Entanglement Entropy in High Energy Collisions

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Q: How do the systems produced in high enegry collisions thermalise/"hydronise"?

Exploratory new approach based on first principles QM w/o assuming any form of thermalisation.
Similar question (formation of black holes/entropy from collisions of pure states)

The wave-function of an incoming projectile hadron (eigenstate of QCD LC Hamiltonian) has many entangled gluon Fock components

After collision with a target, some of these gluons will be decohered and observed as final state particles

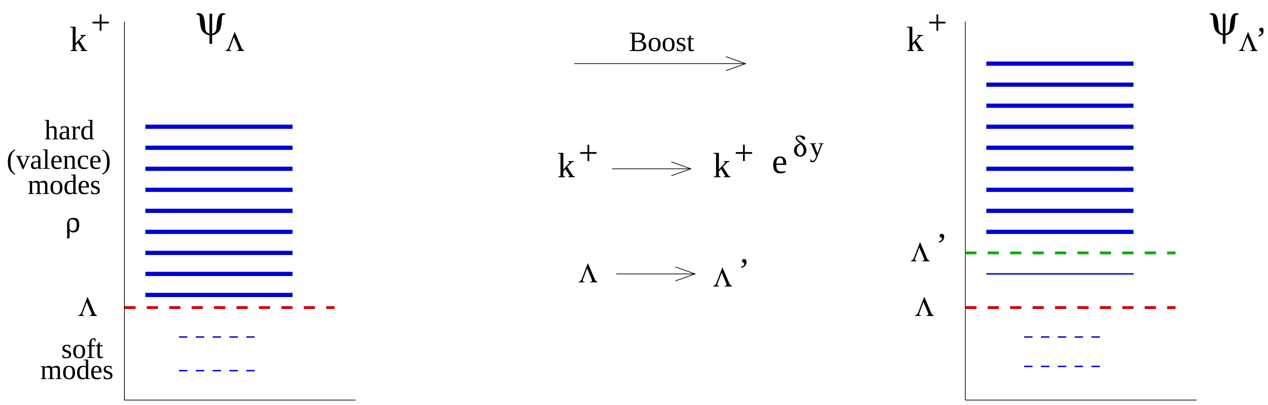
Initially, these gluons had an entanglement entropy (with respect to other Fock components in the incoming wave-function).

This entropy gets "released" by the scattering in the form of final state entropy

We hope to understand the dynamical mechanism of entropy production

We also hope to be able to shad light on the question to what extent the final state of the collision resembles a thermal state.

Light Cone Wave Function



Hard particles with $k^+ > \Lambda$ scatter of the target. Hard (valence) modes are described by the valence density $\rho(x_\perp)$.

The boost opens a window above Λ with the width $\sim \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \sim 1$; gluon emission $\sim \alpha_s \rho$, LO = one gluon, NLO = 2 gluons

In the dense limit $\rho \sim 1/\alpha_s$, we have $\alpha_s \rho \sim 1$, and the number of gluons in the window can be very large.

Denote soft glue creation and annihilation operators as $\mathbf{a}$ and $\mathbf{a}^\dagger$.

$$\mathbf{H_{QCD} \;=\; H(\rho,\, a,\, a^\dagger) \;=\; H_v[\rho] \,+\, H_0[a] \,+\, g\,''\rho\, a'' \,+\, g\,''aaa'' \,+\, g^2\,''aaaa''}$$

The dressed incoming hadron light cone wave function

$$|\boldsymbol{\Psi}_{\mathbf{in}}\rangle_{\mathbf{Y}} \;=\; \boldsymbol{\Omega}_{\mathbf{Y}}(\rho,\, \mathbf{a})\, |\rho\rangle_{\mathbf{valence}} \,\otimes\, |\mathbf{0_a}\rangle_{\mathbf{soft}}$$

$|\Psi_{in}\rangle_Y$ is an eigen-function of the Hamiltonian, $\mathbf{H_{QCD}}\,|\boldsymbol{\Psi}_{\mathbf{in}}\rangle_{\mathbf{Y}} \;=\; \mathbf{E}\,|\boldsymbol{\Psi}_{\mathbf{in}}\rangle_{\mathbf{Y}}$

