Induced color charges, effective $\gamma\gamma g$-vertex in QGP. Applications to heavy-ion collisions

V. Skalozub

Dnipro National University, Ukraine

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Abstract

At the LHC experiment energies the QGP should be created. After the DPT the A_0 condensate (Polyakov's loop) forming out of the gluon g^3 and g^8 fields must be present. These background modifies the spectrum of the (color) charged particles that influences various processes.

Due to violation of the Z_3 symmetry and the Farry theorem for color, in the plasma with A_0 condensate, new type phenomena have to be inspired. Among them the generation of induced color charges $Q^3_{ind.}$ and Q^8_{ind}, the generation of induced gluon-gluon-gluon and gluon-photon-photon vertexes, which result, in particular, in gluon splitting in two photons, inelastic scattering of photons on the plasma. These are the distinguishable signals of the QGP creation.

In the present talk, we present the explicit calculations for these effects.

Outline

- New signals of Deconfinement PT
- QGP, A_0 condensation
- QGP, spontaneous magnetization
- Violation of Farry's theorem in QGP
- Induced charges $Q^3_{ind.}$ and Q^8_{ind}
- Photon dispersion equation
- Effective $\gamma\gamma g$ and g^3 vertexes
- Inelastic scattering of photons in QGP
- Conclusion

1 Deconfinement phase transition (DPT)

Investigations of deconfinement phase of QCD is a hot topic nowadays. Due to asymptotic freedom of non-Abelian gauge field interactions at high temperature $T \geq 150$ MeV quarks are deliberated from hadrons and new matter state - quark-gluon plasma (QGP) - is formed. At lower temperatures quarks are confined inside hadrons. The order parameter of the DPT is the Polaykov loop (PL)

$$P(\vec{x}) = T \exp\left[ig \int dx_4 A_0(\vec{x}, x_4)\right]. \tag{1}$$

It equal 0 at low temperature and $P \neq 0$ at $T > T_d$.

If $A_0(x_4) = const$

$A_0 \neq 0$ **is also the order parameter of the** DPT. The condensation of the A_0 was demonstrated in either lattice simulations or in analytic calculations. $A_0 \neq 0$ violates the $Z(3)$ and gauge symmetries.

Review paper O.A. Borisenko, J. Bohacik, V.V. Skalozub, A_0 **condensate in QCD, Fortschr. Phys. v. 43 (1995) 301.**

Other important order parameter is the temperature dependent chromo (magnetic) fields $H(T) \neq 0$ spontaneously created in the volume of the QGP. This point will not be discussed in this talk. In the literature, numerous applications of the PL in the QGP have been discussed. The combinations of both $A_0 \neq 0$ and $H(T) \neq 0$ were also investigated.

In particular, it was observed that the A_0 is dominant at temperatures not much grater T_d. So, in what follows we consider this case.

We describe some new phenomena and effects taking place due to the A_0 presence.

Spontaneous vacuum magnetization at LHC

Recently (Skalozub, Minaiev (2018)) it was obtained that at LHC experiment energies the QGP should be spontaneously magnetized.

The strengths of the large scale temperature dependent chromomagnetic, $B_3(T)$, $B_8(T)$, and usual magnetic, $H(T)$, fields spontaneously generated after the DPT, were estimated.

The critical temperature for the magnetized plasma is found to be $T_d(H) \sim 110 - 120$ MeV. This is essentially lower compared to the zero field value $T_d(H = 0) \sim 160 - 180$ MeV usually discussed in the literature. Due to contribution of quarks, the color magnetic fields act as the sources generating H. The strengths of the fields are $B_3(T), B_8(T) \sim 10^{18} - 10^{19} G$, $H(T) \sim 10^{16} - 10^{17} G$ for temperatures $T \sim 160 - 220$ MeV.

The presence of strong large scale (color) magnetic fields modifies the spectrum of the (color) charged particles that influence various processes of interest.

2 QGP, A_0 condensate

Quarks interact with electromagnetic field and gluons according the form

$$L^{int.} = \bar{\psi}^a[\gamma_\mu(\partial_\mu \delta^{ab} - ie_f A_\mu \delta^{ab} - ig(Q_\mu \frac{\lambda}{2})^{ab}) - m_f \delta^{ab}]\psi^b, \quad (2)$$

where A_μ is potential of electromagnetic fields, Q_μ is potential of gluon field, e_f is electric charge of quark with flavor f, m_f is quark mass, g is charge of strong interactions, a, b are color indexes.

