpha/4$ → isotherms of chemical equilibrium in (n_N, n_α) plane

below we assume γ=1/6 and study sensitivity of results to cross-term coefficient $a_{N\alpha}$

the only unknown model parameter

Ground state of α-N matter at T=0

Energy per baryon (EPB) as function of n_N, n_α for different α-N couplings ($a_{N\alpha}$)

$E/B = \varepsilon/n_B - m_N \quad (n_B = n_N + 4n_\alpha)$ at T=0 → $\varepsilon = \varepsilon_N^{\rm id}(n_N) + m_\alpha n_\alpha + \Delta f(n_N, n_\alpha)$

➡ the EPB surface changes qualitatively at

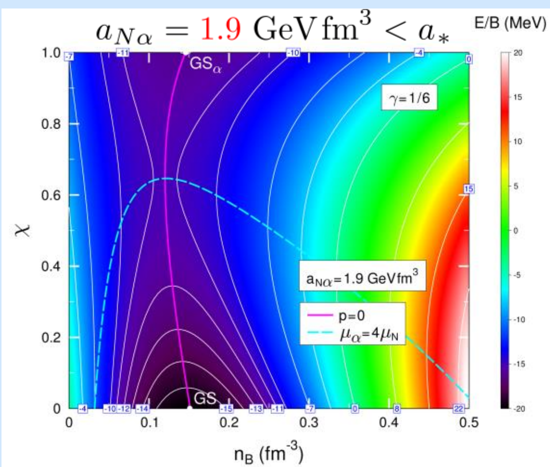
$$\boldsymbol{a_{N\alpha} = a_*} = \frac{1}{2}\left(\frac{m_\alpha - 4\mu_0}{n_0} + \frac{\gamma+2}{\gamma+1} b_N \xi n_0^\gamma\right) \simeq 2.12\ {\rm GeV\,fm^3} \quad (\gamma = 1/6)$$

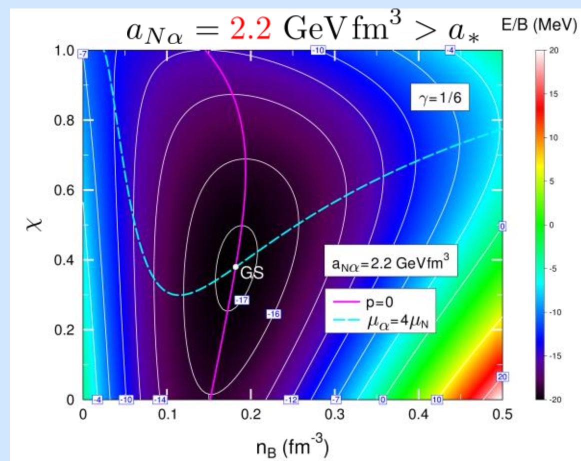
$a_{N\alpha}=a_*$ is found by solving

$$\mu_\alpha = m_\alpha + U_\alpha(n_N = n_0, n_\alpha = 0) = 4\mu_0$$
$$\mu_0 = 923\ {\rm MeV},\ n_0 = 0.15\ {\rm fm}^{-3}$$

$a_{N\alpha} < a_*$ ➡ **two** local **minima** of EPB: 1) stable, without α's + 2) metastable (less bound) without nucleons, separated by energetic barrier → GS without α's (purely nucleonic state with χ=0)

$a_{N\alpha} > a_*$ ➡ **one** local **minimum** of EPB → GS with nonzero fraction of α's





GS: $n_\alpha = 0$ for $a_{N\alpha} < a_*$
$n_\alpha \neq 0$ for $a_{N\alpha} > a_*$

→ n_α and |E/B| increase with $a_{N\alpha}$ (at large $a_{N\alpha}$)

➡ further on we assume that $a_{N\alpha} < a_*$

Choice of model parameters

Coefficients of Skyrme interactions:

γ	$a_N\left(\mathrm{GeV\,fm}^3\right)$	$b_N\left(\mathrm{GeV\,fm}^{3.5}\right)$	$a_\alpha\left(\mathrm{GeV\,fm}^3\right)$	$b_\alpha\left(\mathrm{GeV\,fm}^{3.5}\right)$
1/6	1.17	1.48	3.83	6.67

The only unknown parameter ⇒ $a_{N\alpha}$ (coefficient of Nα attraction)

We assume subcritical values: $a_{N\alpha} < a_* = 2.12\ \mathrm{GeV\,fm}^3$

To study sensitivity to $a_{N\alpha}$ we choose two options:

$a_{N\alpha} = 1\ \mathrm{GeV\,fm}^3$ (set A) and $a_{N\alpha} = 1.9\ \mathrm{GeV\,fm}^3$ (set B)

Comparison with virial EoS (Horowitz et al., 2006) → set B is preferrable

EoS of α-N matter (numerical scheme)

simultaneously solving the equations:

