Searching for Odderon in exclusive reactions

Piotr Lebiedowicz
Institute of Nuclear Physics Polish Academy of Sciences (IFJ PAN)
Cracow, Poland

in collaboration with
Antoni Szczurek (IFJ PAN)
Otto Nachtmann (Univ. Heidelberg)

NEW TRENDS in HIGH-ENERGY PHYSICS
Odessa, Ukraine May 12-18, 2019

Plan

1) Central Exclusive Production (CEP) in *pp* collisions

2) Model for high-energy soft reactions

(tensor-Pomeron and vector-Odderon)

3) Examples of exclusive reactions

- $p\ p \to p\ p\ K^+ K^- K^+ K^-$ (via intermediate $\phi\phi$ state)
- $p\ p \to p\ p\ \phi$

4) Conclusions

Searching for Odderon

- Odderon (C = -1 partner of Pomeron)
  L. Łukaszuk and B. Nicolescu, Lett. Nuovo Cim. 8 (1973) 405

- A hint of the Odderon was seen in ISR results (PRL54 (1985)2180) as a small difference between the differential cross sections of elastic *pp* and $p\bar{p}$ scattering in the diffractive dip region at $\sqrt{s} = 53\ \mathrm{GeV}$.

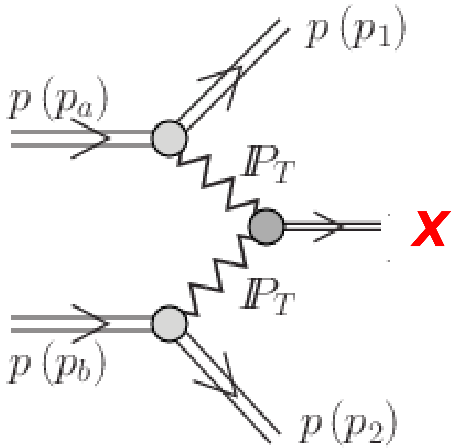
- New data for *pp* elastic scattering at $\sqrt{s} = 2.76\ \mathrm{TeV}$ of TOTEM collaboration:
  Antchev *et al.* [TOTEM collaboration], arXiv:1812.04732, 1812.08610
  → suggestion of presence of Odderon
  → some last year analyses of Nicolescu *et al.*

- It is of great importance to study possible Odderon effects in other reactions than *pp* elastic scattering:
  → central J/Ψ and ϕ production in high-energy *pp* and $p\bar{p}$ collisions
  A.Schafer, L.Mankiewicz, O.Nachtmann, PLB272 (1991) 419; A. Bzdak et al. PRD75 (2007) 094023
  → photoproduction of $f_2(1270)$ and $a_2(1320)$, exclusive neutral pseudoscalar mesons
  Berger, Donnachie, Dosch, Nachtmann, EPJC14 (2000) 673
  → photoproduction and electroproduction of heavy C = +1 quarkonia
  → the asymmetry in the fractional energy of charm versus anticharm jets, which is sensitive to Odderon-Pomeron interference — Brodsky et al., PLB461 (1999) 114
  → observation of charge asymmetry in the $\pi^+ \pi^-$ production — Ginzburg, Ivanov, Nikolaev, EPJC5 (2003) 02
  → ultraperipheral proton-ion collisions — Harland-Lang et al., arXiv:1811.12705; Goncalves et al., arXiv: 1811.07622

  Nice review on Odderon physics: C. Ewerz, arXiv: 0306137

- Central Exclusive Production (CEP) of $\phi\phi$ state offers a very nice way to look for Odderon effects *THIS TALK* *based on P.L., Nachtmann, Szczurek, arXiv:1901.11490*

Central Exclusive Production (CEP) in *pp* collisions



What can we hope to learn from CEP ?

(1) Properties and the coupling of the exchange objects
IP, *O*, *IR*, *γ* to the external protons and the system ***X***

(2) Properties of the system ***X***.
Search for and characterisation of resonances,
e.g. glueballs (gluonic bound states)

From the theory point of view the topics (1) - (2) are,
mainly nonperturbative, QCD problems.

At the moment calculations of CEP reactions from first principles of QCD
are still too difficult for theorists.

We have to resort to models.

Tensor-Pomeron model for high-energy soft reactions

C. Ewerz, M. Maniatis, O. Nachtmann, Ann. Phys. 342 (2014) 31

The main feature of the model is that the Pomeron exchange is described as effective exchange of a symmetric rank 2 tensor:

$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

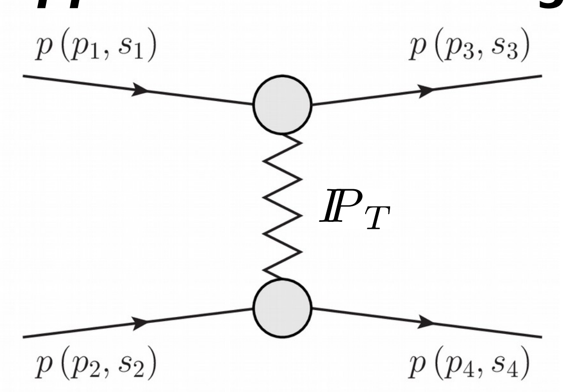
$$i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p) = i\Gamma^{(\mathbb{P}\bar{p}\bar{p})}_{\mu\nu}(p',p) = -i3\beta_{\mathbb{P}NN}F_1\big((p'-p)^2\big)\left\{\frac{1}{2}[\gamma_\mu(p'+p)_\nu + \gamma_\nu(p'+p)_\mu] - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}t$$

$$\alpha_{\mathbb{P}}(0) = 1.0808, \qquad \alpha'_{\mathbb{P}} = 0.25\,\mathrm{GeV}^{-2}$$

$$\beta_{\mathbb{P}NN} = 1.87\ \mathrm{GeV}^{-1}$$

***pp* elastic scattering** (helicity amplitudes)



$$\langle 2s_3, 2s_4|\mathcal{T}|2s_1, 2s_2\rangle = (-i)\bar{u}(p_3,s_3)i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p_3,p_1)u(p_1,s_1)$$
$$\times i\Delta^{(\mathbb{P})\mu\nu,\kappa\lambda}(s,t)$$
$$\times \bar{u}(p_4,s_4)i\Gamma^{(\mathbb{P}pp)}_{\kappa\lambda}(p_4,p_2)u(p_2,s_2)$$

Only 5 out of 16 helicity amplitudes are independent, e.g.

$$\phi_1(s,t) = \langle ++|\mathcal{T}++\rangle$$
$$\phi_3(s,t) = \langle +-|\mathcal{T}|+-\rangle$$
helicity-conserving amplitudes

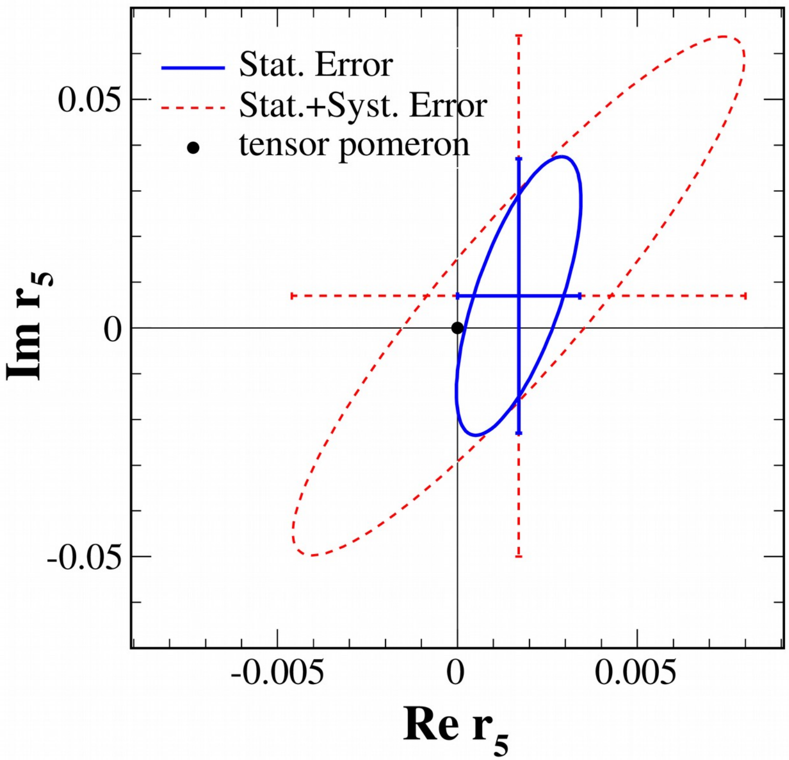
$$\phi_5(s,t) = \langle ++|\mathcal{T}|+-\rangle$$
single-helicity-flip amplitude

Tensor, vector, or scalar Pomeron ?

C. Ewerz, P. L., O. Nachtmann, A. Szczurek, ***Helicity in proton-proton elastic scattering and the spin structure of the pomeron***, PLB 763 (2016) 382

We choose our ansätze for three the effective *IP* propagators and *IPpp* couplings such that at high energies the ϕ_1 and ϕ_3 are the same for all three cases. This gives the same σ_{tot}^{pp}.
The Donnachie-Landshoff (DL) model treats the Pomeron as effective vector exchange and gives a phenomenologically successful fit to σ_{tot}^{pp} and $d\sigma/dt$.
Our ansätze are chosen such that ϕ_1 and ϕ_3 are as in the DL model.