Since quarks carry both electric and strong charges in the QGP the effective interactions of color and white objects are possible due to the quark virtual loops.

The A_0 is an element of the center $Z(3)$ of the $SU(3)$ group. When it is non zero,

both of these symmetries are broken.

The A_0 is a specific classical external fields. It can be introduced by splitting

$Q^a_\mu = (A_0)^a_\mu + (Q^a_\mu)_{rad.}$. of the gluon field potential. In what follows we consider the case $(A_0)^a_\mu = (A_0)_\mu \delta^{a3}$. This is for short.

3 Violation of Farry's theorem in QGP

In the vacuum, the Farry theorem holds:

The amplitudes having odd number of photon(gluon) lines, generated by the fermion loops, equal zero.

It is the consequence of C-parity invariance. The contribution of particles cancels the contribution of antiparticles.

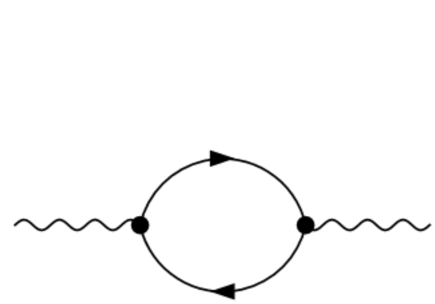
The presence of the A_0 condensate violates this symmetry. So that new type processes are permissible.

In particular,

the diagram with one gluon external line results in an induced color charge in the plasma. This may result in the scattering of quarks on this external charge.

Other interesting object is

Three line vertex - photon-photon-gluon - relates colored and white states. This is new type effective vertex which generates new observable processes - inelastic scattering of photons, splitting (dissociation) of gluons in two photons in the QGP. One of our goals is to calculate



this vertex and investigate these processes in the plasma.

These can be signals of the creation of QGP.

4 Gluon and photon spectra in QGP

Before doing that we have to detect the normal photon and gluon modes presented in the QGP with A_0. This can be done by solving the dispersion equations for these fields.

M. Bordag, V. Skalozub (2019)

Basically, in the plasma the spectra of the excitations can be obtained from the dispersion relations of the type

$$\omega^2 - \vec{k}^2 = Re\Pi(\omega, \vec{k}), \tag{3}$$

where ω and $\vec{k}$ are the frequency and the momentum of the modes.

In the QGP the transverse and the longitudinal excitations present. They are derived from relevant polarization tensors $\Pi(\omega, \vec{k})_T$ and $\Pi(\omega, \vec{k})_L$.

The expression for the photon polarization tensor reads

$$\Pi_{\mu\nu}(k) = -e^2 \sum_{p_4} \int \frac{d^3p}{(2\pi)^3\beta} Tr[\gamma_\mu \frac{(p+k)_\sigma\gamma_\sigma + m}{(p+k)^2+m^2} \gamma_\nu \frac{p_\rho\gamma_\rho + m}{p^2+m^2}]. \quad (4)$$

Here, imaginary time formalism is used. $\gamma_\mu, ...$ are the Dirac matrixes, $p_4 = 2\pi T(l + \frac{1}{2}) + A_0$, $k_\mu = (k_4 = 2\pi T(n), \vec{k})$, and $l, n = 0, \pm1, \pm2, ...$

Such type objects must be calculated in the gluon sector of the model.

As an example, we show the high temperature dispersion equation for the transversal plasma oscillations generated by the gluons

O. K. Kalashnikov, Progr. Theor. Phys. v. 92 (1994) 1207.:

$$(ik_4)^2 = g^2T^2\big[B_2(\frac{x}{2}) + B_2(0)\big]\xi^2\big(\frac{\xi^2}{\xi^2-1} - \frac{\xi}{2}\log\frac{\xi+1}{\xi-1}\big) + i\Gamma. \quad (5)$$

In this formula, $B_2(z) = z^2 - |z| + 1/6$ is the Bernoulli polynomial, $x = A_0/\pi T, \xi = (ik_4 + A_0)/|\vec{k}|$ and Γ is an imaginary part of the expression. It describes the damping of the plasma oscillations.

The similar expression have been obtained for longitudinal oscillations (plasmons) in the high temperature limit $T \to \infty$.