$\mu_N = \widetilde{\mu}_N(T, n_N) + U_N(n_N, n_\alpha)$ (U_i - mean-filed potentials, $\widetilde{\mu}_i$ – chemical pot. of ideal gas, i=N,α)

$$\mu_\alpha = \widetilde{\mu}_\alpha + U_\alpha(n_N, n_\alpha),\ \widetilde{\mu}_\alpha = \begin{cases} \widetilde{\mu}_\alpha(T, n_\alpha), & n_\alpha < n_*(T) \\ m_\alpha, & n_\alpha > n_*(T) \end{cases}$$

→ outside BEC region

→ inside BEC region

$$n_*(T) \equiv g_\alpha \left(\frac{m_\alpha T}{2\pi}\right)^{3/2} \zeta(3/2)$$

$\mu_N = \mu_\alpha/4\ (\equiv \mu)$ → condition of chemical equilibrium

we get $n_N,\ \mu,\ p = p_N^{\rm id} + p_\alpha^{\rm id} + \Delta p$ as functions of $n_\alpha,\ T$

⇒ isotherms in $(n_N, n_\alpha),\ (\mu, p)$ planes

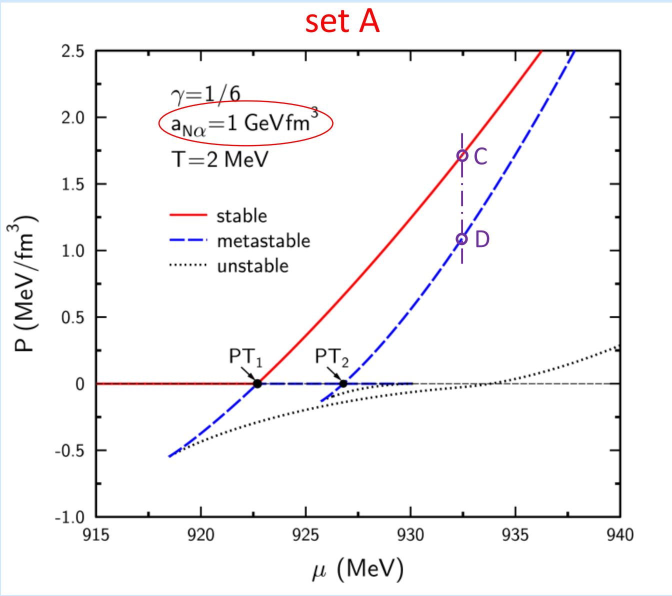
in general, there are several solutions at given T: (in (μ,p) plane) → LGPT

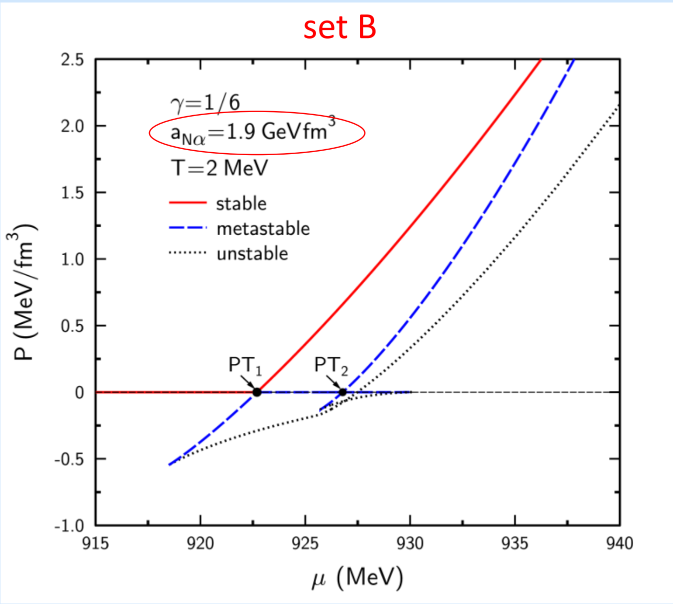
- unstable (spinodal) states $(\det||\partial^2 f/\partial n_i \partial n_j|| < 0)$
- stable/metastable states with larger/smaller pressure at the same μ

using Gibbs conditions (for intersecting branches of p(T, μ))

⇒ we get two LGPTs: stable (with smaller fraction of α) and metastable

Interacting α-N matter: isotherms T=2 MeV in (μ,p) plane





μ - baryon chem. pot.
p - pressure

at given μ states with largest p are favorable (stable)