- Vector exchange IP_V has C = -1 than $\sigma_{tot}^{pp} = -\sigma_{tot}^{\bar{p}p}$ (not a viable option).
- We are left with IP_T and IP_S (both correspond to C=+1 exchanges)
  To decide between them we turn to the STAR experiment (PLB 719 (2013))
  which measured the single spin asymmetry A_N in polarised *pp* elastic scattering.



$$\sqrt{s} = 200\ \text{GeV} \quad 0.003 \leqslant |t| \leqslant 0.035\ \text{GeV}^2$$

$$r_5(s,t) = \frac{2m_p\,\phi_5(s,t)}{\sqrt{-t}\,\text{Im}[\phi_1(s,t)+\phi_3(s,t)]}$$

$$r_5^{IP_T}(s,t) = -\frac{m_p^2}{s}\left[i + \tan\left(\frac{\pi}{2}(\alpha_{IP}(t)-1)\right)\right]$$

$$r_5^{IP_T}(s,0) = (-0.28 - i2.20)\times 10^{-5}$$

$$r_5^{IP_S}(s,t) = -\frac{1}{2}\left[i + \tan\left(\frac{\pi}{2}(\alpha_{IP}(t)-1)\right)\right]$$

$$r_5^{IP_S}(s,0) = -0.064 - i0.500$$ Very far from data !

Only the tensor-Pomeron is compatible with the general rules of QFT and the STAR experimental result

Other applications of the tensor-Pomeron model

$\gamma\, p \to \pi^+ \pi^- p$ *Bolz, Ewerz, Maniatis, Nachtmann, Sauter, Schöning, JHEP 01 (2015) 151*

There will be interference between $\gamma\, p \to (\rho^0 \to \pi^+ \pi^-)\, p$ (IP exchange)
and $\gamma\, p \to (f_2(1270) \to \pi^+ \pi^-)\, p$ (Odderon exchange) processes
and as a consequence $\pi^+ \pi^-$ charge asymmetries.

Photoproduction and low *x* DIS *Britzger, Ewerz, Glazov, Nachtmann, Schmitt, arXiv:1901.08524*

A "vector Pomeron" cannot couple
in the total photoabsorption cross section $\sigma_{\gamma p}$.

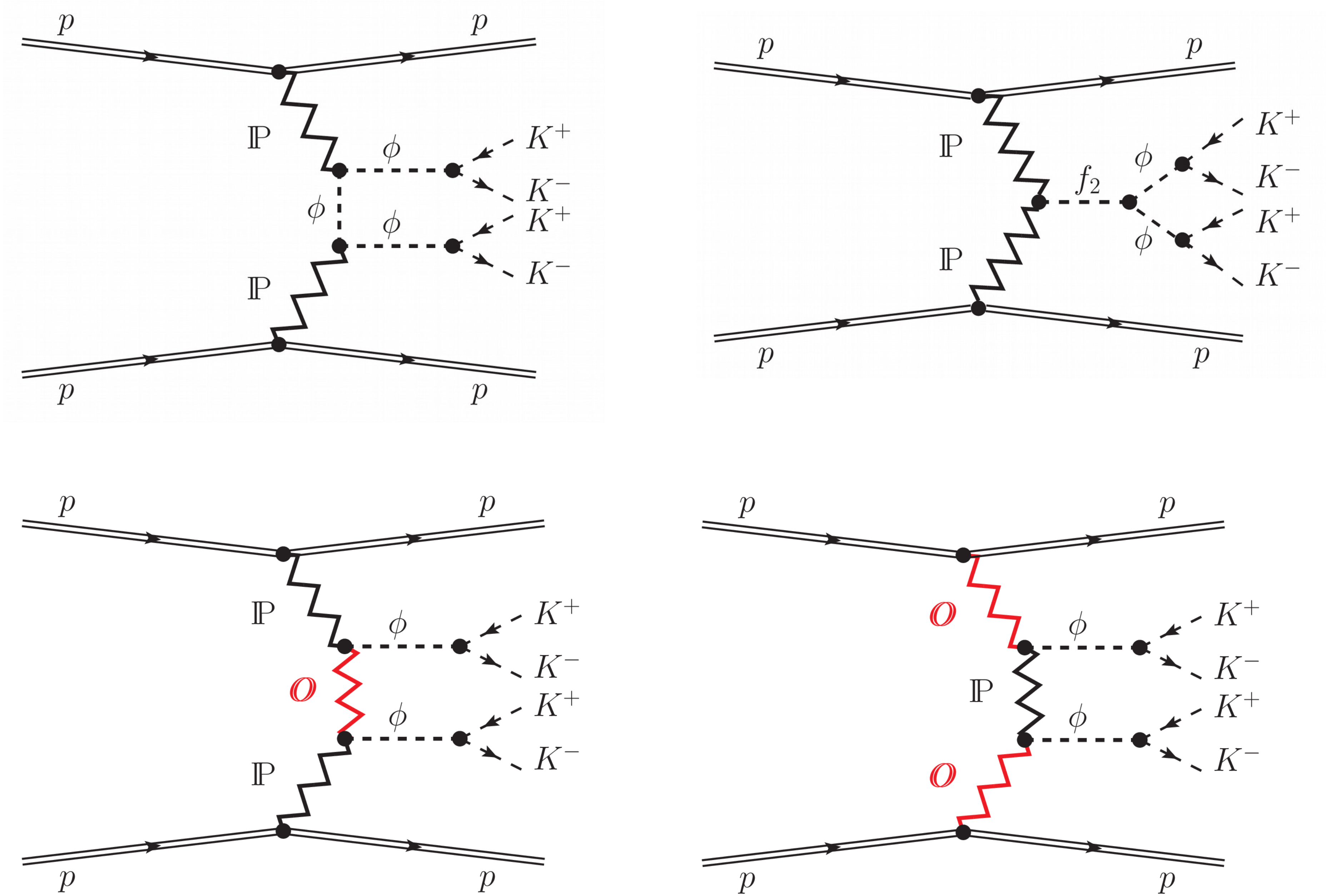
Central Exclusive Production

$p\, p \to p\, p\, X$ *P.L., Nachtmann, Szczurek:*

X:	$\eta,\ \eta',\ f_0$	*Ann. Phys. 344 (2014) 301*
	ρ^0	*PRD91 (2015) 074023*
	$\pi^+ \pi^-,\ f_0,\ f_2$	*PRD93 (2016) 054015, and arXiv:1901.07788*
	$\pi^+ \pi^- \pi^+ \pi^-,\ \rho^0 \rho^0$	*PRD94 (2016) 034017*
	ρ^0 with proton diss.	*PRD95 (2017) 034036*
	$K^+ K^-,\ f_0,\ f_2',\ \phi$	*PRD98 (2018) 014001*
	$p\bar{p}$	*PRD97 (2018) 094027*
	$K^+ K^- K^+ K^-,\ \phi\phi$	*arXiv:1901.11490*

For reactions investigated so far the tensor-Pomeron model works well

$pp \to pp\phi\phi$



$pp \to pp\ (\phi\phi \to K^+K^-K^+K^-)$

- The ϕ(1020) is a narrow resonance and it can be easily identified in K^+K^- spectra
- Some modifications are needed to simulate 2 → 6 reaction with 4K in final state (e.g. smearing of ϕ masses due to their resonance distribution)

$$\sigma_{2\to 6} = [\mathcal{B}(\phi \to K^+K^-)]^2 \int_{2m_K} \int_{2m_K} \sigma_{2\to 4}(..., m_{X_3}, m_{X_4})\, f_\phi(m_{X_3})\, f_\phi(m_{X_4})\, dm_{X_3}\, dm_{X_4}$$

with the branching fraction $\mathcal{B}(\phi(1020) \to K^+K^-) = 0.492$ [PDG]
and the spectral function of ϕ meson:

$$f_\phi(m_{X_i}) = C_\phi \left(1 - \frac{4m_K^2}{m_{X_i}^2}\right)^{3/2} \frac{\frac{2}{\pi} m_\phi^2 \Gamma_\phi}{(m_{X_i}^2 - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2}$$

C_ϕ is found from the condition $\int_{2m_K}^{\infty} f_\phi(m_{X_i}) dm_{X_i} = 1$

- Any differential distribution can be calculated.

To include experimental cuts on produced kaons we perform the decays of ϕ mesons isotropically in the ϕ rest frames and then use relativistic transformations to the overall c.m. frame

$pp \to pp\phi\phi$ (ϕ- exchange continnum)

We consider the 2 → 4 exclusive reaction:

$$p(p_a, \lambda_a) + p(p_b, \lambda_b) \to p(p_1, \lambda_1) + \phi(p_3, \lambda_3) + \phi(p_4, \lambda_4) + p(p_2, \lambda_2)$$

The Born-level amplitude (ϕ-exchange continuum) can be written as the sum

$$\mathcal{M}^{(\phi-\text{exchange})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi} = \mathcal{M}^{(\hat{t})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi} + \mathcal{M}^{(\hat{u})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi}$$

with the $\hat{t}$-channel amplitude:

$$\begin{aligned}\mathcal{M}^{(\hat{t})}_{\rho_3\rho_4} =& (-i)\bar{u}(p_1, \lambda_1) i\Gamma^{(I\!\!Ppp)}_{\mu_1\nu_1}(p_1, p_a) u(p_a, \lambda_a)\, i\Delta^{(I\!\!P)\,\mu_1\nu_1,\alpha_1\beta_1}(s_{13}, t_1) \\ & \times i\Gamma^{(I\!\!P\phi\phi)}_{\rho_1\rho_3\alpha_1\beta_1}(\hat{p}_t, -p_3)\, i\Delta^{(\phi)\,\rho_1\rho_2}(\hat{p}_t)\, i\Gamma^{(I\!\!P\phi\phi)}_{\rho_4\rho_2\alpha_2\beta_2}(p_4, \hat{p}_t) \\ & \times i\Delta^{(I\!\!P)\,\alpha_2\beta_2,\mu_2\nu_2}(s_{24}, t_2)\, \bar{u}(p_2, \lambda_2) i\Gamma^{(I\!\!Ppp)}_{\mu_2\nu_2}(p_2, p_b) u(p_b, \lambda_b)\end{aligned}$$

where $\hat{p}_t = p_a - p_1 - p_3$, $s_{ij} = (p_i + p_j)^2$, $t_1 = (p_1 - p_a)^2$, $t_2 = (p_2 - p_b)^2$