To find Dispersion relations we have to replace $ik_4 \to \omega$. In such a way all the quasi particle states of photons and gluons have been derived.

Photon spectra at high temperature

Let us consider the spectrum in leading order for $T \to \infty$. Here expressions significantly simplify.

For real frequency we get for $T \to \infty$ the equations

$$\begin{aligned} \omega^2 &= k^2 + \Lambda^2 \left(\frac{\omega^2}{k^2} - \frac{1}{2} \left(1 - \frac{\omega^2}{k^2} \right) \frac{\omega}{k} \ln \frac{\omega - k}{\omega + k} \right), \\ k^2 &= -\Lambda^2 \left(2 + \frac{\omega^2}{k^2} \ln \frac{\omega - k}{\omega + k} \right), \end{aligned} \tag{6}$$

where we defined

$$\Lambda^2 = -\frac{2e^2}{\pi^2} \int_0^\infty dp\, p\, n_s = \frac{2e^2}{\pi^2} \left(\frac{\pi^2 T^2}{12} - \frac{A_0^2}{4} \right) = e^2 \left(\frac{T^2}{6} - \frac{A_0^2}{2\pi^2} \right), \tag{7}$$

n_s is Fermi's distribution factor. The first line corresponds to the transversal photons and second for longitudinal ones.

The A_0 condensate stabilizes the infrared behavior of the plasma and has a lover energy as compared to the empty vacuum case.

5 Induced charge in QGP

Generation of the strong charge due to one-line non-zero diagram.

I. Baranov, V. Skalozub (2018)

Its quark loop contribution can be calculate from the expression

$$Q^{quark}_{induced} = -g \sum_{p_4} \int \frac{d^3p}{(2\pi)^3 \beta} Tr \gamma_4 [\frac{\lambda^3}{2} \frac{(p+k)_\sigma \gamma_\sigma + m_f}{(p+k)^2 + m_f^2}]. \qquad (8)$$

Here, the momentum $p = (p_4 = p_4 \pm A_0, \vec{p})$, $p_4 = 2\pi T(l + 1/2), l = 0, \pm 1, ...,$ $\beta = 1/T$.

Similar expressions can be calculated from tadpole gluon diagram having charged gluon loop.

These also hold for the color charge Q_8.

The resulting induced charge changes the coupling constant of gluons in the QGP.

We obtain in the high temperature limit ($\beta \to 0$)

$$Q^{quark}_{3ind.} = -gA_0\Big(\frac{T^2}{3} - \frac{m^3}{T} + O(1/T^3)\Big). \qquad (9)$$

In the presence of the induced charge the Slavnov-Taylor identity reads

$$\hat{p}_\mu \Pi^{\perp}_{\mu\nu}(\hat{p}_4, \vec{p}) = -gJ^3_\nu, \quad J^3_\nu = -2igQ_{3ind.}u_\nu. \qquad (10)$$

The induced current is

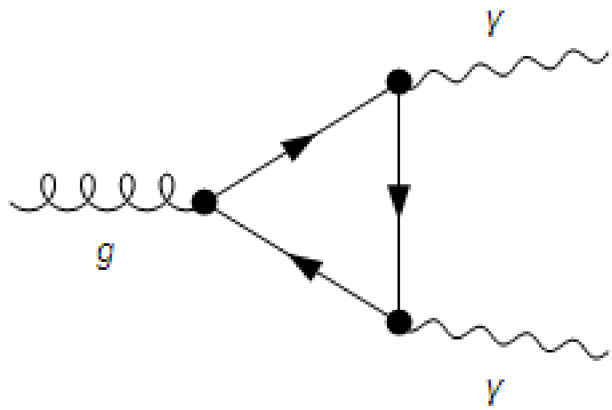
$$J^3_\nu = -2igQ_{3ind.}u_\nu, \qquad (11)$$

u_ν is plasma velocity.