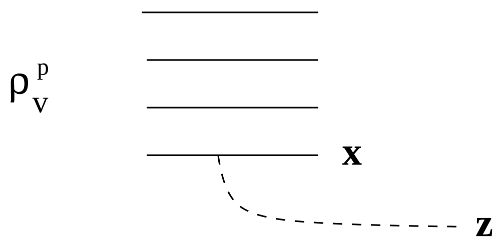
The major challenge is to find $\boldsymbol{\Omega}$ that diagonalises the Hamiltonian

$$\boldsymbol{\Omega}^\dagger\; \mathbf{H}\; \boldsymbol{\Omega} \;=\; \mathbf{H_{diagonal}}$$

LCWF in Dilute Limit

Gluon coherent field operator in the dilute limit

$$\Omega_Y(\rho \to 0) \equiv \mathcal{C}_Y = \mathrm{Exp}\left\{ \mathbf{i} \int d^2z\, b_i^a(z) \int_{e^{Y_0}\Lambda}^{e^{Y}\Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z) \right) \right] \right\}$$



Emission amplitude is given by the Weizsaker-Williams field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(x - z)_i}{(x - z)^2} \rho^a(x)$$

The operator $\mathcal{C}$ dresses the valence charges by a cloud of the WW gluons

Density Matrix of soft modes

The wave function coming into the collision region at time $t = 0$

$$|\mathbf{\Psi_{in}}\rangle \; = \; \mathbf{\Omega_Y} \, |\rho, \mathbf{0_a}\rangle \; .$$

Define the reduced density matrix of soft modes

$$\hat{\rho} \; = \; \int \mathbf{D}\rho \, \mathbf{W}[\rho] \, |\mathbf{\Psi_{in}}\rangle \, \langle \mathbf{\Psi_{in}}|$$

McLerran-Venugopalan model for dense systems:

$$\mathbf{W^{MV}}[\rho] \; = \; \mathcal{N} \, \exp\left[-\int_k \frac{\mathbf{1}}{\mathbf{2}\mu^2(\mathbf{k})} \rho(\mathbf{k})\rho(-\mathbf{k})\right] \qquad \text{where } \mathbf{Q_s^2} = \frac{\mathbf{g}^4}{\pi}\mu^2$$

Q_s denotes Saturation Scale – a typical semi-hard transverse momentum in a dense nucleus. At the same time Q_s measures average gluon density.

"Dilute/Dense mix approximation": $\Omega = \mathcal{C}$ and $W = W^{MV}$ (Gaussian), $\hat{\rho}$ is computable analytically

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML, arXiv:1503.07126

$$\hat{\rho} = \sum_n \frac{1}{n!} e^{-\frac{1}{2}\phi_i M_{ij}\phi_j} \left[\prod_{m=1}^{n} M_{i_m j_m} \phi_{i_m} |0\rangle \langle 0| \phi_{j_m} \right] e^{-\frac{1}{2}\phi_i M_{ij}\phi_j}$$

Here we have introduced compact notations:

$$\phi_{\mathbf{i}} \equiv \left[\mathbf{a}_{\mathbf{i}}^{\dagger\mathbf{a}}(\mathbf{x}) + \mathbf{a}_{\mathbf{i}}^{\mathbf{a}}(\mathbf{x}) \right] \; ; \qquad \mathbf{M}_{\mathbf{ij}} \equiv \frac{\mathbf{g}^2}{4\pi^2} \int_{\mathbf{u},\mathbf{v}} \mu^2(\mathbf{u},\mathbf{v}) \frac{(\mathbf{x}-\mathbf{u})_{\mathbf{i}}}{(\mathbf{x}-\mathbf{u})^2} \frac{(\mathbf{y}-\mathbf{v})_{\mathbf{j}}}{(\mathbf{y}-\mathbf{v})^2} \delta^{\mathbf{ab}}$$

M **bears two polarisation, colour, and coordinate indices, collectively denoted as $\{ij\}$.**

Entanglement Entropy

Alex Kovner and ML, arXiv:1506.05394

Entanglement Entropy of soft modes

$$\sigma^{\mathbf{E}} \;=\; -\,\mathbf{tr}[\hat{\rho}\ln\hat{\rho}]$$

How to calculate $\ln$? **The "replica trick":**

$$\ln\hat{\rho} = \lim_{\epsilon\to\mathbf{0}}\frac{\mathbf{1}}{\epsilon}\left(\hat{\rho}^{\epsilon}-\mathbf{1}\right)$$

Calculate ρ^N and take $N\to 0$. N copies of the field - replicas.

