Regge Cuts and NNLLA BFKL

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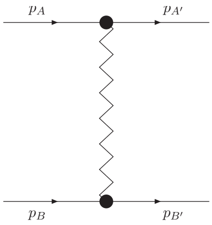
Contents

One of remarkable properties of QCD is the Reggeization of all elementary particles in perturbation theory, which is very important for theoretical description of high energy processes. The gluon Reggeization is especially important because it determines the high energy behaviour of non-decreasing with energy cross sections. In particular, it appears to be the basis of the famous BFKL (Balitskii-Fadin-Kuraev-Lipatov) equation, which was first derived in non-Abelian theories with spontaneously broken symmetry
F. V.S., Kuraev E.A., Lipatov L.N., 1975
and whose applicability in QCD was then shown
Balitsky I.I., Lipatov L.N., 1978

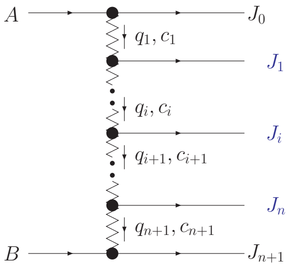
Introduction

For elastic scattering processes $A+B \to A' + B'$ in the Regge kinematical region: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s) the Reggeization means that scattering amplitudes with the gluon quantum numbers in the t-channel can be presented as



$$\mathcal{A}_{AB}^{A'B'} = \Gamma_{A'A}^{c} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^{c} \; ;$$

$\Gamma^{c}_{P'P}$–particle-particle-Reggeon (PPR) vertices or scattering vertices (”c” are colour indices); $j(t) = 1 + \omega(t)$ – Reggeon trajectory.
The Reggeization means definite form not only of elastic amplitudes, but of inelastic amplitudes in the multi-Regge kinematics (MRK) as well. It can be presented by the picture



and written as

$$\Re\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{J_i}(q_i, q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)$$

$$\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

Here $\gamma_{c_i c_{i+1}}^{J_i}(q_i, q_{i+1})$ – the Reggeon-Reggeon-particle (RRP) or production vertices.

MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with s) invariant masses of any pair of the jets.

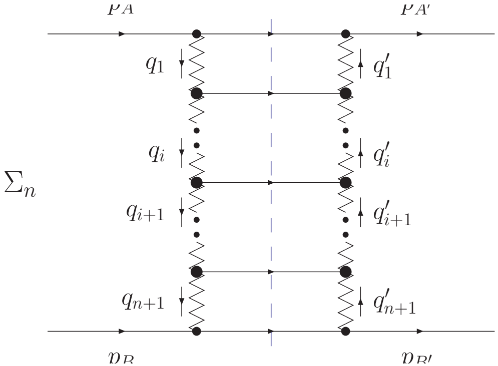
The MRK gives dominant contributions to cross sections of QCD processes at high energy $\sqrt{s}$. In the LLA only a gluon can be produced. In the NLA one has to account production of $Q\bar{Q}$ and GG jets.

Introduction

The BFKL equation was derived for summation of radiative corrections to elastic scattering amplitudes.
The corrections were calculated using the *s*-channel unitarity and analyticity and the pole Regge form of amplitudes with negative signature and adjoint representation of the gauge group in cross channels.
The unitarity was used for calculation of discontinuities of elastic amplitudes, and analyticity for their full restoration.
It is extremely important that both in the leading logarithmic approximation (LLA) and in the next-to-leading one (NLLA) the amplitudes used in the unitarity relations are determined by the Regge pole contributions and have a simple factorized form (pole Regge form).
Due to this, the Reggeization provides a simple derivation of the BFKL equation in the LLA and in the NLLA.

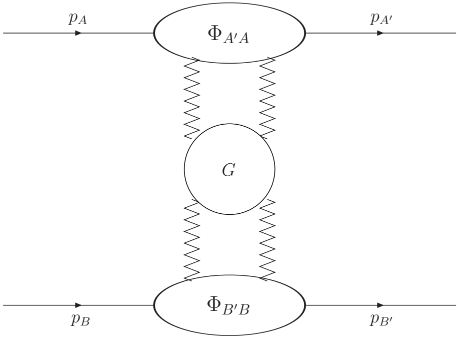
Amplitudes of processes with all possible quantum numbers in the t–channel are calculated using s-channel unitarity and analyticity .