Absorptive corrections should be included

$$\mathcal{M}_{pp\to pp\phi\phi} = \mathcal{M}^{Born}_{pp\to pp\phi\phi} + \mathcal{M}^{pp-rescattering}_{pp\to pp\phi\phi}$$

$$\mathcal{M}^{pp-rescattering}_{pp\to pp\phi\phi}(s, \vec{p}_{1\perp}, \vec{p}_{2\perp}) = \frac{i}{8\pi^2 s}\int d^2\vec{k}_\perp \mathcal{M}^{Born}_{pp\to pp\phi\phi}(s, \vec{p}_{1\perp} - \vec{k}_\perp, \vec{p}_{2\perp} + \vec{k}_\perp)\, \mathcal{M}^{I\!\!P-exch.}_{pp\to pp}(s, -\vec{k}^2_\perp)$$

here $\vec{k}_\perp$ is the transverse momentum carried around the loop

$pp \to pp\phi\phi$ (ϕ- exchange continnum)

- Our ansatz for the effective propagator and proton vertex function

$$i\Delta^{(\mathbb{P})}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{\mathbb{P}})^{\alpha_{\mathbb{P}}(t)-1}$$

$$i\Gamma^{(\mathbb{P}pp)}_{\mu\nu}(p',p) = -i3\beta_{\mathbb{P}NN}F_1(t)\left\{\frac{1}{2}\left[\gamma_\mu(p'+p)_\nu + \gamma_\nu(p'+p)_\mu\right] - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

where $\beta_{\mathbb{P}NN} = 1.87\ \mathrm{GeV}^{-1}$

$$\alpha_{\mathbb{P}}(t) = \alpha_{\mathbb{P}}(0) + \alpha'_{\mathbb{P}}t$$

$$\alpha_{\mathbb{P}}(0) = 1.0808, \qquad \alpha'_{\mathbb{P}} = 0.25\,\mathrm{GeV}^{-2}$$

- For the $\mathbb{P}\phi\phi$ vertex we have

$$i\Gamma^{(\mathbb{P}\phi\phi)}_{\mu\nu\kappa\lambda}(k',k) = iF_M((k'-k)^2)\left[2a_{\mathbb{P}\phi\phi}\,\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k',-k) - b_{\mathbb{P}\phi\phi}\,\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k',-k)\right]$$

with two rank-four tensor functions: Ewerz, Maniatis, Nachtmann, Ann. Phys. 342 (2014) 31

$$\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k_1,k_2) = \left[(k_1\cdot k_2)g_{\mu\nu} - k_{2\mu}k_{1\nu}\right]\left[k_{1\kappa}k_{2\lambda} + k_{2\kappa}k_{1\lambda} - \frac{1}{2}(k_1\cdot k_2)g_{\kappa\lambda}\right]$$

$$\begin{aligned}\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k_1,k_2) = &\ (k_1\cdot k_2)(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa}) + g_{\mu\nu}(k_{1\kappa}k_{2\lambda} + k_{2\kappa}k_{1\lambda}) \\ &- k_{1\nu}k_{2\lambda}g_{\mu\kappa} - k_{1\nu}k_{2\kappa}g_{\mu\lambda} - k_{2\mu}k_{1\lambda}g_{\nu\kappa} - k_{2\mu}k_{1\kappa}g_{\nu\lambda} \\ &- \left[(k_1\cdot k_2)g_{\mu\nu} - k_{2\mu}k_{1\nu}\right]g_{\kappa\lambda}\end{aligned}$$

- We take $F_1(t) = \dfrac{4m_p^2 - 2.79\,t}{(4m_p^2 - t)(1 - t/m_D^2)^2}\,, \quad F_M(t) = \dfrac{1}{1 - t/\Lambda_0^2}$

Photoproduction of $\phi(1020)$ meson

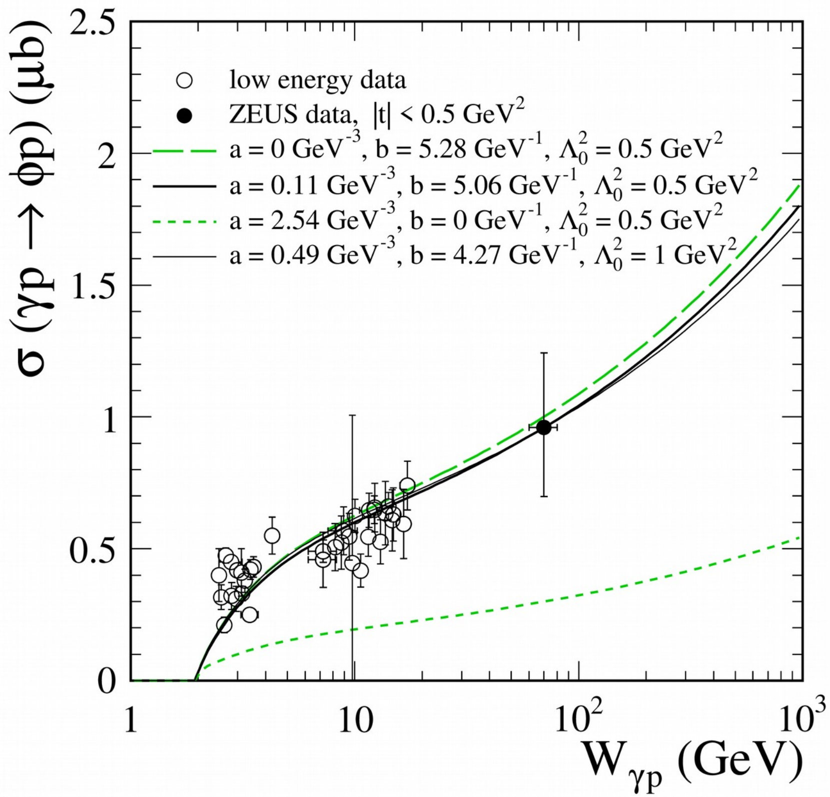
In this framework application of vector meson-dominance (VMD) model is straightforward and do not lead to gauge-invariance problem.

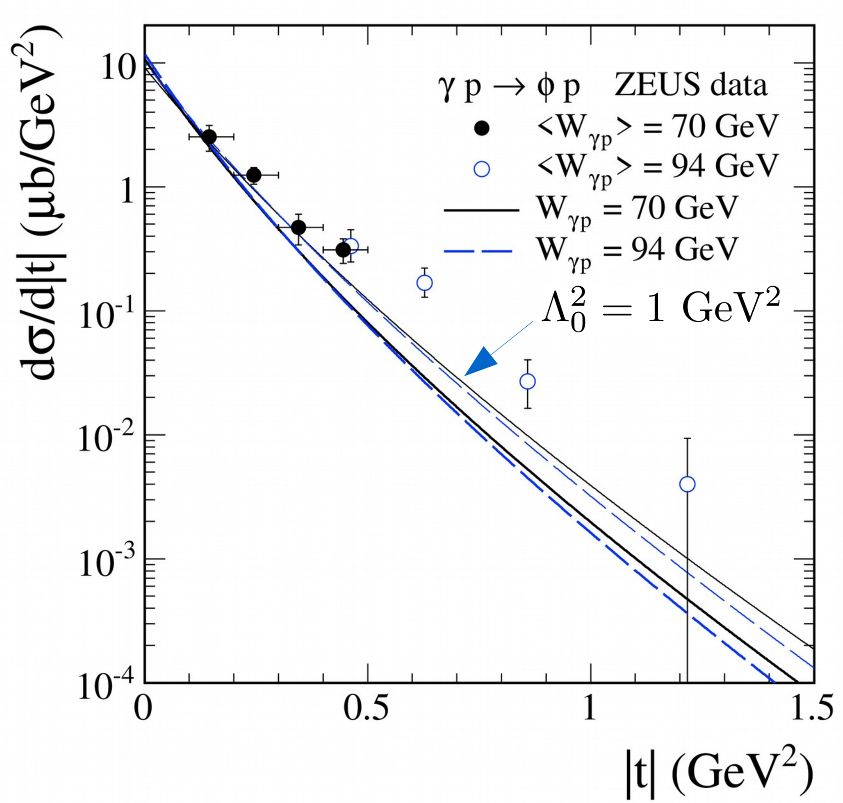
$$\mathcal{M}_{\gamma p \to \phi p}(s,t) \cong ie\frac{m_\phi^2}{\gamma_\phi}\,\Delta_T^{(\phi)}(0)\,(\epsilon^{(\phi)\,\mu})^*\epsilon^{(\gamma)\,\nu}\left[2a_{I\!P\phi\phi}\,\Gamma^{(0)}_{\mu\nu\kappa\lambda}(p_\phi,-q) - b_{I\!P\phi\phi}\,\Gamma^{(2)}_{\mu\nu\kappa\lambda}(p_\phi,-q)\right]$$
$$\times 3\beta_{I\!PNN}\,\frac{1}{2s}(-is\alpha'_{I\!P})^{\alpha_{I\!P}(t)-1}\,(p_2+p_b)^\kappa(p_2+p_b)^\lambda\,\delta_{\lambda_2\lambda_b}F_1(t)F_M(t)$$

With assumption (based on the additive quark model)

$\sigma_{tot}(\phi(\epsilon^{(m)}),p) = \sigma_{tot}(K^+,p) + \sigma_{tot}(K^-,p) - \sigma_{tot}(\pi^-,p)$ for transversely polarised ϕ (m = ±1)

we get $2m_\phi^2\,a_{I\!P\phi\phi} + b_{I\!P\phi\phi} = 4(2\beta_{I\!PKK} - \beta_{I\!P\pi\pi}) = 5.28\,\mathrm{GeV}^{-1}$





$pp \to pp\phi\phi$ (ϕ- exchange continnum)

In the high-energy approximation we can write

$$\mathcal{M}^{(\phi-\text{exchange})\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi} = 2(p_1+p_a)_{\mu_1}(p_1+p_a)_{\nu_1}\,\delta_{\lambda_1\lambda_a}\,F_1(t_1)\,F_M(t_1)$$
$$\times \Big\{ V^{\rho_3\rho_1\mu_1\nu_1}(s_{13},t_1,\hat{p}_t,p_3)\,\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}_t)\,V^{\rho_4\rho_2\mu_2\nu_2}(s_{24},t_2,-\hat{p}_t,p_4)\,\Big[\hat{F}_\phi(\hat{p}_t^2)\Big]^2$$
$$+ V^{\rho_4\rho_1\mu_1\nu_1}(s_{14},t_1,-\hat{p}_u,p_4)\,\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}_u)\,V^{\rho_3\rho_2\mu_2\nu_2}(s_{23},t_2,\hat{p}_u,p_3)\,\Big[\hat{F}_\phi(\hat{p}_u^2)\Big]^2 \Big\}$$
$$\times\, 2(p_2+p_b)_{\mu_2}(p_2+p_b)_{\nu_2}\,\delta_{\lambda_2\lambda_b}\,F_1(t_2)\,F_M(t_2)$$

where

$$V_{\mu\nu\kappa\lambda}(s,t,k_2,k_1) = \frac{1}{4s}\,3\beta_{I\!PNN}\,(-is\alpha'_{I\!P})^{\alpha_{I\!P}(t)-1}\Big[2a_{I\!P\phi\phi}\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k_1,k_2) - b_{I\!P\phi\phi}\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k_1,k_2)\Big]$$

The amplitude contains a form factor taking into account the off-shell dependences of the intermediate ϕ-mesons

$$\hat{F}_\phi(\hat{p}^2) = \exp\left(\frac{\hat{p}^2 - m_\phi^2}{\Lambda^2_{off,E}}\right)$$

where the cut-off parameter $\Lambda_{off,E}$ could be adjusted to experimental data.