The result

$$\sigma^{\mathbf{E}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tr}\left\{\ln\frac{\mathbf{M}}{\pi}+\sqrt{\mathbf{1}+\frac{\mathbf{4M}}{\pi}}\ln\left[\mathbf{1}+\frac{\pi}{\mathbf{2M}}\left(\mathbf{1}+\sqrt{\mathbf{1}+\frac{\mathbf{4M}}{\pi}}\right)\right]\right\}$$

Translationally invariant limit ($\mu = const$):

$$\mathbf{M^{ab}_{ij}(p) = g^2 \mu^2 \frac{p_i p_j}{p^4} \delta^{ab}}$$

For small M, or the UV contribution (formally UV divergent)

$$\sigma^{E}_{UV} = \mathbf{tr}\left[\frac{\mathbf{M}}{\pi} \ln \frac{\pi \mathbf{e}}{\mathbf{M}}\right] = \frac{\mathbf{Q_s^2}}{4\pi \mathbf{g^2}}(\mathbf{N_c^2} - 1)\mathbf{S}\left[\ln^2 \frac{\mathbf{g^2 \Lambda^2_{UV}}}{\mathbf{Q_s^2}} + \ln \frac{\mathbf{g^2 \Lambda^2_{UV}}}{\mathbf{Q_s^2}}\right]$$

$\mathbf{\Lambda_{UV} \sim M e^{Y_0} \gg M}$, where eikonal approximation breaks down

The large M, IR contribution is

$$\sigma^{E}_{IR} \simeq \frac{1}{2} \mathbf{tr}[\ln \frac{\mathbf{e^2 M}}{\pi}] == \frac{3(\mathbf{N_c^2} - 1)}{8\pi \mathbf{g^2}} \mathbf{S Q_s^2}$$

But not quite what we would like to know.

We need to address scattering process

Density Matrix of produced soft gluons

The wave function coming into the collision region at time $t = 0$

$$|\mathbf{\Psi_{in}}\rangle \;=\; \mathbf{\Omega_Y}\,|\rho, \mathbf{0_a}\rangle\;.$$

The system emerges from the collision region with the wave function

$$|\mathbf{\Psi_{out}}\rangle \;=\; \hat{\mathbf{S}}\,\mathbf{\Omega_Y}\;|\rho, \mathbf{0_a}\rangle\;.$$

Eikonal scattering approximation

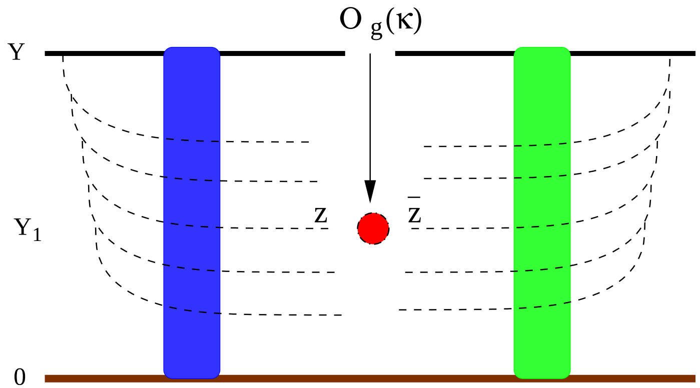
The system keeps evolving after the collision to the asymptotic time $t \to +\infty$, at which point measurements are made

The final state density matrix

$$\hat{\rho}_{\mathbf{P}} \;=\; \int \mathbf{D}\rho\,\mathbf{W}[\rho]\,\mathbf{\Omega_Y^\dagger}\,|\mathbf{\Psi_{out}}\rangle\,\langle\mathbf{\Psi_{out}}|\,\mathbf{\Omega_Y} \;=\; \hat{\rho}[\mathbf{M^P} \;\to\; \mathbf{M}]$$

Here extra Ω corresponds to final state radiation. It could be also viewed as change of basis projecting into eigenstates of free Hamiltonian.

Single inclusive gluon production



The observable

$$\hat{\mathcal{O}}_{\rm g} \;\sim\; \mathbf{a}_{\rm i}^{\dagger\,\rm a}(\mathbf{k})\,\mathbf{a}_{\rm i}^{\rm a}(\mathbf{k})$$

$$\frac{\mathbf{dN}}{\mathbf{d^2kd}\eta} \;=\; \langle\, \mathbf{tr_a}[\hat{\rho}_{\rm p}\,\hat{\mathcal{O}}_{\rm g}]\rangle_{\rm T} \;=\; \int_{\mathbf{x},\mathbf{y}} \mathbf{e}^{\mathbf{ik(x-y)}}\,\langle \mathbf{M}^{\rm P}_{\rm ii}\rangle_{\rm T}$$

$$\mathbf{M}^{\rm P}_{\rm ij} \equiv \mathbf{g}^2 \int_{\mathbf{u},\mathbf{v}} \mu^2(\mathbf{u},\mathbf{v}) \frac{(\mathbf{x}-\mathbf{u})_{\rm i}}{(\mathbf{x}-\mathbf{u})^2}\frac{(\mathbf{y}-\mathbf{v})_{\rm j}}{(\mathbf{y}-\mathbf{v})^2}\,[(\mathbf{S}(\mathbf{u})-\mathbf{S}(\mathbf{x}))(\mathbf{S}^\dagger(\mathbf{v})-\mathbf{S}^\dagger(\mathbf{y}))]^{\rm ab}$$