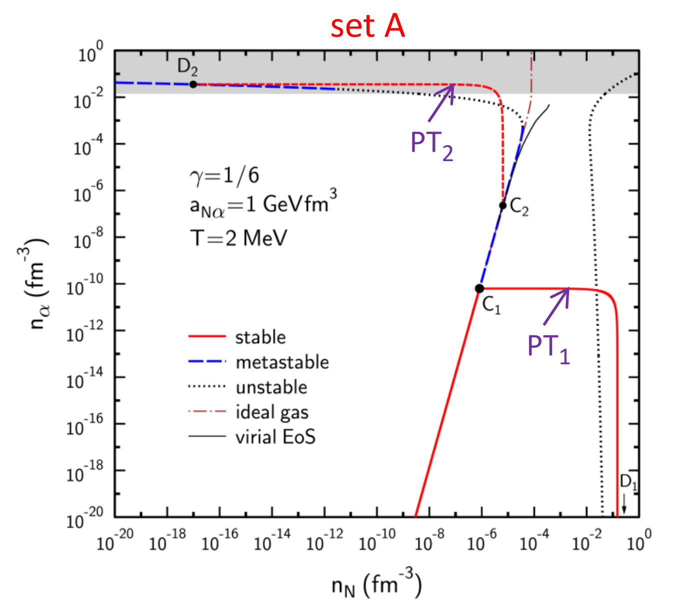
e.g. C → stable state,
D → metastable

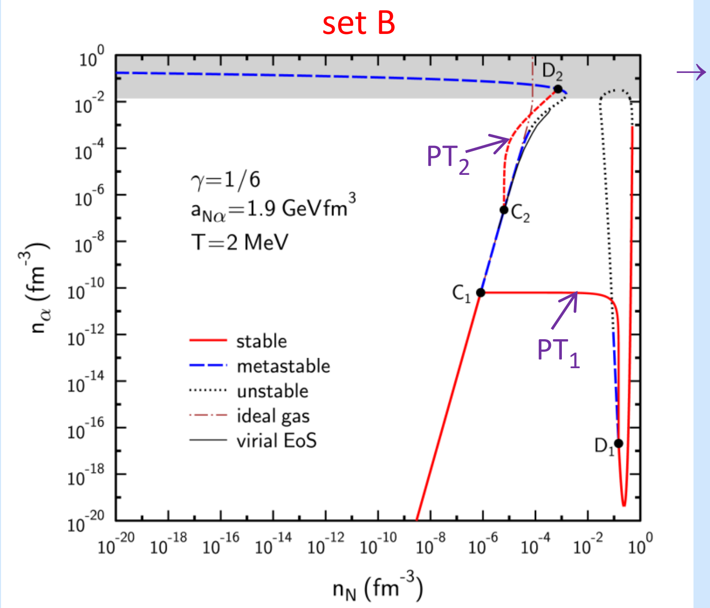
→ jumps of pressure slopes (=n_B) at the LGPT points $PT_{1,2}$

- stable (PT_1) and metastable (PT_2) liquid-gas phase transitions (at T=2 MeV)
- low sensitivity to $a_{N\alpha}$ in the (μ,p) plane

Isotherms T=2 MeV in (n_N, n_α) plane

reasonable values: $n_\alpha \lesssim 0.1 \text{ fm}^{-3}$





→ **shaded domains: BEC region $n > n_*(T)$**

C_1D_1→MP states of PT$_1$ (stable, without BEC)

C_2D_2→MP states of PT$_2$ (metastable, D_2=state with BEC)

virial EoS: Horowitz et al (2006)

- suppression of α's at large nucleon densities → similar to Mott effect
- fractions of α's are small (large) for stable (metastable) LGPT
- states with BEC are metastable
- set B is preferable (closer to virial EoS as compared to set A)

Horowitz et al. (2006)

Phase diagram of α-N matter (stable states)

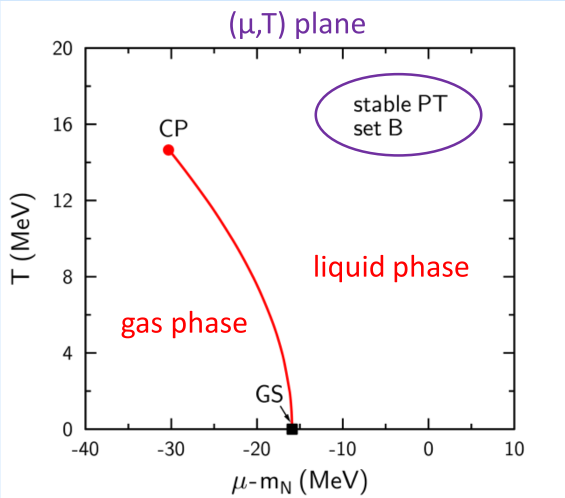
Characteristics of critical point (CP):

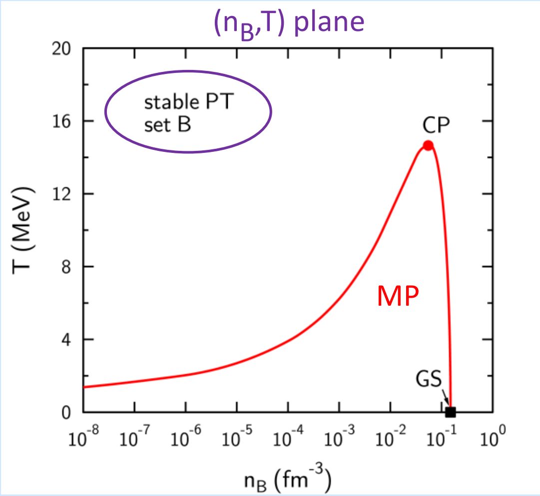
	T_{CP} (MeV)	n_{BCP} (fm^{-3})	χ_{CP}
set A	15.4	$4.8\cdot10^{-2}$	$2.5\cdot10^{-4}$
set B	14.7	$5.3\cdot10^{-2}$	$6.9\cdot10^{-2}$

