Eikonal Approximation in High-Energy Physics

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Charged High-Energy Particle Scattering

In the eikonal approximation of quantum electrodynamics:

$$4\pi^2 \frac{d\sigma}{d^2 q_\perp} = \left| \int d^2\rho \; e^{i\frac{\vec{q}_\perp \vec{\rho}}{\hbar}} \left(e^{\frac{i}{\hbar}\Phi(\vec{\rho})} - 1 \right) \right|^2 \qquad (1)$$

where $\vec{q}_\perp$ is transverse component of the transmitted to external field $U(\vec{\rho}, z)$ impulse and

$$\Phi(\vec{\rho}) = -\frac{1}{v} \int_{-\infty}^{\infty} dz\, U(\vec{\rho}, z) \qquad (2)$$

For scattering in a medium the potential energy $U(\vec{\rho}, z)$ is the sum of potentials of all medium atoms:

$$U(\vec{\rho}, z) = \sum_{n=1}^{N} u(\vec{\rho} - \vec{\rho}_n, z - z_n) \qquad (3)$$

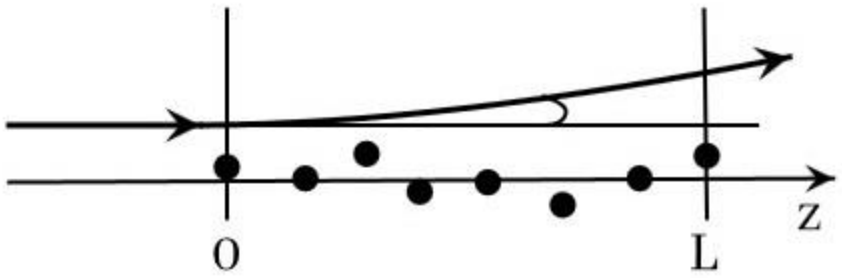
where $\vec{r}_n = (\vec{\rho}_n, z_n)$ is n-th atom position in the medium along x, y, z axes.

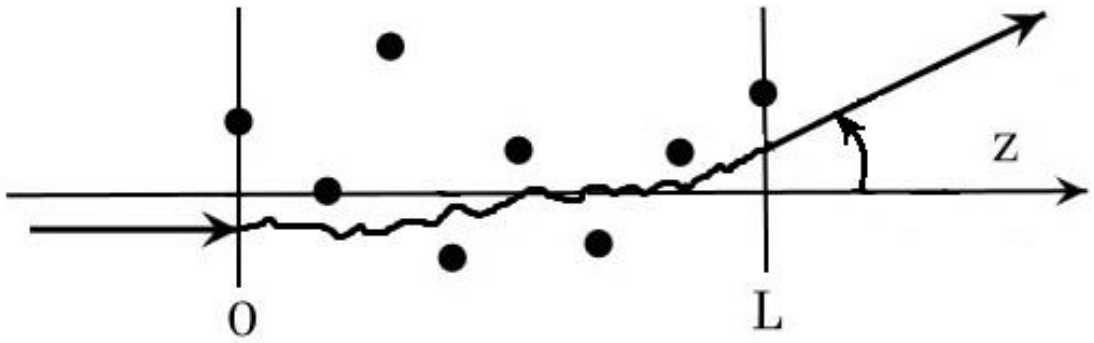
Atoms in a medium can be placed regularly as in a crystal) or chaotically (as in amorphous media). Formula (1) should be averaged over atoms positions in the medium.

$$4\pi^2 \langle \frac{d\sigma}{d^2 q_\perp} \rangle = \int d^2\rho \, d^2\rho' \, e^{\frac{i}{\hbar}\vec{q}_\perp(\vec{\rho}-\vec{\rho}')} F(\vec{\rho}, \vec{\rho}') \qquad (4)$$

where

$$\langle F(\vec{\rho}, \vec{\rho}') \rangle = \prod_{n=1}^{N} \int d^2 u \, f(\vec{\rho}_n) \exp\left\{ \frac{i}{\hbar} \left[\chi(\vec{\rho} - \vec{\rho}_n) - \chi(\vec{\rho}' - \vec{\rho}_n) \right] \right\} \qquad (5)$$





Atoms Distribution Function in Different Medias

Amorphous media :

$$f(\vec{\rho}_1) = \frac{1}{L_y L_x} \qquad (6)$$

where L_y и L_x are linear target sizes along *y* and *x* axes.