$pp \to pp\phi\phi$ (ϕ-exchange continnum)

We should take into account the fact that the exchanged intermediate object is not a simple spin-1 particle (ϕ meson) but may correspond to a Regge exchange, that is, the reggeization of the intermediate ϕ meson is necessary.

$$\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}) \to \Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}) \left(\exp(i\phi(s_{34}))\, \frac{s_{34}}{s_{\rm thr}} \right)^{\alpha_\phi(\hat{p}^2)-1} \qquad \text{Eq. (3.21)}$$

where $s_{34} = (p_3 + p_4)^2 = \mathrm{M}^2_{\phi\phi}$, $s_{\rm thr} = 4m^2_\phi$

We assume for the ϕ Regge trajectory (from Collins book)

$$\alpha_\phi(\hat{p}^2) = \alpha_\phi(0) + \alpha'_\phi\, \hat{p}^2$$
$$\alpha_\phi(0) = 0.1\,, \quad \alpha'_\phi = 0.9\ \mathrm{GeV}^{-2}$$

In order to have the correct phase behaviour we introduced the function $\exp(i\phi(s_{34}))$ with

$$\phi(s_{34}) = \frac{\pi}{2} \exp\left(\frac{s_{\rm thr} - s_{34}}{s_{\rm thr}} \right) - \frac{\pi}{2}$$

This procedure of reggeization assures agreement with mesonic physics in the $\phi\phi$ system close to threshold, $s_{34} = 4m^2_\phi$ (no suppression), and it gives the Regge behaviour at large s_{34}.

$pp \to pp\phi\phi$ (ϕ-exchange continnum)

- Regge formalism applies when $|\hat{p}_t^2|, |\hat{p}_u^2| \ll s_{34}$

  At the threshold ($M_{\phi\phi} = 2m_\phi$) both $|\hat{p}_t^2|$ and $|\hat{p}_u^2|$ are not very small.

- Another idea of reggeization:
  At $\mathrm{Y_{diff}} = \mathrm{Y}_3 - \mathrm{Y}_4 = 0$ (i.e. for $|\hat{p}^2| \sim s_{34}$) reproduce meson physics, suggested by Harland-Lang, Khoze, Ryskin.

We propose a formula for the ϕ propagator which interpolates between the regions of low $\mathrm{Y_{diff}}$, where we use the standard ϕ propagator, and of high $\mathrm{Y_{diff}}$ where we use the reggeized form:

$$\Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p}) \to \Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p})\, F(\mathrm{Y_{diff}}) + \Delta^{(\phi)}_{\rho_1\rho_2}(\hat{p})\, [1 - F(\mathrm{Y_{diff}})] \left(\exp(i\phi(s_{34}))\, \frac{s_{34}}{s_{\mathrm{thr}}} \right)^{\alpha_\phi(\hat{p}^2)-1}$$

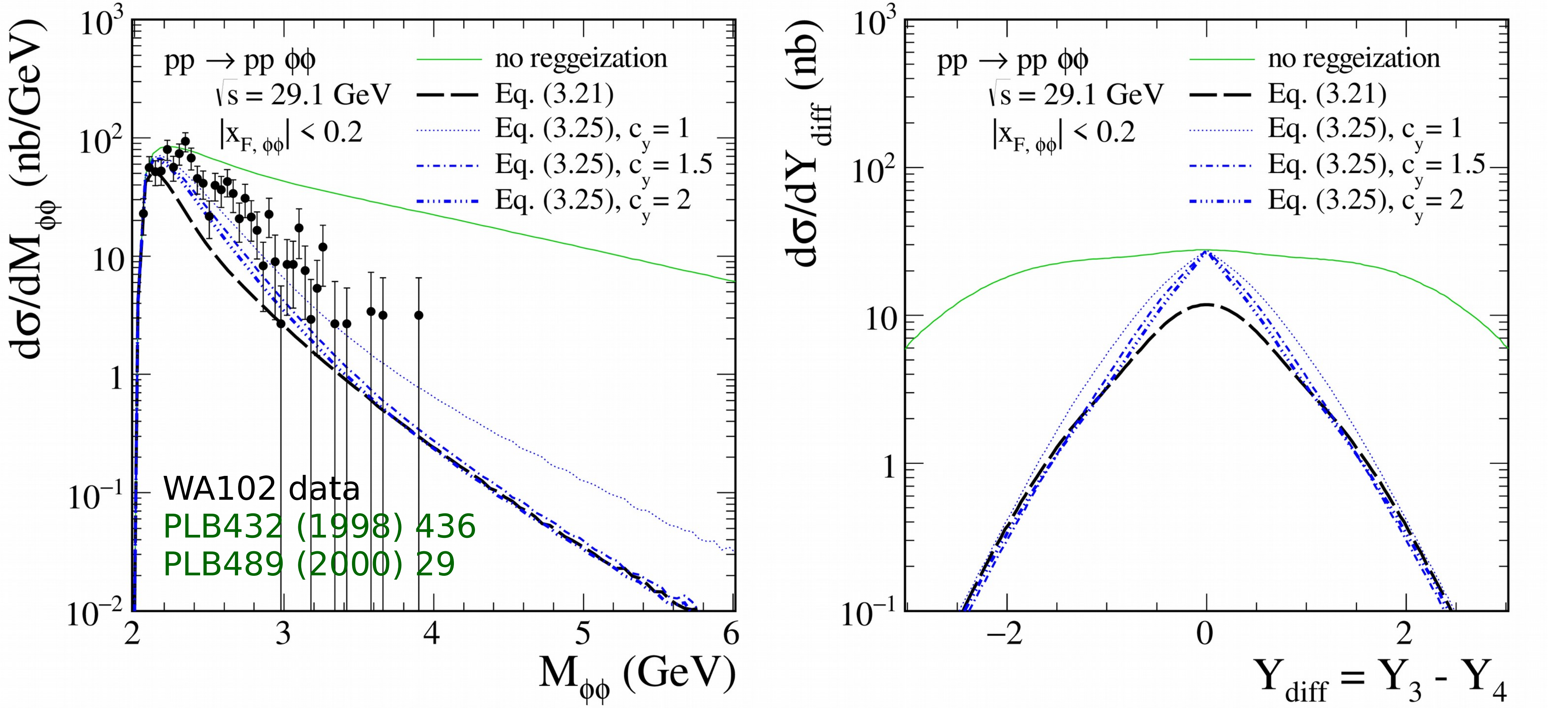
with a simple function

Eq. (3.25)

$$F(\mathrm{Y_{diff}}) = \exp\left(-\mathrm{c_y}|\mathrm{Y_{diff}}|\right)$$

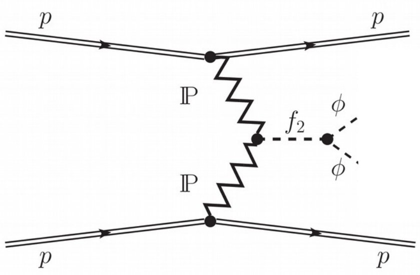
Here $\mathrm{c_y}$ is an unknown parameter which measures how fast one approaches to the Regge regime.

$pp \to pp\phi\phi$ (ϕ-exchange continnum)



- The distributions in $\phi\phi$ invariant mass and in Y_{diff}, the rapidity distance between the two $\phi\phi$ mesons, for the ϕ-exchange continuum contribution
- The green solid line corresponds to the non-reggeized contribution.
  The results for the two prescription of reggeization are shown by the black and blue lines
- The absorption effects calculated at the amplitude level (*pp* nonperturbative interactions) are included

Table 1: A list of resonances, up to a mass of 2500 MeV, that decay into a vector meson pair. The meson masses m and their total widths Γ are taken from PDG. For $\eta(2100)$ and $X(2500)$ the information is taken from BESIII experiment (arXiv:1602.01523). In the columns: • indicates rather established particles, (?) denotes the states that need further experimental confirmation.