→ found from

$$(\partial p/\partial n_B)_T = 0$$

$$(\partial^2 p/\partial n_B^2)_T = 0$$





squares:
ground state (GS)

$$T = 0,\ \varepsilon/n_B = \min$$

parameters of GS coincide with those for pure nucleon matter (χ=0):

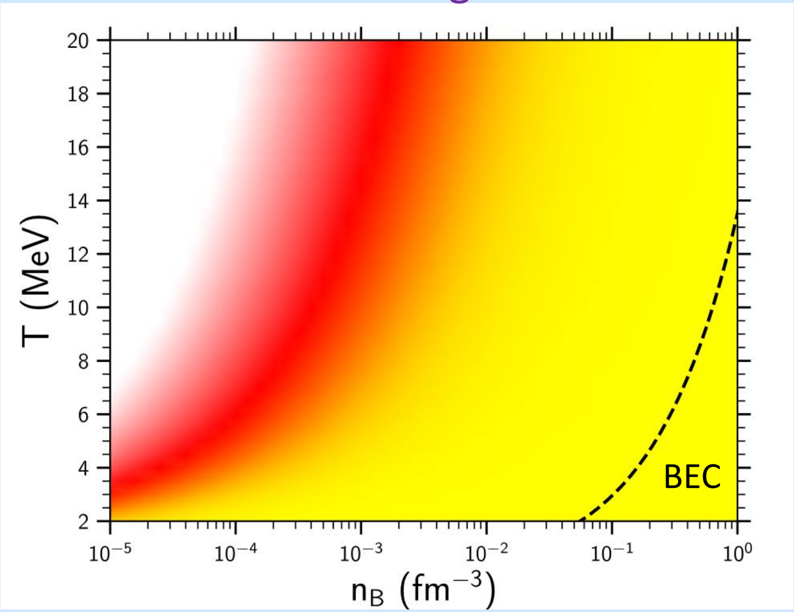
$$\mu = m_N - 15.9\ \text{MeV}$$

$$n_B = 0.15\ \text{fm}^{-3}$$

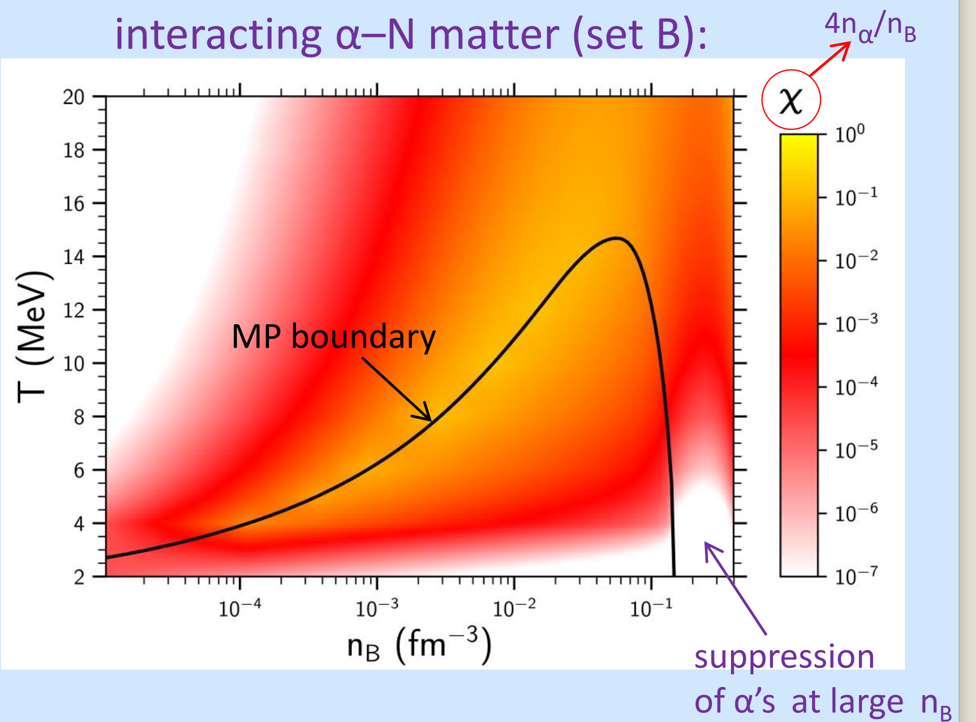
position of critical point (CP) only slightly changes with $a_{N\alpha}$