The s-channel discontinuity



The amplitudes are presented in the form :

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}.$$



Introduction

Impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \to A'$ $B \to B'$,
G – Green's function for two interacting Reggeized gluons,

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

$\hat{\mathcal{K}}$ – BFKL kernel, $Y = \ln(s/s_0)$,

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r$$

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.
The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory.
The kernel is universal (process independent).

Introduction

Validity of the pole Regge form is proved now in all orders of perturbation theory in the coupling constant g both in the LLA and in the NLLA.

The pole Regge form is violated in the NNLLA.

The first observation of the violation was done

Del Duca V., Glover E.W.N., 2001

at consideration of the high-energy limit of the two-loop amplitudes for gg, gq and qq scattering. The discrepancy appears in non-logarithmic terms.

If the pole Regge form would be correct in the NNLLA, they should satisfy a definite condition (factorization condition), because three amplitudes should be expressed in terms of two Reggeon-Particle-Particle vertices.

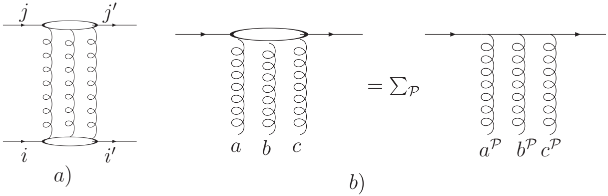
Detailed consideration of the terms responsible for breaking of the pole Regge form in two-loop and three-loop amplitudes of elastic scattering in QCD was performed by

Del Duca V., Falcioni G., Magnea L., Vernazza L., 2013-2015.

In particular, the non-logarithmic terms violating the pole Regge form at two-loops were recovered and not satisfying the factorization condition single-logarithmic terms at three loops were found using the techniques of infrared factorization.
It is necessary to say that, in general, breaking the pole Regge form is not a surprise.
It is well known that Regge poles in the complex angular momenta plane generate Regge cuts. Moreover, in amplitudes with positive signature the Regge cuts appear already in the LLA. In particular, the BFKL Pomeron is the two-Reggeon cut in the complex angular momenta plane. But in amplitudes with negative signature Regge cuts must be at least three-Reggeon ones and can appear only in the NNLLA.
It was natural to expect that the observed violation of the pole Regge form can be explained by existence of the cut.

Appearance of three-Reggeon cuts

It was shown
Lipatov L.N., F. V.S., 2016
Caron-Huot S., Gardi E., Vernazza L., 2017
using different approaches
that this is actually so.
Due to the signature conservation the cut with negative signature must be the three-Reggeon one.



Appearance of three-Reggeon cuts

Corresponding matrix elements can be written as the sum over permutations σ of products of colour factors and colour-independent amplitudes $\left(C^{(0)\sigma}\right)^{a'a}_{b'b}$ and colour-independent amplitudes $M^{(0)\sigma}_{ij}$. For the adjoint representation and negative signature it can be written as

$$\mathcal{T}^c_{a'a}\mathcal{T}^c_{b'b}\sum_\sigma G^{(0)\sigma}_{ij}M^{(0)\sigma}_{ij} \ ,$$

with

$$G^{(0)\sigma}_{ij} = \frac{1}{(N_c^2-1)T_iT_j}\,\mathrm{Tr}\left(\mathcal{T}^{c_1}\mathcal{T}^{c_2}\mathcal{T}^{c_3}\mathcal{T}^{c}\right)\ \mathrm{Tr}\left(\mathcal{T}^{c_1^\sigma}\mathcal{T}^{c_2^\sigma}\mathcal{T}^{c_3^\sigma}\mathcal{T}^{c}\right) \ ,$$

where $\mathcal{T}^a$ are the colour group generators in the corresponding representations, $[\mathcal{T}^a,\mathcal{T}^b] = if_{abc}\mathcal{T}^c$; $\mathcal{T}^a_{i'i} = T^a_{i'i} = -if_{abc}$ for gluons and $\mathcal{T}^a_{i'i} = t^a_{i'i}$ for quarks; $\mathrm{Tr}(\mathcal{T}_i^a\mathcal{T}_i^b) = T_i\delta_{ab}$, $T_q = 1/2$, $T_g = N_c$.