Meson	$I^G J^{PC}$	m (MeV)	Γ (MeV)	$\phi\phi$	$K^{*0}\bar{K}^{*0}$	$\rho^0\rho^0$	$\omega\omega$
• $f_1(1285)$	0^+1^{++}	1281.9 ± 0.5	22.7 ± 1.1			Seen	
• $f_0(1370)$	0^+0^{++}	$1200 - 1500$	$200 - 500$			Dominant	Not seen
• $f_0(1500)$	0^+0^{++}	1504 ± 6	109 ± 7			Seen	
$f_2(1565)$	0^+2^{++}	1562 ± 13	134 ± 8			Seen	Seen
$f_2(1640)$	0^+2^{++}	1639 ± 6	99^{+60}_{-40}				Seen
• $f_0(1710)$	0^+0^{++}	1723^{+6}_{-5}	139 ± 8				Seen
$\eta(1760)$	0^+0^{-+}	1751 ± 15	240 ± 30			Seen	Seen
$f_2(1910)$	0^+2^{++}	1903 ± 9	196 ± 31			Seen	Seen
• $f_2(1950)$	0^+2^{++}	1944 ± 12	472 ± 18		Seen		
• $f_2(2010)$	0^+2^{++}	2011^{+60}_{-80}	202 ± 60	Seen			
$f_0(2020)$	0^+0^{++}	1992 ± 16	442 ± 60			Seen	Seen
$f_0(2100)$	0^+0^{++}	2101 ± 7	224^{+23}_{-21}	Seen (?)			
$\eta(2100)$	0^+0^{-+}	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	Seen (?)			
• $f_4(2050)$	0^+4^{++}	2018 ± 11	237 ± 18				Seen
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$	2231.1 ± 3.5	23^{+8}_{-7}	Not seen			
$\eta(2225)$	0^+0^{-+}	2221^{+13}_{-10}	185^{+40}_{-20}	Seen (?)			
• $f_2(2300)$	0^+2^{++}	2297 ± 28	149 ± 40	Seen			
$f_4(2300)$	0^+4^{++}	2320 ± 60	250 ± 80			Seen	Seen
• $f_2(2340)$	0^+2^{++}	2345^{+50}_{-40}	322^{+70}_{-60}	Seen			
$X(2500)$	0^+0^{-+}	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	Seen (?)			

The nature of these resonances is not understood at present and a tensor glueball has still not been clearly identified. According to lattice-QCD simulations, the lightest tensor glueball has a mass between 2.2 and 2.4 GeV.

$pp \to pp\phi\phi$ *via* IP IP $\to f_2 \to \phi\phi$

- Now we consider the amplitude through *s*-channel f_2 meson exchange

- f_2(2010), f_2(2300) and f_2(2340) mesons could be considered as potential candidates

$$\mathcal{M}^{(\mathbb{P}\mathbb{P}\to f_2\to\phi\phi)\,\rho_3\rho_4}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\phi\phi} = (-i)\,\bar{u}(p_1,\lambda_1) i\Gamma^{(\mathbb{P}pp)\,\mu_1\nu_1}(p_1,p_a)u(p_a,\lambda_a)\; i\Delta^{(\mathbb{P})}_{\mu_1\nu_1,\alpha_1\beta_1}(s_1,t_1)$$
$$\times\, i\Gamma^{(\mathbb{P}\mathbb{P}f_2)\,\alpha_1\beta_1,\alpha_2\beta_2,\rho\sigma}(q_1,q_2)\; i\Delta^{(f_2)}_{\rho\sigma,\alpha\beta}(p_{34})\; i\Gamma^{(f_2\phi\phi)\,\alpha\beta\rho_3\rho_4}(p_3,p_4)$$
$$\times\, i\Delta^{(\mathbb{P})}_{\alpha_2\beta_2,\mu_2\nu_2}(s_2,t_2)\;\bar{u}(p_2,\lambda_2) i\Gamma^{(\mathbb{P}pp)\,\mu_2\nu_2}(p_2,p_b)u(p_b,\lambda_b)$$

where $s_1 = (p_1+p_3+p_4)^2$, $s_2 = (p_2+p_3+p_4)^2$, $q_1 = p_a - p_1$, $q_2 = p_b - p_2$, $t_1 = q_1^2$, $t_2 = q_2^2$, and $p_{34} = q_1 + q_2 = p_3 + p_4$.

- The $I\!PI\!Pf_2$ vertex, including a form factor, can be written as

Lebiedowicz, Nachtmann, and Szczurek PRD93 (2016) 054015

$$i\Gamma^{(\mathbb{P}\mathbb{P}f_2)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = \left(i\Gamma^{(\mathbb{P}\mathbb{P}f_2)(1)}_{\mu\nu,\kappa\lambda,\rho\sigma}\big|_{\text{bare}} + \sum_{j=2}^{7} i\Gamma^{(\mathbb{P}\mathbb{P}f_2)(j)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2)\big|_{\text{bare}} \right) \tilde{F}^{(\mathbb{P}\mathbb{P}f_2)}(q_1^2,q_2^2,p_{34}^2)$$

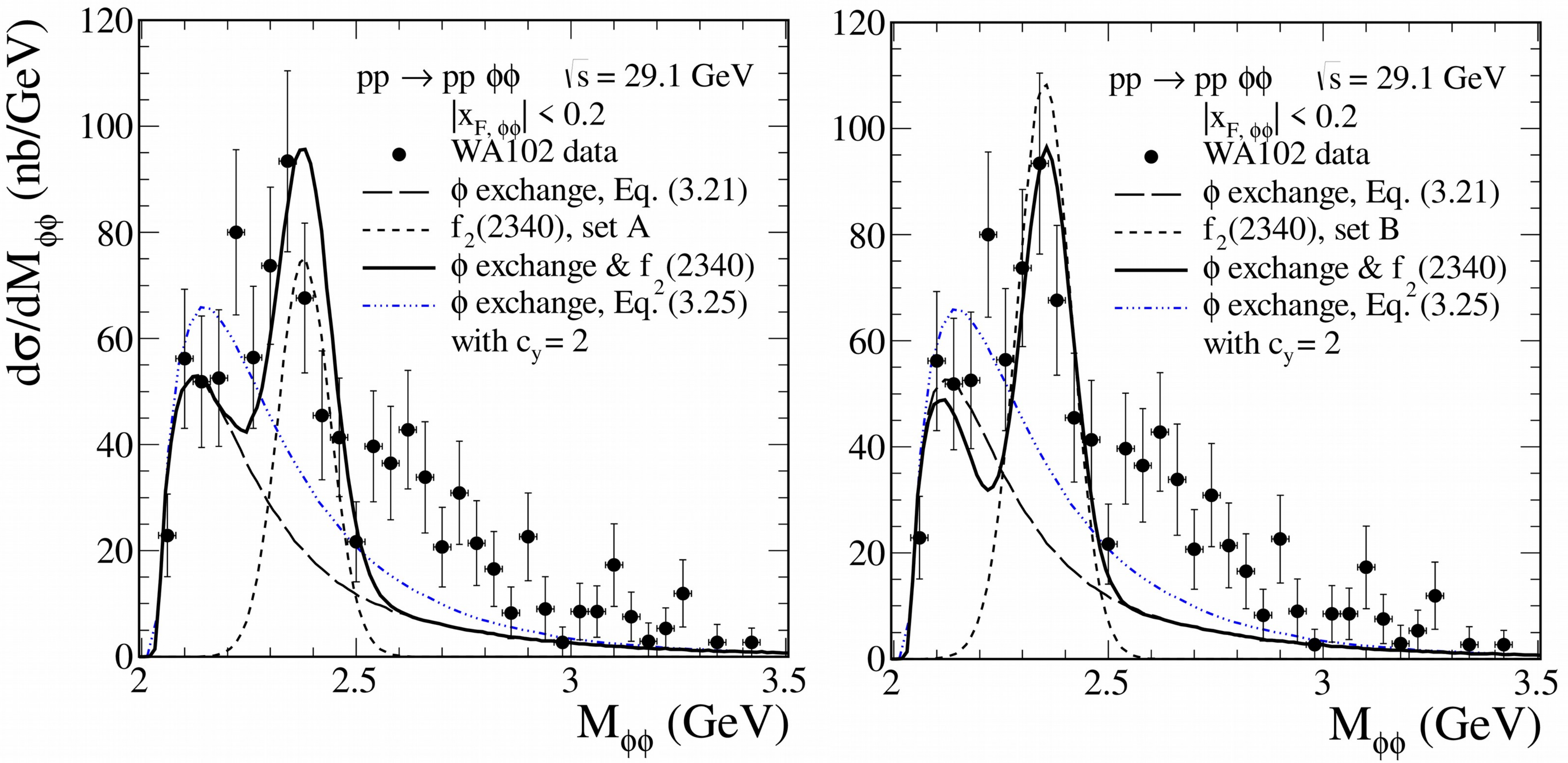
$$i\Gamma^{(I\!PI\!Pf_2)(1)}_{\mu\nu,\kappa\lambda,\rho\sigma} = 2i\, g^{(1)}_{I\!PI\!Pf_2} M_0\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1}\, R_{\rho\sigma\rho_1\sigma_1}\, g^{\nu_1\alpha_1}\, g^{\lambda_1\rho_1}\, g^{\sigma_1\mu_1}$$

$$R_{\mu\nu\kappa\lambda} = \frac{1}{2}g_{\mu\kappa}g_{\nu\lambda} + \frac{1}{2}g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{4}g_{\mu\nu}g_{\kappa\lambda}$$

- For the $f_2\phi\phi$ vertex we take (in analogy to $f_2\gamma\gamma$ vertex)