6 Effective $\gamma\gamma G$ vertexes in QGP

Explicit form for the photon-photon-gluon vertex, its dominant terms are

M. Bordag, V. Skalozub (2019)



$$\Pi_{\mu\nu\lambda}(k^1,k^2,k^3) = \delta(k^1+k^2+k^3)(-e^2 g\Lambda)\sum_{p_4}\int \frac{d^3p}{(2\pi)^3\beta}(\Gamma^{(1)}_{\mu\nu\lambda}+\Gamma^{(2)}_{\mu\nu\lambda}), \tag{12}$$

$\Lambda = -16A_0 m_f^2,$

$$\Gamma^{(1)}_{\mu\nu\lambda} = \frac{\delta_{\mu\nu}\delta_{\lambda 4}+\delta_{\mu\lambda}\delta_{\nu 4}+\delta_{\lambda\nu}\delta_{\mu 4}}{D^2(p)D^2(p,k^1)D^2(p,k^3)}, \tag{13}$$

and

$$\Gamma^{(2)}_{\mu\nu\lambda} = \frac{-2S_{\mu\nu\lambda}}{D^2(p)D^2(p,k^1)D^2(p,k^3)}\Big(\frac{(p+k^3)_4}{D^2(p,k^3)} + \frac{(p-k^1)_4}{D^2(p,k^1)} + \frac{p_4}{D^2(p)}\Big), \tag{14}$$

where $D^2(p) = p^2 + m_f^2$, $D^2(p,k^1) = (p-k^1)^2 + m_f^2$, $D^2(p,k^3) = (p+k^3)^2 + m_f^2$,

$$S_{\mu\nu\lambda} = \delta_{\mu\nu}(p+k^1+k^3)_\lambda + \delta_{\lambda\nu}(p-k^1-k^3)_\mu + \delta_{\mu\lambda}(p-k^1+k^3)_\nu. \tag{15}$$

In the above formulas, k^1, k^3 are momenta of ingoing photons and $k^2 = -(k^1+k^2)$ is momentum of ingoing color neutral gluon $Q^{a=3}$.

All the other three-vertexes composing photons and gluons are zero.

So, we have a possibility for direct interaction of color and white worlds.

7 Loop integration in $\Gamma^{\nu}_{\mu\lambda}$

We perform loop integration in high temperature limit $\beta \to 0$. Denote as $\tilde{P} = (\tilde{P}_4 = p_4 - A_0, \vec{p})$.

In the high temperature limit, the large values of p give the leading contribution. Therefore we present $D_i(\tilde{P})$ as:

$$\begin{aligned} D(\tilde{P}) &= \tilde{P}_4^2 + \epsilon_p^2 = \tilde{P}^2, \\ D(\tilde{P} - k^1) &= \tilde{P}^2\Big(1 - \frac{2\tilde{P}\cdot k^1 - k_1^2}{\tilde{P}^2}\Big), \\ D(\tilde{P} + k^3) &= \tilde{P}^2\Big(1 + \frac{2\tilde{P}\cdot k^3 + k_3^2}{\tilde{P}^2}\Big). \end{aligned} \tag{16}$$

Here, $k_1^2 = (k_4^1)^2 + \vec{k}_1^2, k_3^2 = (k_4^3)^2 + \vec{k}_3^2$. At high temperature and $\tilde{P}^2 \to \infty$ the k-dependent terms are small. So, we can expand in these parameters and obtain for the integrand Intd. in Eq.(12)

$$Intd. = \frac{N_1}{(\tilde{P}^2)^3}\Big[1 + \sum_{i=1}^{4} A_i\Big], \tag{17}$$

where

$$A_1 = -2\frac{(\tilde{P}\cdot q)}{\tilde{P}^2}, \quad A_2 = -\frac{k_3^2 - k_1^2}{\tilde{P}^2} \tag{18}$$

$$A_3 = -4\frac{(\tilde{P}\cdot k^1)(\tilde{P}\cdot k^3)}{\tilde{P}^2}, \quad A_4 = 4\frac{(\tilde{P}\cdot k^1)^2 + (\tilde{P}\cdot k^3)^2}{\tilde{P}^2} \tag{19}$$

and vector $q_\mu = (q_4, \vec{q})$.

Scattering of photons on the potential Q_4^3

We set velocity $u_\nu = (1, \vec{0}), \nu = 4$. The numerators are

$$N_1 - > \delta_{\mu\lambda}(\tilde{P} + q)_4, \quad N_2 - > \delta_{\mu\lambda}(\tilde{P} - q)_4, \tag{20}$$

We have to calculate the series of two types:

$$S_1^{(n)} = \frac{1}{\beta} \sum_{p_4} \frac{p_4 - A_0}{(\tilde{P}^2)^n}, \quad S_2^{(n)} = \frac{1}{\beta} \sum_{p_4} \frac{q_4}{(\tilde{P}^2)^n}, n = 3, 4, 5. \tag{21}$$