$\langle\ldots\rangle_T$ corresponds to averaging over target fields S and is equivalent to event-by-event statistical ensemble average

Produced Entropy

$$\sigma^{P} \;=\; -\,\langle\, \mathbf{tr}[\hat{\rho}_{\mathbf{p}}\ \ln \hat{\rho}_{\mathbf{p}}]\,\rangle_{\mathbf{T}}$$

$$\sigma^{P} = \frac{1}{2}\langle \mathbf{tr}\left\{ \ln\frac{\mathbf{M^P}}{\pi} + \sqrt{\mathbf{1} + \frac{\mathbf{4M^P}}{\pi}}\,\ln\left[\mathbf{1} + \frac{\pi}{\mathbf{2M^P}}\left(\mathbf{1} + \sqrt{\mathbf{1} + \frac{\mathbf{4M^P}}{\pi}}\right)\right]\right\}\rangle_{\mathbf{T}}$$

T-averaging is complicated Let expand σ^P around $\bar{\mathrm{M}} \equiv \langle \mathrm{M^P}\rangle_{\mathrm{T}}$ (dilute projectile limit)

$$\bar{\mathbf{M}}_{\mathbf{ij}} \;=\; \delta^{ab}\frac{\mathbf{Q_s^2}\pi}{\mathbf{g^2}}\int_{\mathbf{z}}\frac{(\mathbf{x}-\mathbf{z})_{\mathbf{i}}}{(\mathbf{x}-\mathbf{z})^2}\frac{(\mathbf{y}-\mathbf{z})_{\mathbf{j}}}{(\mathbf{y}-\mathbf{z})^2}[\mathbf{P_A}(\mathbf{x},\mathbf{y}) + \mathbf{1} - \mathbf{P_A}(\mathbf{x},\mathbf{z}) - \mathbf{P_A}(\mathbf{z},\mathbf{y})]$$

$P_A(x,y) \equiv \langle tr[S(x)S^{\dagger}(y)]\rangle_T$ - S-matrix of an adjoint dipole

$\bar{M}$ is *almost* single inclusive gluon, but it is not summed over ij

$$\sigma^{\mathbf{P}} = \mathbf{tr}\left[\frac{\bar{\mathbf{M}}}{\pi}\ln\frac{\pi\mathbf{e}}{\bar{\mathbf{M}}}\right] - \frac{\mathbf{1}}{\mathbf{2}\pi}\mathbf{tr}\left[\left\{\langle(\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\ (\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\rangle_{\mathbf{T}}\right\}\bar{\mathbf{M}}^{-1}\right]\ldots.$$

First term is $almost$ $-n\ln n$, where n is a multiplicity per unit rapidity ($dN/d\eta$)

it depends on the production probabilities of longitudinally and transversely (with respect to the direction of their transverse momentum) polarized gluons separately

Second term - $almost$ correlated part of double inclusive gluon production.

Correlations between gluons decrease entropy of the produced state.

For a parametrically large number of produced particles ($\alpha_s dN/d\eta \sim 1$), the entropy is parametrically of order $1/\alpha_s$

"Temperature" of produced system

We can naturally define temperature through:

$$\mathbf{T^{-1} \; = \; \frac{d\sigma}{dE_T}}$$

$$\mathbf{E_\perp \;\propto\; \int d^2k\, |k|\, M^P(k) \;\propto\; (N_c^2 - 1)\, S\, \frac{Q_P^2}{g^2} Q_T}$$

Keeping only mean field term in the entropy:

$$\mathbf{T \; = \; \frac{\pi}{2} \langle k_T \rangle}$$

.

$$\mathbf{\langle k_T \rangle \; = \; E_\perp / N_{total}} \qquad\qquad \mathbf{N_{total} \; = \; \int d^2k\, M^P(k)}$$

Summary/Outlook

- What I reported is just a pilot project on "Quasi-Thermodynamics"

- Energy evolution of the density matrix/entanglement entropy. Decoherence inside parton cascade

  For a fixed distribution of valence charges ρ (event-by-event), our projectile density matrix is that of a pure state. Applying JIMWLK evolution to the density matrix (or to EE), we can demonstrate that the energy evolution generates mixing/increases entropy.