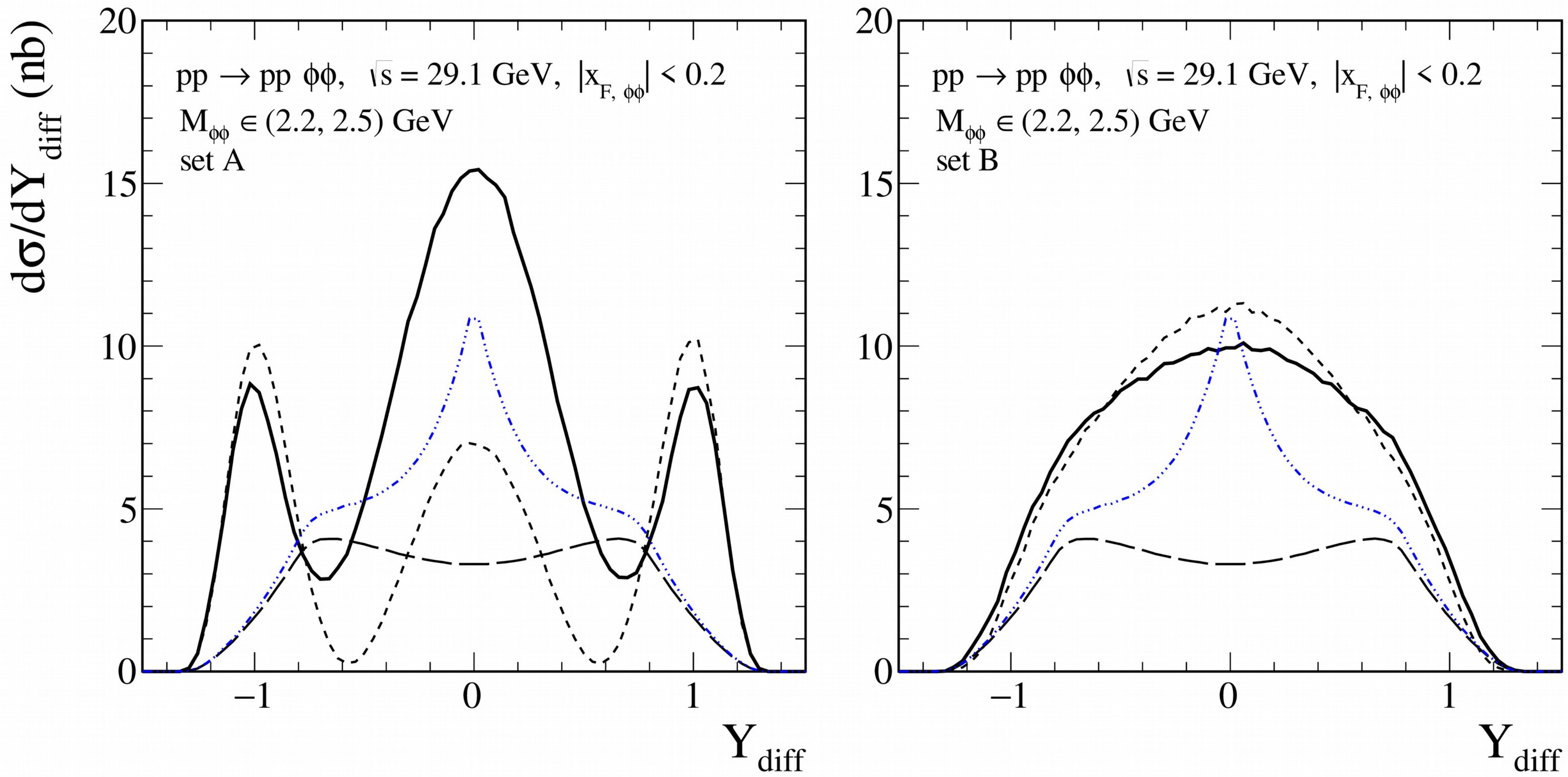
$$i\Gamma^{(f_2\phi\phi)}_{\mu\nu\kappa\lambda}(p_3,p_4) = i\frac{2}{M_0^3} g'_{f_2\phi\phi}\,\Gamma^{(0)}_{\mu\nu\kappa\lambda}(p_3,p_4)\,F'^{(f_2\phi\phi)}(p_{34}^2)$$
$$-i\frac{1}{M_0} g''_{f_2\phi\phi}\,\Gamma^{(2)}_{\mu\nu\kappa\lambda}(p_3,p_4)\,F''^{(f_2\phi\phi)}(p_{34}^2)$$

$pp \to pp\phi\phi$



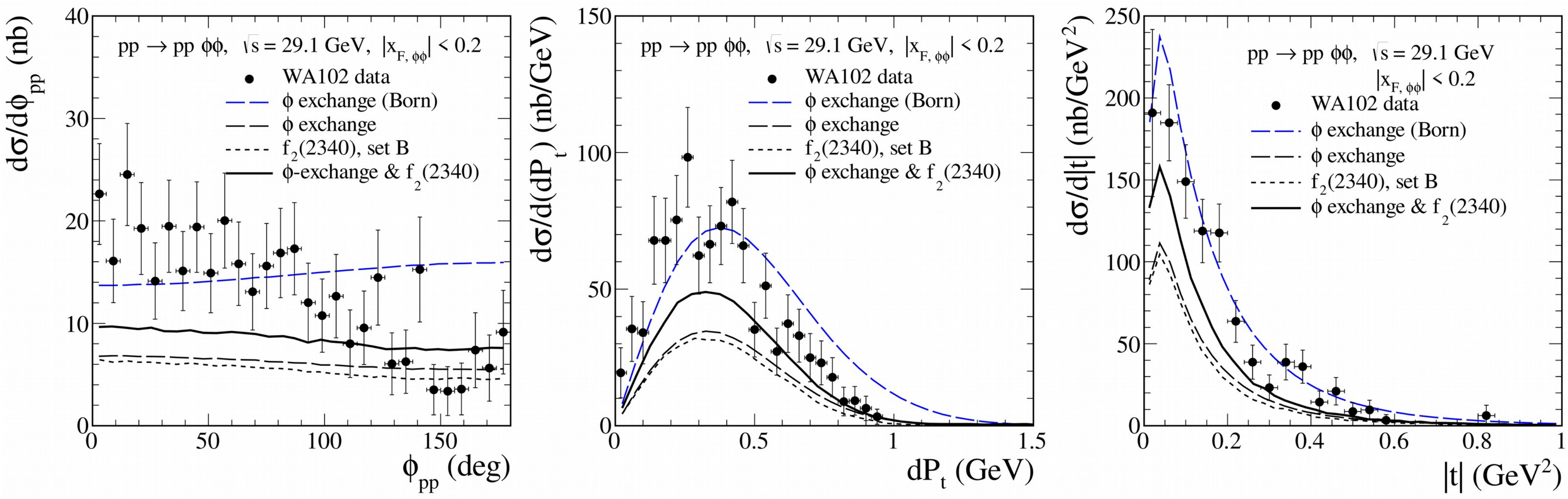
- We show results for two sets of parameters: set A (left panel) $g^{(1)}_{I\!PI\!Pf_2} g'_{f_2\phi\phi} \neq 0$ and set B (right panel) $g^{(1)}_{I\!PI\!Pf_2} g''_{f_2\phi\phi} \neq 0$
- The interference of the continuum and resonance contributions depends on subtle details (choice of the couplings for resonant term, reggeization)

$pp \to pp\phi\phi$



- We have checked that the shapes of Y_{diff} distributions do not depend significantly on the choice of the IP IP f_2 vertex coupling
- The distribution in Y_{diff} can be used to determine the $f_2(2340) \to \phi\phi$ coupling → using results expected from LHC measurements, in particular, if they cover a wider range of rapidities