These functions can be calculated from the $S_1^{(1)}$ and $S_2^{(1)}$ by computing a number of derivatives over ϵ_p^2.

$$S_1^{(1)} = \frac{1}{\beta} \sum_{p_4} \frac{p_4 - A_0}{\tilde{P}^2} = -\frac{1}{2} \frac{\sin(A_0\beta)}{\cos(A_0\beta) + \cosh(\epsilon_p\beta)}. \tag{22}$$

The function $S_2^{(1)}$ is

$$S_2^{(1)} = \frac{1}{\beta} \sum_{p_4} \frac{q_4}{\tilde{P}^2} = -\frac{1}{2\epsilon_p} \frac{\sinh(\epsilon_p\beta)}{\cos(A_0\beta) + \cosh(\epsilon_p\beta)}. \tag{23}$$

the A_i expressions:

$$A_1 = -2\frac{(p_4 - A_0)q_4}{\tilde{P}^2}, \tag{24}$$

$$A_3 = -\frac{4}{\tilde{P}^2}\Big[(1 - \frac{\epsilon_p^2}{\tilde{P}^2})k_4^1 k_4^3 + \frac{(\vec{p}\cdot\vec{k}_1)(\vec{p}\cdot\vec{k}_3)}{\tilde{P}^2}\Big], \tag{25}$$

$$A_4 = \frac{4}{\tilde{P}^2}\Big[(1 - \frac{\epsilon_p^2}{\tilde{P}^2})((k_4^1)^2 + (k_4^3)^2) + \frac{(\vec{p}\cdot\vec{k}_1)^2 + (\vec{p}\cdot\vec{k}_3)^2}{\tilde{P}^2}\Big], \tag{26}$$

The resulting amplitude consists of the terms

$$M_1 = 2\delta_{\mu\lambda}\frac{p_4 - A_0}{(\tilde{P}^2)^3}(1 + A_2 + A_3 + A_4) \tag{27}$$

and

$$M_2 = -4\delta_{\mu\lambda}\frac{(p_4 - A_0)q_4^2}{(\tilde{P}^2)^4}. \tag{28}$$

The next step is integration over d^3p.

Integration in leading order

We present integration Eqs.(27),(28) in leading in $T \to \infty$ approximation considering the first term in Eq.(27)

$$S_3 = -A_0\beta \, \frac{Sech(\beta\epsilon_p/2)^4}{64p^3}(-2\beta\epsilon_p + \beta\epsilon_p Cosh(\beta\epsilon_p) + Sinh(\beta\epsilon_p)). \quad (29)$$

Now, we calculate the integral

$$I_3 = \int\limits_{-\infty}^{\infty} d^3p \; S_3 = 4\pi \int\limits_{0}^{\infty} p^2 dp \; S_3(p) \quad (30)$$

In leading order $\epsilon_p\beta = p\beta$, for $p\beta = y$ we obtain for (29),

$$I_3 = -\frac{A_0\pi\beta}{16} \int\limits_{0}^{\infty} \frac{dy}{y} Sech(y/2)^4(-2y + yCosh(y) + Sinh(y)). \quad (31)$$

Note that the integral is convergent. The S_3 small y expansion is

$$S_3(y \to 0) = -A_0\beta^4(\frac{1}{96} + \frac{17y^2}{3840}). \quad (32)$$

Numeric integration in Eq.(31) gives

$$I_3 = -A_0\pi\beta \; (0.3348). \quad (33)$$

In such a way all other integrations in Eqs.(27), (28) can be carried out.

As a result, the explicit high temperature limits for scattering amplitude can be calculated.

The most important points:

1. The vertex is not transversal

2. It relates transversal and longitudinal modes of photons and gluons

In particular, new phenomena such as scattering of photons on the QGP as an effective vertex become possible. All the necessary ingredients to investigate these are calculated. These are the spectra of photons and gluons in the QGP, the effective charges.

There are two sorts of the processes of interest:

1) Scattering of photons on the plasma as on the external filed generated due to quark current and induced color charge. Radiation of photon pairs from plasma.

2) Scattering on the real gluon excitations in the plasma.

In these processes the plasma exhibits itself via the effective vertex and therefore the inelastic (or even elastic) scattering may be realized. Specific values for these cases depend on the characteristics of QGP.