Fraction of α in (n_B,T) plane (stable states)

ideal α–N gas:



interacting α–N matter (set B):



- strong influence of interaction: non-monotonic density behavior of χ
- maximum values of χ (~10-20%) are reached near the left boundary of MP

(larger fractions can be achieved for metastable states)

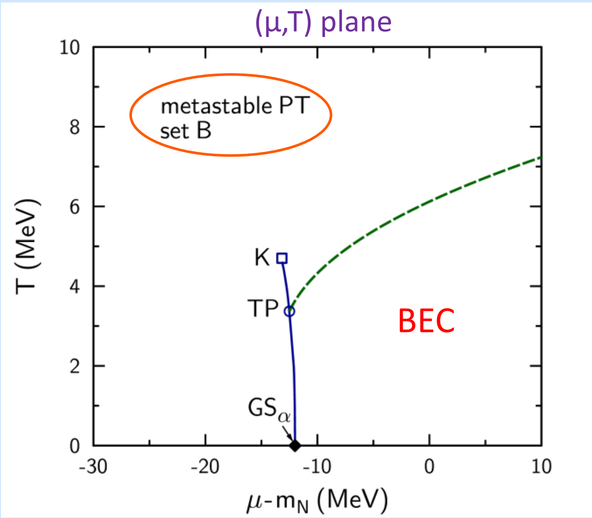
Phase diagram of α-N matter (metastable states)

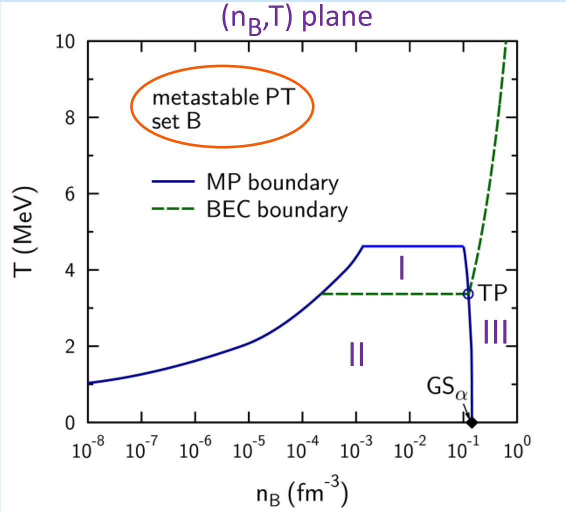
characteristics of metastable PT:

↑ 'remnant' of LGPT in pure α -matter

	T_K (MeV)	n_{BK} (fm^{-3})	χ_K	T_{TP} (MeV)
set A	7.6	$(1.2\text{-}2.6)\cdot 10^{-2}$	0.14–1.0	3.5
set B	4.6	$1.3\cdot 10^{-3}$-0.1	0.46–0.86	3.4

K – end point, TP – triple point (intersection of LGPT and BEC lines)





larger concentration of α's (χ) as compared to stable LGPT

metastable LGPT disappears 'abruptly' at T=TK (with nonzero jump of n_B)

diamonds show GS of pure α-matter

⇒ strong sensitivity to $a_{N\alpha}$

Conclusions

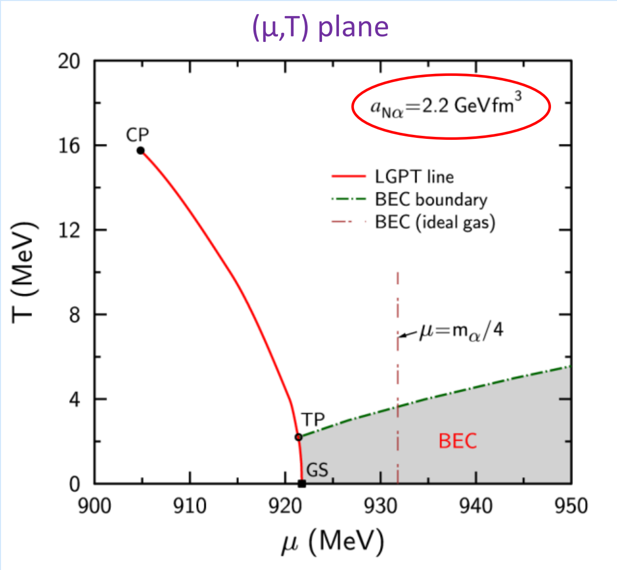
- Simultaneous description of LGPT and BEC in mean-field model
- Strong sensitivity of results to αN attractiive strength
- Two first-order phase transitions (stable and metastable) are found for iso-symmetric matter
- Strong suppression of α-cluster abundance at large nucleon densities
- States with α condensate are metastable