Appearance of three-Reggeon cuts

It was shown that the terms violating the pole factorization have σ-independent colour coefficients, so that momentum factors for them summed up to the eikonal amplitude

$$A^{eik} = \sum_{\sigma} M_{ij}^{(0)\sigma} = g^2 \frac{s}{t} \left(\frac{-4\pi^2}{3} \right) g^4 \, \vec{q}^{\,2} A_{\perp}^{(3)} \,,$$

$$A_{\perp}^{(3)} = \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^2}$$

$$= 3 C_\Gamma^2 \frac{4}{\epsilon^2} \frac{(\vec{q}^{\,2})^{2\epsilon}}{\vec{q}^{\,2}} \frac{\Gamma^2(1+2\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)} \,,$$

$$C_\Gamma = \frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)} = \frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}} (1 - \epsilon^2 \zeta(2) + 2\epsilon^3 \zeta(3) + \ldots..) \,,$$

The "infrared" ϵ, $\epsilon = (D-4)/2$, D is the space-time dimension, is used.

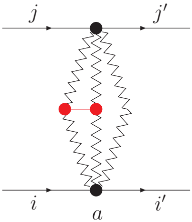
Three loops

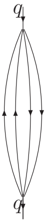
Separation of the pole and cut contributions is impossible in the two-loop approximation because of the ambiguity of the allocation of the part of the amplitudes violating the factorization. The separation becomes possible in higher loops, due to different energy dependence of the pole and cut contributions. Energy dependence of the pole contribution is determined by the Regge factor of the Reggeized gluon $\exp(\omega(t) \ln s)$, where $\omega(t)$ is the gluon trajectory, whereas for the three-Reggeon cut it is

$$e^{[(\hat{\omega}_1+\hat{\omega}_2+\hat{\omega}_3+\hat{\mathcal{K}}_r(1,2)+\hat{\mathcal{K}}_r(1,3)+\hat{\mathcal{K}}_r(2,3)) \ln s]} ,$$

where $\hat{\mathcal{K}}_r(m,n)$ is the real part of the BFKL kernel describing interaction between Reggeons m and n.

Three loops





Three loops

The calculation of the three-loop corrections
F.V.S.
shows that the violation of the pole Regge form, analysed in this approximation with the help of the infrared factorization, can be explained by the pole and cut contributions. In other words, the restrictions imposed by the infrared factorization on the parton scattering amplitudes with the adjoint representation of the colour group in the *t*-channel and negative signature can be fulfilled in the NNLLA at two and three loops if besides the Regge pole contribution there is the Regge cut contribution

$$A^{eik} C_{ij}^{C} \left(1 - (C_R + C_3) \ln s\right) \ ,$$

Three loops

$$A_\perp^{(3)} C_R = -g^2 N_c C_\Gamma \frac{2}{\epsilon} \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^{2-2\epsilon}}$$

$$= -g^2 N_c C_\Gamma \frac{4}{3\epsilon} (\vec{q}^{\,2})^\epsilon \frac{\Gamma(1-3\epsilon)\Gamma(1+2\epsilon)\Gamma(1+3\epsilon)}{\Gamma(1-\epsilon)\Gamma(1-2\epsilon)\Gamma(1+\epsilon)\Gamma(1+4\epsilon)} A_\perp^{(3)} \, ,$$

$$A_\perp^{(3)} C_3 = g^2 N_c C_\Gamma \frac{4}{\epsilon} \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2 (l_1 + l_2)^{2\epsilon}}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^{2}}$$

$$= g^2 N_c C_\Gamma \frac{32}{9\epsilon} (\vec{q}^{\,2})^\epsilon \frac{\Gamma(1-3\epsilon)\Gamma(1-\epsilon)\Gamma^2(1+3\epsilon)}{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)\Gamma(1+4\epsilon)} A_\perp^{(3)} \, .$$

and

$$C_{gg}^C = -\frac{3}{2} \, , \quad C_{gq}^C = -\frac{3}{2} \, , \quad C_{qq}^C = \frac{3(1-N_c^2)}{4N_c^2} \, .$$

Three loops

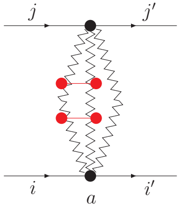
It should be noted that this result is limited to three loops and can not be considered as a proof that in the NNLLA the only singularities in the J plane are the Regge pole and the three-Reggeon cut. Moreover, the explanation of the violation of the pole Regge form given in
Caron-Huot S., Gardi E., Vernazza L., 2017
differs from described above. In this paper, besides the cut with the vertex of interaction with particles i having the colour structure

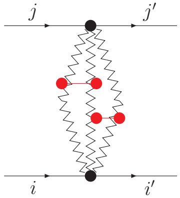
$$C^{(0)c}_{a'a} = \mathcal{T}^c_{a'a} \frac{1}{3!(N_c^2-1)T_i T_j} \mathrm{Tr} \sum_\sigma \left(\mathcal{T}^{c_1^\sigma} \mathcal{T}^{c_2^\sigma} \mathcal{T}^{c_3^\sigma} \mathcal{T}^c \right) ,$$

the Reggeon-cut mixing is introduced. Actually, in the three-loop approximation the mixing is not required. Whether mixing is necessary can be verified in the four-loop approximation.