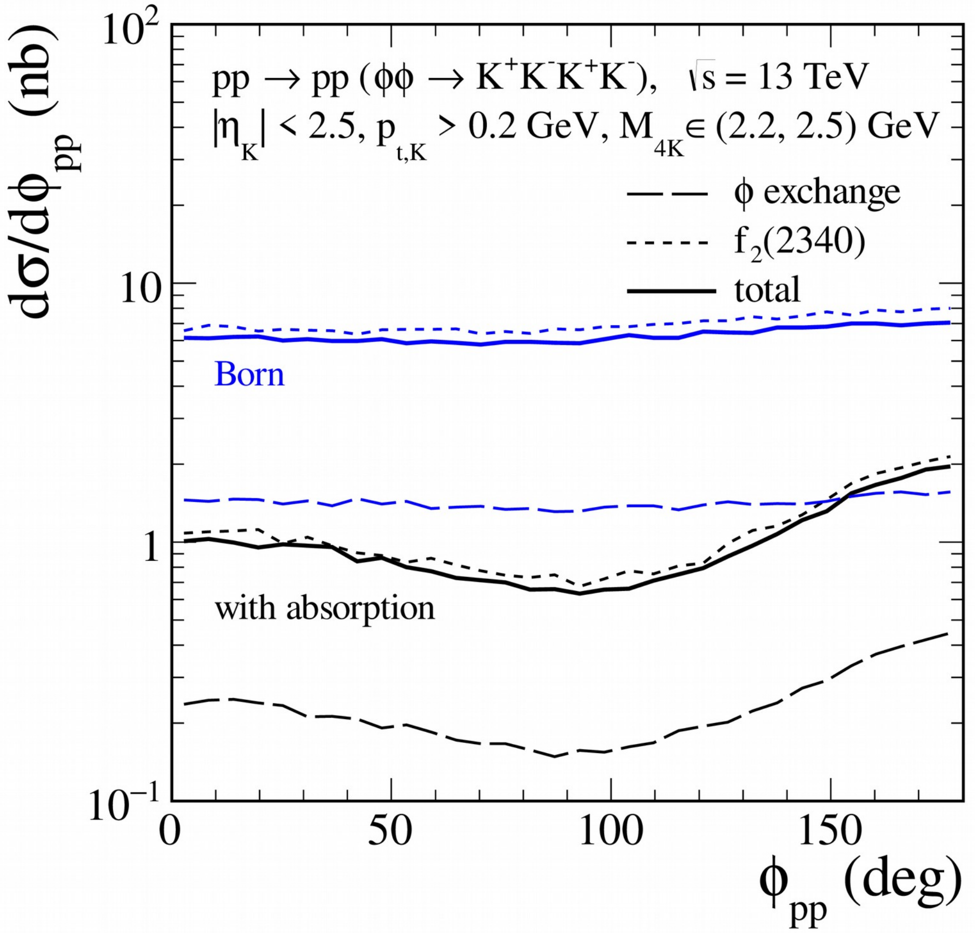
$pp \to pp\phi\phi$

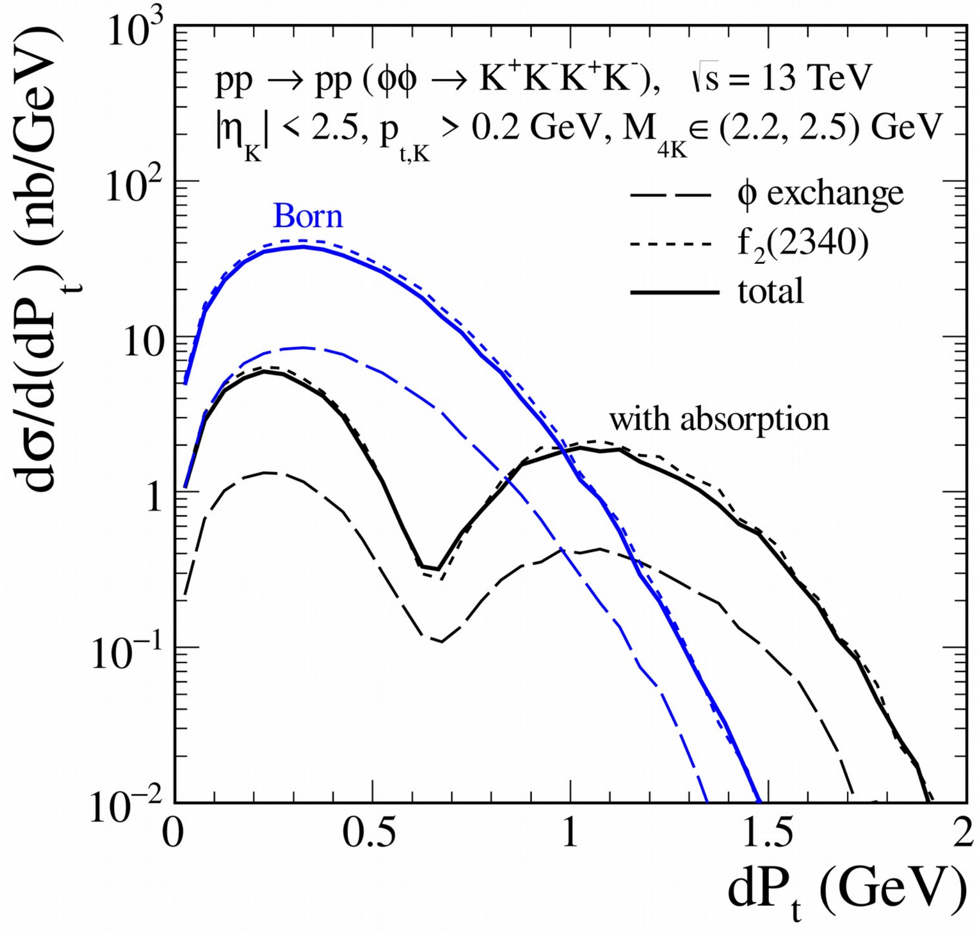


Here $\mathrm{dP_t}$ is the "glueball-filter variable" $\boldsymbol{dP_t} = \boldsymbol{q_{t,1}} - \boldsymbol{q_{t,2}} = \boldsymbol{p_{t,2}} - \boldsymbol{p_{t,1}}$, $\mathrm{dP_t} = |\boldsymbol{dP_t}|$ and ϕ_{pp} is the azimuthal angle between the transverse momentum vectors $\boldsymbol{p_{t,1}}$, $\boldsymbol{p_{t,2}}$ of the outgoing protons.

- Quite a different pattern can be seen for the Born case and for the case with absorption. The ratio of full and Born cross sections is $S_g \sim 0.4$ (WA102 kinematics).
- Glueball candidates should be prominent for $\mathrm{dP_t} \to 0$.

$pp \to pp\ (\phi\phi \to K^+K^-K^+K^-)$





The ratio of full and Born cross sections is S_g ~0.2 (LHC kinematics).

Absorption effect leads to significant modification of these distributions.
This effect could be verified in future experiments when both protons are measured, e.g. by the ATLAS-ALFA and CMS-TOTEM experimental groups.

$pp \to pp\ (\phi\phi \to K^+K^-K^+K^-)$

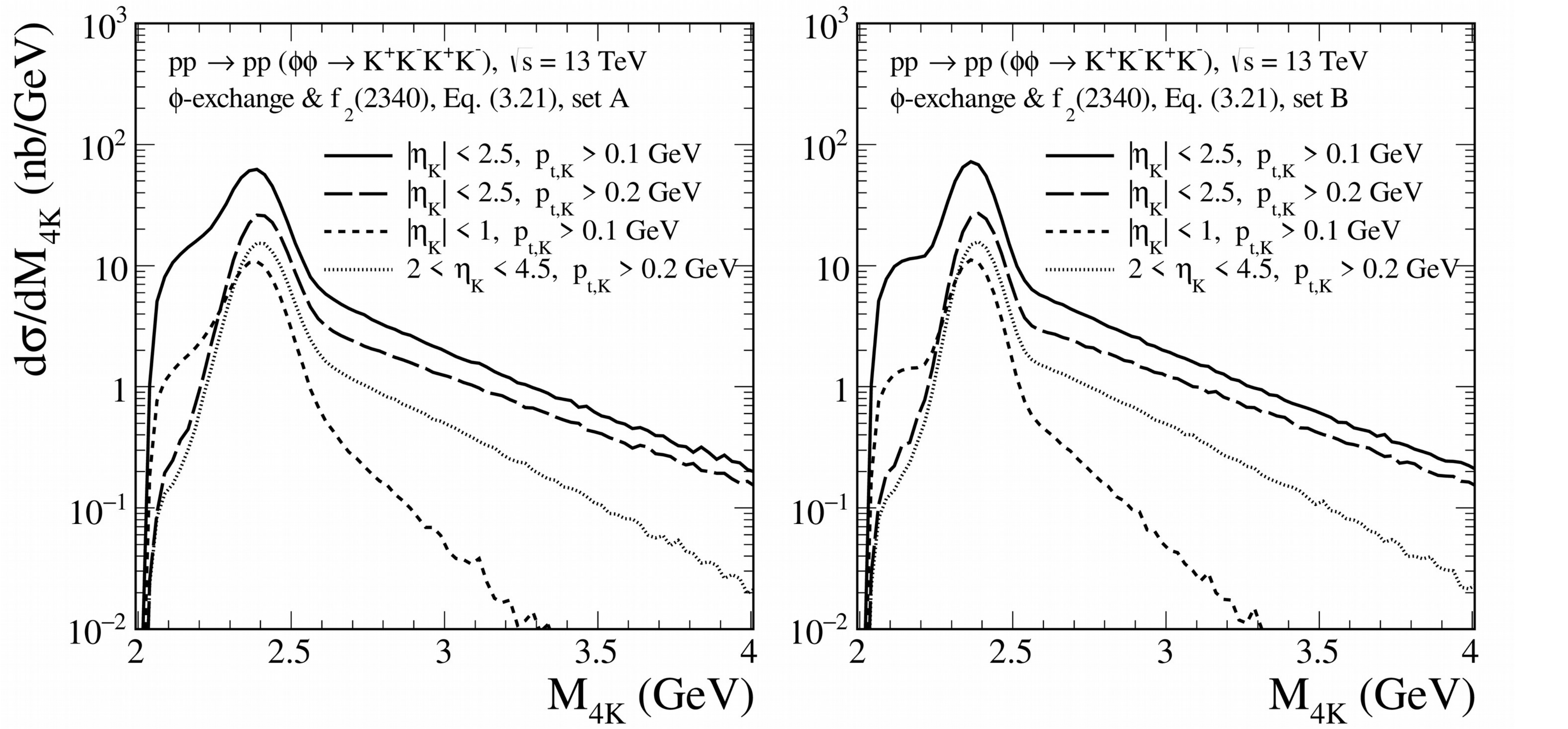
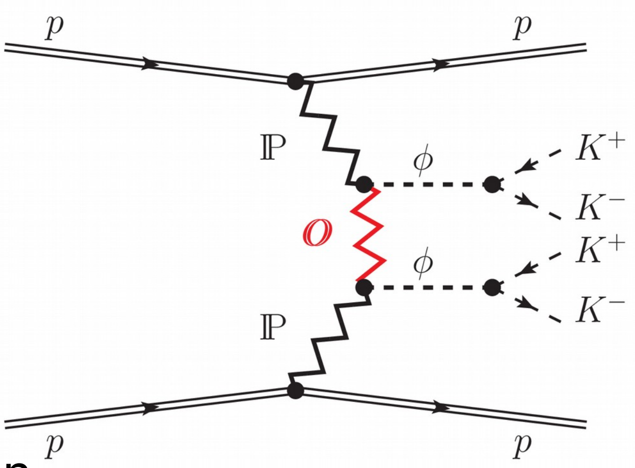


Table 1: The integrated **cross sections in nb** for the $pp \to pp(4K)$ reaction. The absorption effects are included here.

$\sqrt{s}$, TeV	Cuts	Total	ϕ exchange	$f_2(2340)$ (set B)
13	$\lvert\eta_K\rvert < 1$, $p_{t,K} > 0.1$ GeV	2.11	0.83	2.00
13	$\lvert\eta_K\rvert < 2.5$, $p_{t,K} > 0.1$ GeV	16.16	8.30	12.80
13	$\lvert\eta_K\rvert < 2.5$, $p_{t,K} > 0.2$ GeV	5.75	2.67	4.47
13	$2 < \eta_K < 4.5$, $p_{t,K} > 0.2$ GeV	3.06	1.26	2.62

Continuum with Odderon exchange



- The amplitude as for exchange, but we have to make:

$$i\Delta^{(\phi)}_{\mu\nu}(\hat{p}) \to i\Delta^{(\mathbb{O})}_{\mu\nu}(s_{34},\hat{p}^2)$$

$$i\Gamma^{(I\!\!P\phi\phi)}_{\mu\nu\kappa\lambda}(k',k) \to i\Gamma^{(I\!\!P\mathbb{O}\phi)}_{\mu\nu\kappa\lambda}(k',k)$$