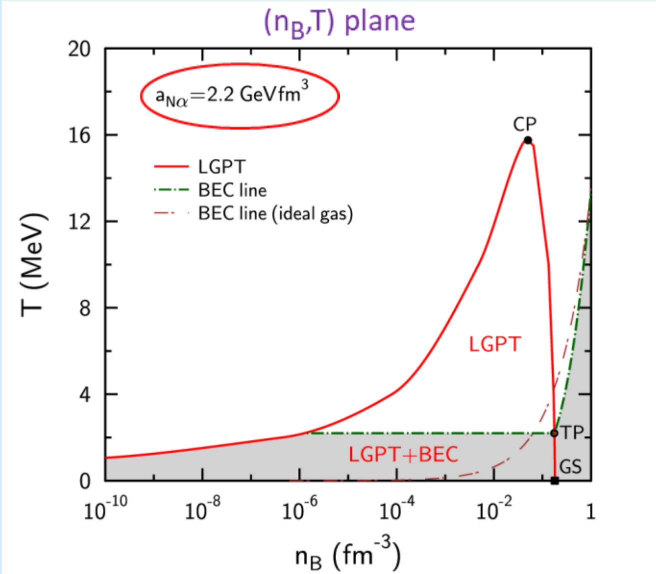
Outlook

- We are going to study the role of α-clusters in asymmetric (neutron star) matter
- Search for metastable LGPT and BEC in heavy-ion collisions: select events with larger fraction of α's

Phase diagram of α-N matter (supercritical $a_{N\alpha}$)

$a_{N\alpha} > 2.12$ GeV fm^3





squares:
ground state (GS)

$n_B \simeq 0.18$ fm^{-3}

$\chi = 4n_\alpha/n_B \simeq 38\%$

$E/B \simeq$ 17.2 MeV

full circles:
critical point (CP)

$T \simeq 15.8$ MeV

$n_B \simeq 0.051$ fm^{-3}

$\chi \simeq 21\%$

- one stable LGPT + stable BEC states (shaded regions) are predicted
- nonzero fraction of α's in ground state

Ground state of α-N matter at T=0

we calculate energy per baryon (EPB) as function of n_N, n_α for different couplings $a_{N\alpha}$

$$E/B = \varepsilon/n_B - m_N \quad (n_B = n_N + 4n_\alpha)$$ at T=0 $\rightarrow$ $\varepsilon = \varepsilon_N^{\rm id}(n_N) + m_\alpha n_\alpha + \Delta f(n_N, n_\alpha)$

➡ the EPB surface changes qualitatively at

$$a_{N\alpha} = a_* = \frac{1}{2}\left(\frac{m_\alpha - 4\mu_0}{n_0} + \frac{\gamma+2}{\gamma+1} b_N \xi n_0^\gamma\right) \simeq 2.12\ \mathrm{GeV\,fm^3} \quad (\gamma = 1/6)$$

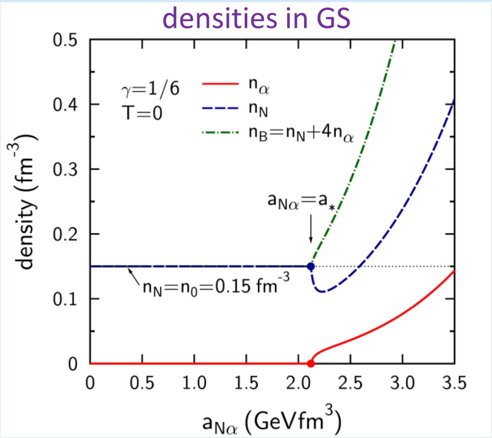
$a_{N\alpha} = a_*$ is found by solving

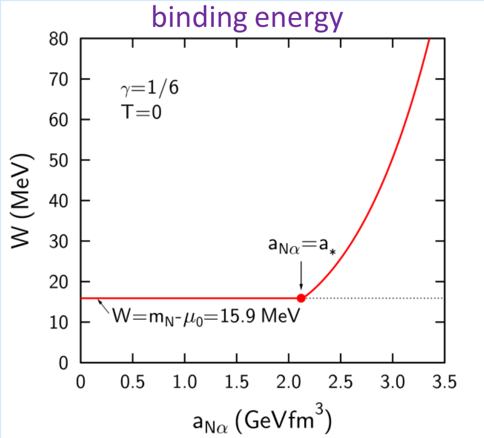
$$\mu_\alpha = m_\alpha + U_\alpha(n_N = n_0, n_\alpha = 0) = 4\mu_0$$

$$\mu_0 = 923\ \mathrm{MeV},\ n_0 = 0.15\ \mathrm{fm}^{-3}$$

$a_{N\alpha} < a_*$ ➡ **two** local **minima** of EPB: 1) stable, without α's + 2) metastable (less bound) without nucleons, separated by energetic barrier $\rightarrow$ GS without α's (i.e. purely nucleonic state)

$a_{N\alpha} > a_*$ ➡ **one** local **minimum** of EPB $\rightarrow$ GS with nonzero fraction of α's