Scattering of photons in the QGP has to be estimated by two parameters - induced charge and deviation of of the photon beams from an initial direction.

Other important expected process is splitting of the gluon field G^3, G^8 generated by the induced charge $Q^3_{ind.}, Q^8_{ind.}$ in two photons which have to move along the direction of the plasma motion.

These processes are basically different from the scattering of photons on chaotically moving particles of usual plasma.

8 Scattering due to $\Gamma^{\nu}_{\mu\lambda}$ vertex

Assume that $\bar{Q}^3_4$ is a classical gluon field generated in plasma due to the induced charge presence. We could consider scattering of classical electromagnetic waves on the plasma produced by the vertex $\Gamma^{\nu}_{\mu\lambda}$ calculated above.

Here, we calculate scattering of photons on this potential. Let the momenta of ingoing photon be k^1_μ and outgoing one k^3_λ.

The matrix element of the process reads

$$M = (2\pi)^4 \delta(k^1 + k^2 - k^3) \ \frac{e^{\sigma_1}_{\mu}}{\sqrt{2\omega_1}} \, \bar{Q}^3_4 \, \Gamma^4_{\mu\lambda} \, \frac{e^{\sigma_3}_{\lambda}}{\sqrt{2\omega_3}}. \tag{34}$$

Here, ${e^{\sigma_1}}_\mu, {e^{\sigma_2}}_\lambda$ are amplitudes and polarizations of photons, ω_1, ω_3 - corresponding energies. For not polarized beams,

$$\sum_{\sigma_1} e^{\sigma_1}_{\mu} e^{\sigma_1}_{\mu'} = \delta_{\mu\mu'} \ \sum_{\sigma_3} e^{\sigma_3}_{\lambda} e^{\sigma_3}_{\lambda'} = \delta_{\lambda\lambda'}, \tag{35}$$

, we obtain for the probability

$$P = MM^{+} = (\bar{Q}^3_4)^2 \ \Gamma^4_{\mu\lambda} \Gamma^4_{\mu\lambda} \frac{C}{4\omega_1\omega_3} \ \delta(k^1 + k^2 - k^3), \tag{36}$$

where C is the not relevant now number In this expression, accounting for the momentum conservation, $\omega_3 = [(\omega^1_x)^2 + (\omega^1_y)^2 + (\omega^1_z + k^2_z)^2]^{1/2}$. The explicit form of $\Gamma^4_{\mu\lambda}$ is known. It is calculated separately.

9 Toy model for $\bar{Q}_4^3$

Consider the plane of QGP $\bar{Q}_4^3 = \phi(z)$, $\phi(z = -L/2) = 0, phi(z = L/2) = 0$. Plasma is inside box $\frac{-L}{2} < z < \frac{L}{2}$. $-\infty < y, x < \infty$.

Solution to Eq.

$$(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial k^2} + m_D^2)\phi = -Q^{ind.} \tag{37}$$

reads

$$\phi = d + ae^{-(\epsilon t - kx)} + be^{(\epsilon t - kx)}. \tag{38}$$

Here $d = \frac{Q^{ind.}}{m_d^2}$.

Applying confinement condition we obtain

$$\phi(z) = \frac{Q^{ind.}}{m_d^2}\big(1 - \frac{\cos(k_z z)}{\cos(\frac{k_z L}{2})}\big). \tag{39}$$

The most important that this potrntial is staic. So, color longitudinal classical gluon fields in the QGP are static classical fields. No dinamical plasmons exisit if the indiced charge present.

10 Conclusion

According to basic principles of QCD, the QGP has to be either magnetized with strong long range temperature dependent magnetic fields $B^3(T), B^8(T), H(T)$ (that lowers the deconfiniment transition temperature T_d) or charged with color induced charges $Q^3_{ind.}, Q^8_{ind.}$.

Due to violation of the Farry theorem, in the QGP new type phenomena have to be generated. Among them the deviation of the photon beam from its initial direction and the change of the frequency. Generation of induced color charges, gluon splitting in two photons. These are the distinguishable signals of the QGP creation.

Similar processes may take place in dense nuclear matter. Here the role of the A_0 plays the chemical potential μ related with the matter formation. The role of gluon field Q_3 play neutral ρ meson and photon fields. In the matter the induced electric charge has to appear. This may serve as a signal for the nuclear matter creation. As s result, inelastic scattering of photons on the matter is expected.