The chain of atoms :

$$f(\vec{u}) = \frac{1}{\pi\overline{u^2}} e^{-\vec{u}^2/\overline{u^2}} \qquad (7)$$

where $\overline{u^2}$ is average squared deviation of atom position in the chain. $\overline{u^2}$ in a crystal is small compared with the atom potential screening radius *R*.

The crystalline plane of atoms:

$$f = \frac{1}{L_y}\frac{1}{\pi\overline{u^2}} e^{-u_x^2/\overline{u^2}} \qquad (8)$$

Using series expansion ($\bar{\chi} \ll 1$) we will obtain the following:

$$4\pi^2 \langle \frac{d\sigma}{d^2 q_\perp} \rangle = \int d^2\rho \, d^2\rho' \, e^{\frac{i}{\hbar}\vec{q}_\perp(\vec{\rho}-\vec{\rho}')} F(\vec{\rho}, \vec{\rho}') \quad (4)$$

$$\langle \boldsymbol{F}(\vec{\boldsymbol{\rho}}, \vec{\boldsymbol{\rho}}') \rangle = \boldsymbol{exp}\left\{ \boldsymbol{N} \left[\boldsymbol{i}\overline{(\chi - \chi')} - \frac{\mathbf{1}}{\mathbf{2}} \overline{(\chi - \chi')^{\mathbf{2}}} - \frac{\boldsymbol{i}^{\mathbf{2}}}{\mathbf{2}} (\overline{\chi - \chi'})^{\mathbf{2}} + \cdots \right] \right\} \quad \mathbf{(9)}$$

where $\chi = \chi(\vec{\rho} - \vec{u})$, $\chi' = \chi(\vec{\rho}' - \vec{u})$ and

$$\overline{(\chi - \chi')} = \int d^2u \, f(\vec{u}) \, (\chi - \chi') \quad (10)$$

$$\overline{(\chi - \chi')^2} \int d^2u \, f(\vec{u}) \, (\chi - \chi')^2 \quad (11)$$

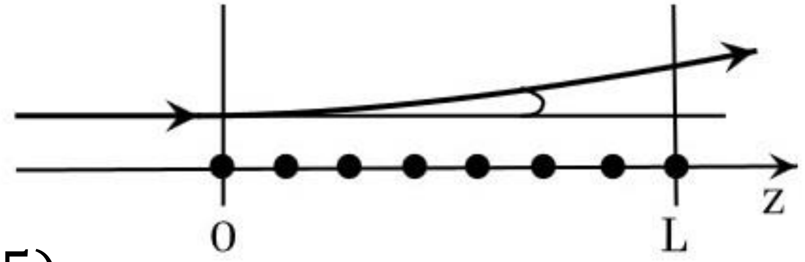
and $\chi(\vec{\rho} - \vec{\rho}_n) = -\frac{1}{v} \int_{-\infty}^{\infty} dz \, u(\vec{\rho} - \vec{\rho}_n, z),$ (12)

The Scattering on the Atoms Chain in the Born Approximation

Let the particles fall on the chain along its axis and $N\bar{\chi} \ll 1$, then

$$4\pi^2 \langle\frac{d^2\sigma}{dq_\perp^2}\rangle = \int d^2\rho \; d^2\rho' e^{i\vec{q}_\perp(\vec{\rho}-\vec{\rho}\prime)}[N(\overline{\chi\chi'} - \bar{\chi}\bar{\chi}') + N^2\bar{\chi}\bar{\chi}'] \qquad (13)$$

$$\boldsymbol{4\pi^2 \langle\frac{d^2\sigma}{dq_\perp^2}\rangle = N\left(1 - e^{-\frac{q_\perp^2\overline{u^2}}{2}}\right)\left|\chi_{q_\perp}\right|^2 + N^2 e^{-\frac{q_\perp^2\overline{u^2}}{2}}\left|\chi_{q_\perp}\right|^2} \quad (14)$$



where $\chi_{q_\perp} = \int d^2\rho \, e^{i\vec{q}_\perp\vec{\rho}}\chi(\vec{\rho})$ (15)