GS: $n_\alpha = 0$ for $a_{N\alpha} < a_*$

$n_\alpha \neq 0$ for $a_{N\alpha} > a_*$

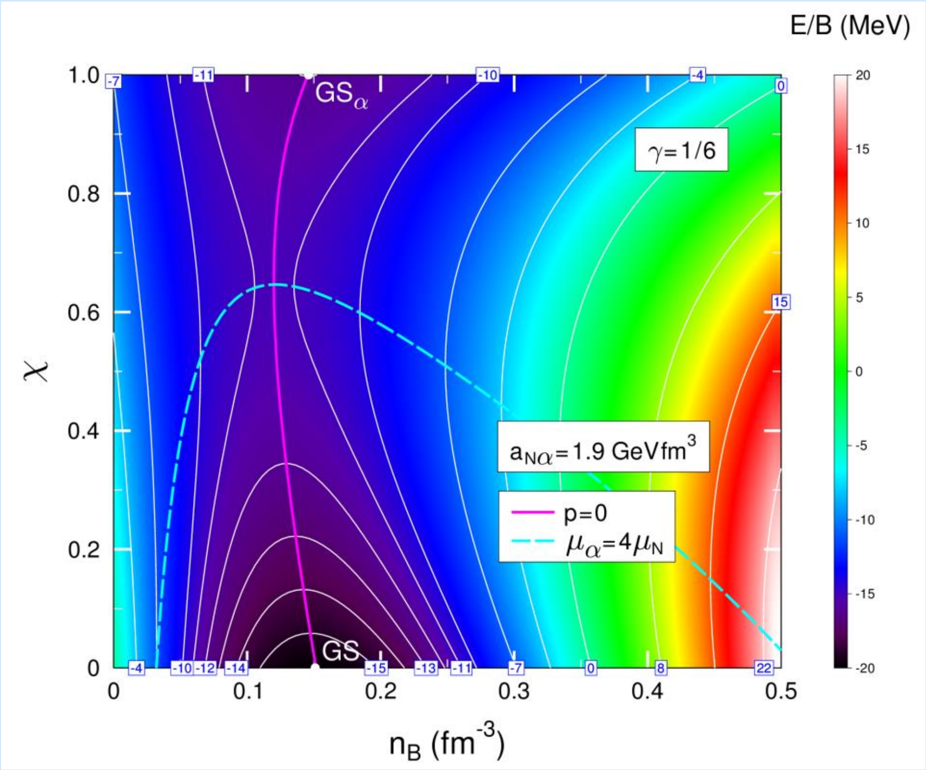
$\rightarrow$ n_α and W increase with $a_{N\alpha}$ (at large $a_{N\alpha}$)

Contours of binding energy in (n_B, χ) plane (T=0, set B)

subthreshold case

$a_{N\alpha} = 1.9\ \mathrm{GeV fm}^3 < a_*$

binding energy per baryon of cold α-N matter $E/B = \varepsilon/n_B - m_N$



$\chi = 4n_\alpha/n_B$

two local minima of E/B:

true GS (absolute minimum)

$$\frac{E}{B} = -15.9\ \mathrm{MeV}, \quad n_B = 0.15\ \mathrm{fm}^{-3}, \quad \chi = 0$$

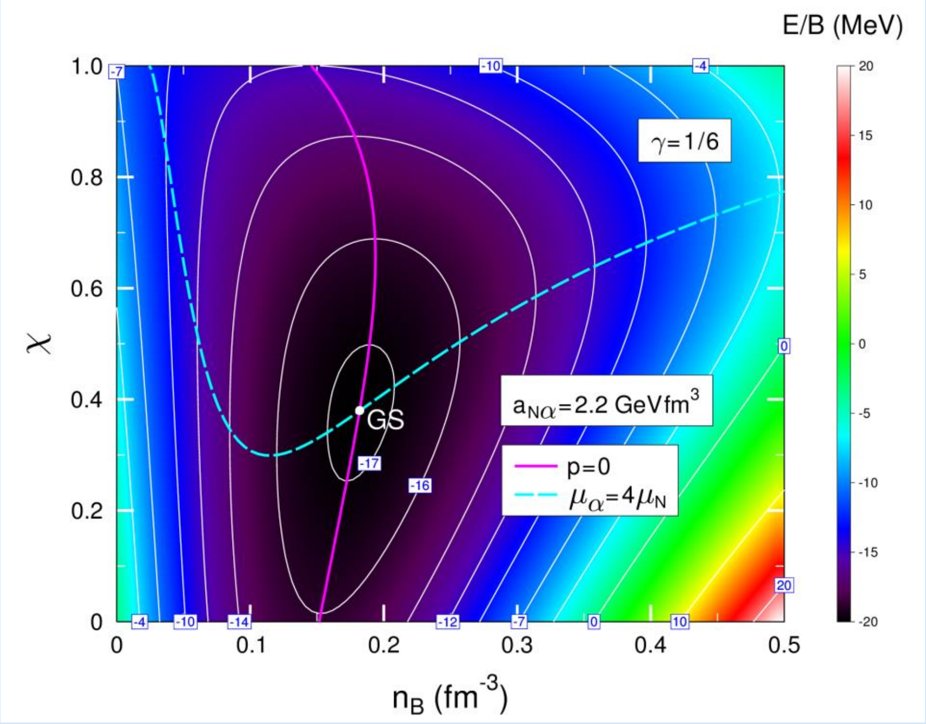
and metastable GS$_\alpha$ (shallower minimum)