Four loops

In the four loops there are three types of corrections. The first (simplest) ones come from account of the Regge factors of each of three Reggeons. The second type of the corrections are given by the products of the trajectories and real parts of the BFKL kernel, and the third come from account of Reggeon-Reggeon interactions.





Four loops

With the help of the integral representation of the trajectory

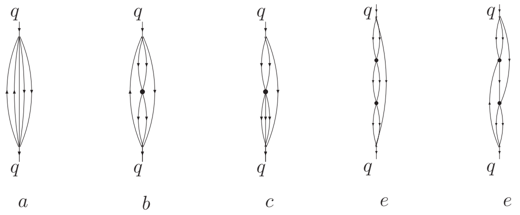
$$\omega(t) = -g^2 N_c \vec{q}^{\,2} \int \frac{d^{2+2\epsilon} l}{2(2\pi)^{(3+2\epsilon)} \vec{l}^{\,2} (\vec{q} - \vec{l})^2}$$

and the explicit form of the real part of the kernel describing interaction between two Reggeons with transverse momenta $\vec{l}_1$ and $\vec{l}_2$ and colour indices c_1 and c_2

$$\left[\mathcal{K}_r(\vec{q}_1, \vec{q}_2; \vec{k})\right]_{c_1 c_2}^{c_1' c_2'} = -T^a_{c_1 c_1'} T^a_{c_2 c_2'} \frac{g^2}{(2\pi)^{D-1}} \left[\frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\, 2} + \vec{q}_2^{\,2} \vec{q}_1^{\,\prime\, 2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right],$$

Four loops

all the types of the corrections are expressed through the integrals in the transverse momentum space corresponding to the diagrams



Four loops

$$I_i = \int \frac{d^{2+2\epsilon}l_1\, d^{2+2\epsilon}l_2\, d^{2+2\epsilon}l_3}{(2\pi)^{3(3+2\epsilon)}\vec{l}_1^{\,2}\vec{l}_2^{\,2}\vec{l}_3^{\,2}} F_i \delta^{2+2\epsilon}(\vec{q}-\vec{l}_1-\vec{l}_2-\vec{l}_3)\;,$$

$$F_a = f_1(\vec{l}_1)f_1(\vec{l}_2)\;,\quad F_b = f_1(\vec{l}_1)f_1(\vec{l}_1)\;,\quad F_c = f_2(\vec{l}_1+\vec{l}_2)\;,$$

$$F_d = f_1(\vec{l}_1+\vec{l}_2)f_1(\vec{l}_1+\vec{l}_2)\;,\quad F_e = f_1(\vec{q}-\vec{l}_1)f_1(\vec{q}-\vec{l}_3)\;,$$

$$f_1(\vec{k}) = \vec{k}^{\,2}\int \frac{d^{2+2\epsilon}l}{(2\pi)^{(3+2\epsilon)}\vec{l}^{\,2}(\vec{l}-\vec{k})^2}\;,\quad f_2(\vec{k}) = \int \frac{d^{2+2\epsilon}l f_1(\vec{l})}{(2\pi)^{(3+2\epsilon)}\vec{l}^{\,2}(\vec{l}-\vec{k})^2}\;.$$

These integrals enter in the total four-loop correction with different colour factors in the approaches with or without the Reggeon-cut mixing. The question of whether the four-loop amplitudes of elastic scattering in QCD are given by the Regge pole and cut contributions, with or without mixing, can be solved by comparing of these corrections with the result obtained using the infrared factorization.

Summary

- The BFKL equation was derived assuming the pole Regge form of amplitudes with gluon quantum numbers in cross channels and negative signature.
- It is proved now in all orders of perturbation theory that this form is valid both in the leading and in the next-to-leading logarithmic approximations.
- However, this form is violated in the NNLLA.
- It was shown that the observed violation can be explained by the cut contributions.
- But the assertion that the QCD amplitudes with gluon quantum numbers in cross-channels and negative signature are given in the NNLLA by the contributions of the Regge pole and the three-Reggeon cut is only a hypotheses, and as yet there is no general proof of it, it should be checked in each order of the perturbation theory.