If $\overline{u^2} \to \infty$ coherent effect in scattering vanishes and formula (14) becomes equal to the Born theory result for fast particles scattering in amorphous media :

$$4\pi^2 \langle\frac{d^2\sigma}{dq_\perp^2}\rangle = N\left|\chi_q\right|^2 \qquad (16)$$

The Born approximation scope of application for scattering on the chain of atoms:

$$\frac{N|\chi|}{\hbar} = N\frac{Ze^2}{\hbar v} \ll 1 \qquad (17)$$

where $N = L/a$, L is the atoms chain length, a is the distance between atoms in the chain, $Z|e|$ is atom nucleus charge and v is the particle velocity.

The Scattering on the Atoms Chain in the Eikonal Approximation

The main contribution to the scattering cross section is made by $\vec{\rho}$, which are close to $\vec{\rho}'$.

Let $\vec{\Delta} = \vec{\rho} - \vec{\rho}'$, then:

$$\overline{\chi - \chi'} = \vec{\Delta}\frac{\partial}{\partial\vec{\rho}'}\overline{\chi(\vec{\rho}')} + \frac{1}{2}\Delta_i\Delta_j\frac{\partial^2\overline{\chi(\vec{\rho}')}}{\partial x_i' x_j'} \quad (18), \quad \overline{(\chi - \chi')^2} = \overline{\left(\vec{\Delta}\frac{\partial}{\partial\vec{\rho}'}\chi(\vec{\rho}')\right)^2} \quad (19)$$

Omitting components proportional to the square of $\vec{\Delta}$ in (11) we will have the following scattering cross section:

$$\left\langle\frac{d^2\sigma}{dq_\perp^2}\right\rangle = \int d^2\rho'\,\delta\left(\vec{q}_\perp + N\overline{\frac{\partial}{\partial\vec{\rho}'}\chi(\vec{\rho}')}\right) \quad (20)$$

This cross section expression is similar to the cross section obtained on the basis of classical scattering theory:

$$d\sigma_{cl} = d^2b = \frac{\partial(b_x, b_y)}{\partial(\theta_x, \theta_y)}do = \frac{1}{\left|\partial(\theta_x, \theta_y)/\partial(b_x, b_y)\right|}do \quad (21)$$

where $\vec{b}$ s impact parameter and $\left|\partial\vec{\theta}/\partial\vec{b}\right|$ is the determinant of deviation function in external field $\vec{\theta} = \vec{\theta}(\vec{b})$.

$$\vec{q}_\perp = p\vec{\theta} = -N\overline{\frac{\partial}{\partial\vec{\rho}'}\chi(\vec{\rho}')}\bigg|_{\vec{\rho}'=\vec{\rho}_*'} \quad (22)$$

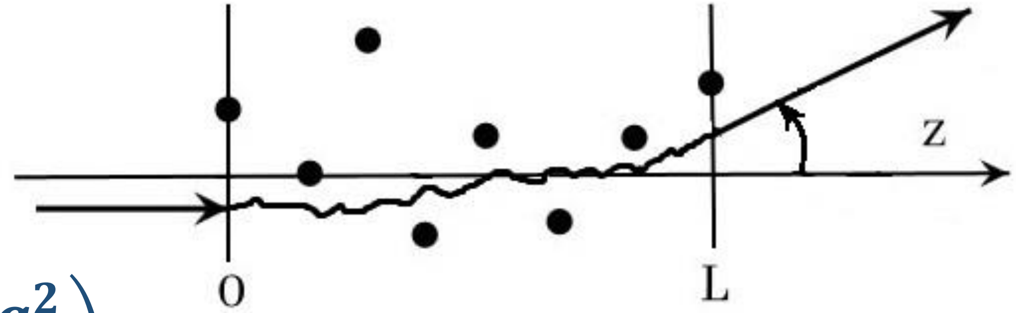
where $\vec{\rho}_*' = \vec{b}$ is solution of the equation (22) for determined value of $\vec{q}_\perp$.