$$\frac{E}{B} = -12\ \mathrm{MeV}, \quad n_B = 0.144\ \mathrm{fm}^{-3}, \quad \chi = 1$$

the line $\mu_\alpha=4\mu_N$ corresponds to energetic barrier (E/B=max), separating two minima of E/B

Contours of binding energy (T=0, $a_{N\alpha}$=2.2 GeV fm^3)

above threshold a_*



qualitatively different E/B surface with single local minimum:

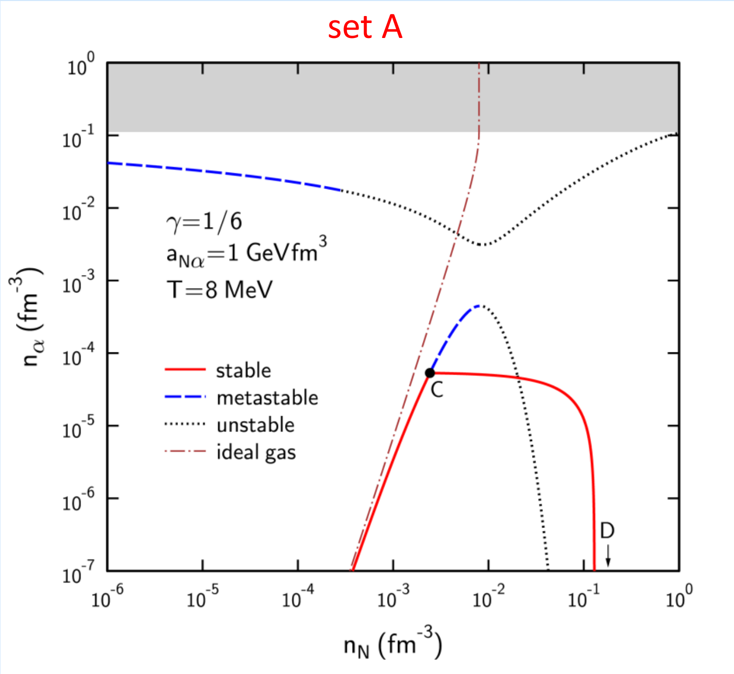
$$\frac{E}{B} \simeq -17.2 \text{ MeV}, \quad n_B \simeq 0.18 \text{ fm}^{-3}, \quad \chi \simeq 0.38$$

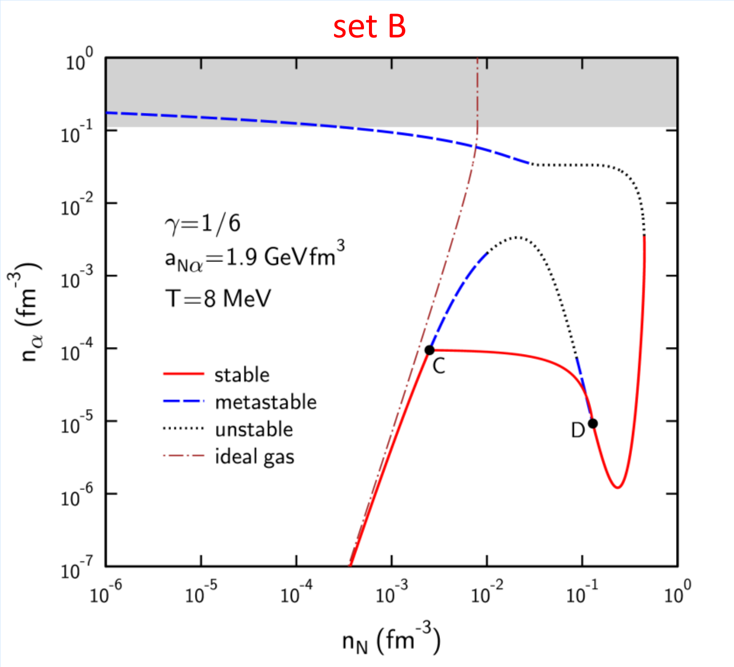
this new GS is stronger bound and has nonzero α fraction as compared to normal nuclear matter

the condition $\mu_\alpha=4\mu_N$ holds along the line E/B=min

Presumably, subthreshold values $a_{N\alpha} < a_* \simeq 2.12 \text{ GeV fm}^3$ are more reasonable

Isotherms T=8 MeV in (n_N,n_α) plane





only one (stable) LGPT is predicted for T=8 MeV