  N. Armesto, F. Dominguez, A. Kovner, ML, and V. Skokov, 1901.08080 [hep-ph] (JHEP)

- In the dense regime : $\boldsymbol{\Omega}(\rho \sim \mathbf{1}/\alpha_{\rm s}) = \mathcal{C}\,\mathbf{B}$ $\qquad$ $\mathbf{B}$ is a Bogolyubov operator

$$\mathbf{B} = \exp[\,\mathcal{B}(\rho)\,(\mathbf{a}^2 + \mathbf{a}^{\dagger\,2}) + \cdots]$$

  CGC wavefunction is a squeezed state

  Altinoluk, Kovner, ML, Peressutti, Wiedemann (2007-2009)

- **Entropy production as a function of time:** $\hat\rho(\mathbf{t}) \;\; \rightarrow \;\; \sigma^{\mathbf{P}}(\mathbf{t})$

$$\hat\rho(\mathbf{t}) \;=\; \mathbf{\Omega}^\dagger\,\mathbf{U}(\mathbf{t})\,\hat{\mathbf{S}}\,\mathbf{\Omega}|\mathbf{0}\rangle\langle\mathbf{0}|\mathbf{\Omega}^\dagger\,\hat{\mathbf{S}}^\dagger\,\mathbf{U}^\dagger(\mathbf{t})\,\mathbf{\Omega}; \qquad\qquad \mathbf{U}(\mathbf{t}) \;=\; \mathbf{e}^{\mathbf{iH_{QCD}\,t}}$$

This turns out to produce time-independent EE, the one we have computed earlier

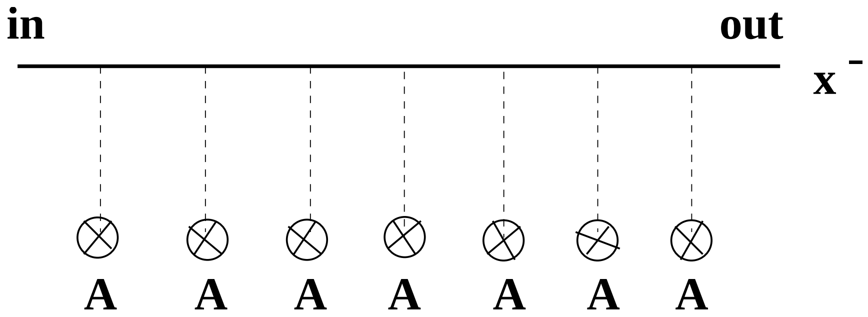
New idea is to introduce decoherence at event-by-event (fixed ρ) level by turning on some "white noise" variable ξ coupled to energy.

$$\hat\rho(\mathbf{t}) \;=\; \mathbf{tr}_\xi\,[\mathbf{e}^{-\frac{\xi^2}{2t^2}}\,\mathbf{\Omega}^\dagger\,\mathbf{e}^{\mathbf{iH_0}\xi}\,\hat{\mathbf{S}}\,\mathbf{\Omega}|\mathbf{0}\rangle\langle\mathbf{0}|\mathbf{\Omega}^\dagger\,\hat{\mathbf{S}}^\dagger\;\mathbf{e}^{-\mathbf{iH_0}\xi}\,\mathbf{\Omega}]$$

The new variable ξ can be effectively thought of as an energy resolution scale which could be ascribed to the experimental apparatus. what we have effectively done is to artificially extend the Hilbert space by augmenting it through the ξ subspace.

A. Kovner, ML, and M. Serino, Phys. Lett. B 792, 4 (2019)

Eikonal scattering approximation



Eikonal scattering is a color rotation
Eikonal factor does not depend on rapidity

In the light cone gauge ($A^+ = 0$) the large target field component is $A^- = \alpha^t$.

$$S(x) = \mathcal{P} \exp\left\{ i \int dx^+ \, T^a \, \alpha_t^a(x, x^+) \right\} . \qquad \text{"}\Delta\text{"} \alpha^t = \rho^t \quad (YM)$$

$$|in\rangle = |z, b\rangle \, ; \qquad |out\rangle = |z, a\rangle \, ; \qquad |out\rangle = S \, |in\rangle$$