The Scattering in the Amorphous Media in the Eikonal Approximation

For amorphous media, components in (9) proportional to $\overline{\chi - \chi'}$ vanish, since

$$\overline{\chi - \chi'} \approx \vec{\Delta} \frac{\partial}{\partial \vec{\rho}'} \int \frac{d^2 u}{S} \chi(\vec{\rho}' - \vec{u}) = -\vec{\Delta} \int \frac{d^2 u}{S} \frac{\partial}{\partial \vec{u}} \chi(\vec{\rho}' - \vec{u}) = 0 \qquad (23)$$

$$\overline{(\chi - \chi')^2} \approx \frac{\Delta^2}{4\pi S} \int \kappa d\kappa \, \kappa^2 |\chi_\kappa|^2 \qquad (24)$$



Then the cross section (4) is:

$$\left\langle \frac{d^2\sigma}{dq_\perp^2} \right\rangle = S \frac{1}{\pi \overline{q_\perp^2}} exp\left(-\frac{q_\perp^2}{\overline{q_\perp^2}} \right) \qquad (25)$$

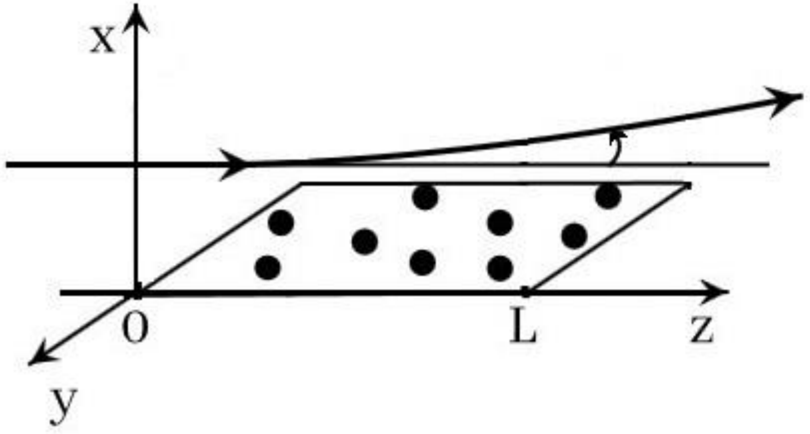
where $\overline{q_\perp^2}$ is average squared transmitted impulse:

$$\overline{q_\perp^2} = Ln \int d\sigma_1(q_\perp) q_\perp^2 \qquad (26)$$

Here L is target thickness, n is atoms density in the target and $d\sigma_1(q_\perp)$ is the cross section for scattering on one atom in the Born approximation, $N = nLS$.

Formula (25) is analogous to the result of Bethe-Molier theory obtained on the basis of **kinetic equation method.**

The Scattering on the Crystalline Plane with Amorphous Distribution of Atoms



This case is interesting with the fact that the particles scattering in longitudial and transverse diretions relatively to the plane sagnificantly differs: in the transverse direction correlations in sequential collisions are essential, while in the longitudinal direction scattering is similar to the scattering in amorphous media.

$$2\pi \langle\frac{d^2\sigma}{dq_\perp^2}\rangle = L_y \int dx'\, \delta\left(q_x + N_{xy} \overline{\frac{\partial}{\partial x'} \chi(x' - u_x, u_y)} \right) e^{-\frac{q_y^2}{\langle q_y^2\rangle}} \sqrt{\frac{4\pi}{\langle q_y^2\rangle}} \qquad (27)$$

where $\langle q_y^2\rangle$ is average squared transmitted impulse *y*-component.

Conclusion

The coherent and incoherent scattering processes of fast charged particles in crystals were considered from a unified point of view on the example of particles falling along one chain of atoms and one crystallographic plane in the eikonal and Born approximations. It was shown that our result in the Born approximation is consistent with the main results of the theory of M.L. Ter-Mikaelyan [1] about the processes of coherent radiation of ultrarelativistic electrons in a crystal, the scattering cross section splits into cross sections of coherent and incoherent scattering. If the Born approximation condition of applicability is violated, the scattering pattern is more complex. In this case, the separation of the scattering cross section into coherent and incoherent components is generally absent. For scattering on one crystalline plane, it is shown that the scattering cross section in this case has a different structure in the scattering direction along the crystalline plane and in the transverse direction. The results indicate the need for a more detailed analysis of the inclusion of incoherent scattering processes in the simulation of electromagnetic processes in crystals at high energies.

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Thank You for Attention!