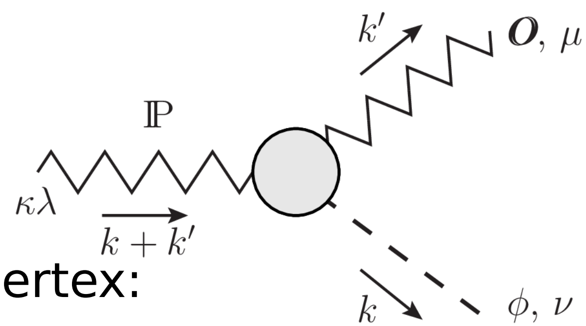
- Our ansatz for the effective propagator of C = -1 Odderon

$$i\Delta^{(\mathbb{O})}_{\mu\nu}(s,t) = -ig_{\mu\nu}\frac{\eta_{\mathbb{O}}}{M_0^2}(-is\alpha'_{\mathbb{O}})^{\alpha_{\mathbb{O}}(t)-1}\,, \qquad M_0 = 1\ \mathrm{GeV}$$

$$\alpha_{\mathbb{O}}(t) = \alpha_{\mathbb{O}}(0) + \alpha'_{\mathbb{O}}\,t$$

we shall assume representative values for Odderon parameters

$$\eta_{\mathbb{O}} = -1\,, \alpha'_{\mathbb{O}} = 0.25\ \mathrm{GeV}^{-2}\,, \quad \alpha_{\mathbb{O}}(0) = 1.05$$



- For the $I\!\!P\mathbb{O}\phi$ vertex we use an ansatz analogous to $I\!\!P\phi\phi$ vertex:

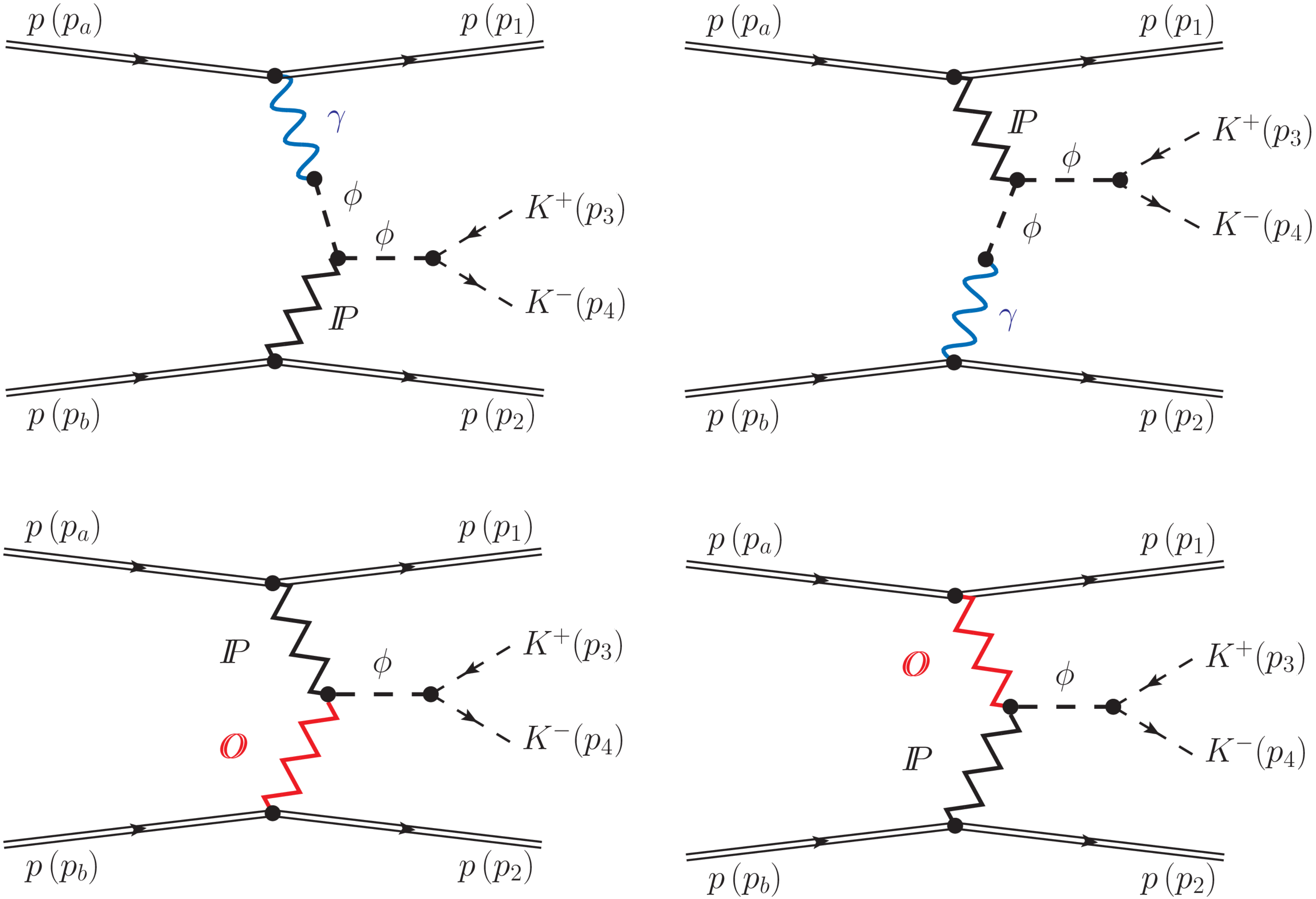
$$i\Gamma^{(I\!\!P\mathbb{O}\phi)}_{\mu\nu\kappa\lambda}(k',k) = iF^{(I\!\!P\mathbb{O}\phi)}((k+k')^2,k'^2,k^2)\left[2\,a_{I\!\!P\mathbb{O}\phi}\,\Gamma^{(0)}_{\mu\nu\kappa\lambda}(k',k) - b_{I\!\!P\mathbb{O}\phi}\,\Gamma^{(2)}_{\mu\nu\kappa\lambda}(k',k)\right]$$

In practical calculations we take the factorized form for the $I\!\!P\mathbb{O}\phi$ form factor

$$F^{(I\!\!P\mathbb{O}\phi)}((k+k')^2,k'^2,k^2) = F((k+k')^2)\,F(k'^2)\,F^{(I\!\!P\mathbb{O}\phi)}(k^2)\,,$$

$$F(k^2) = \frac{1}{1-k^2/\Lambda^2_{\mathrm{odd}}}\,, \qquad F^{(I\!\!P\mathbb{O}\phi)}(0,0,m_\phi^2) = 1$$

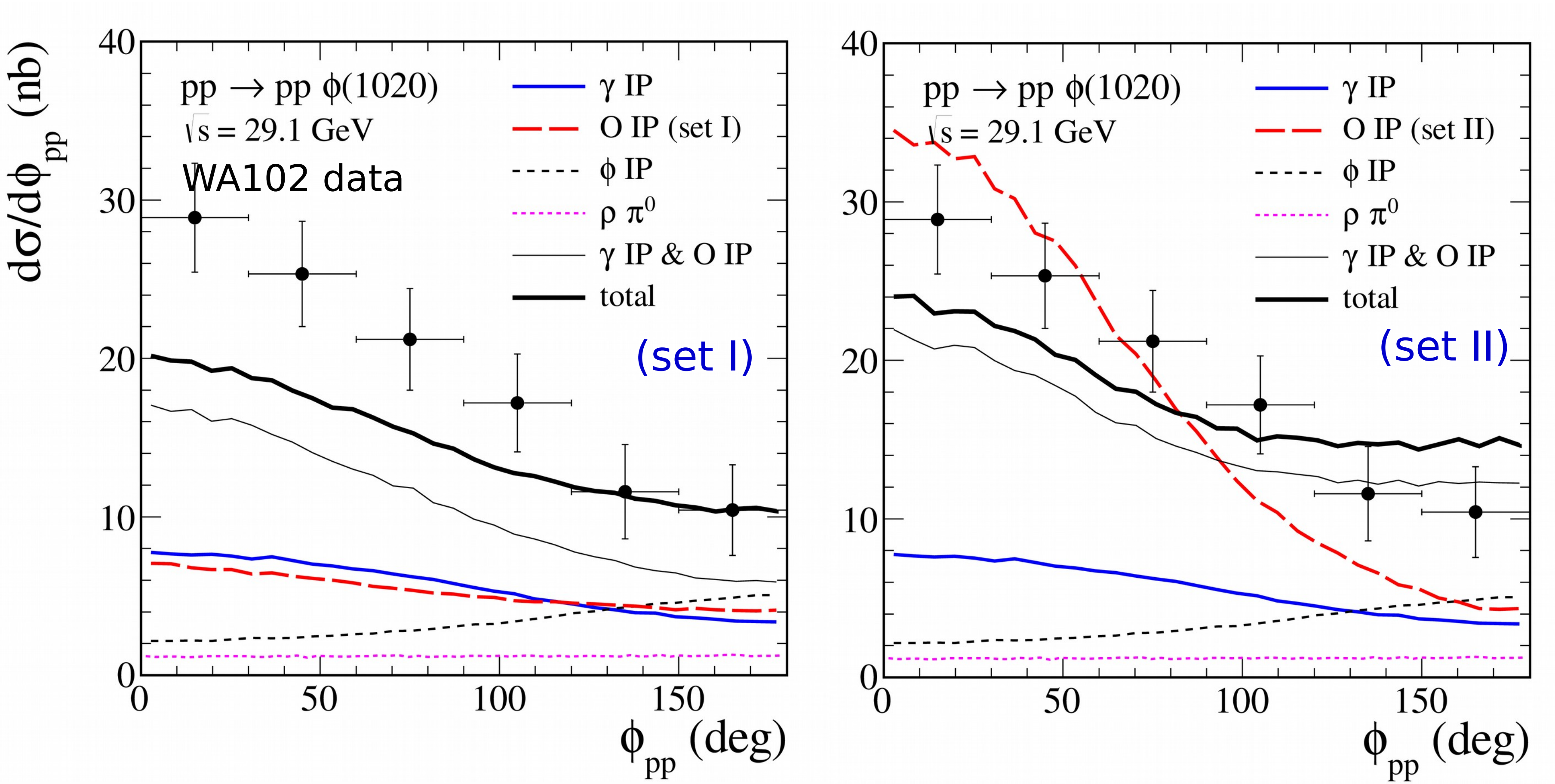
$pp \to pp\phi$



$$\beta_{\mathbb{O}NN} = 0.1\ \beta_{I\!PNN}$$

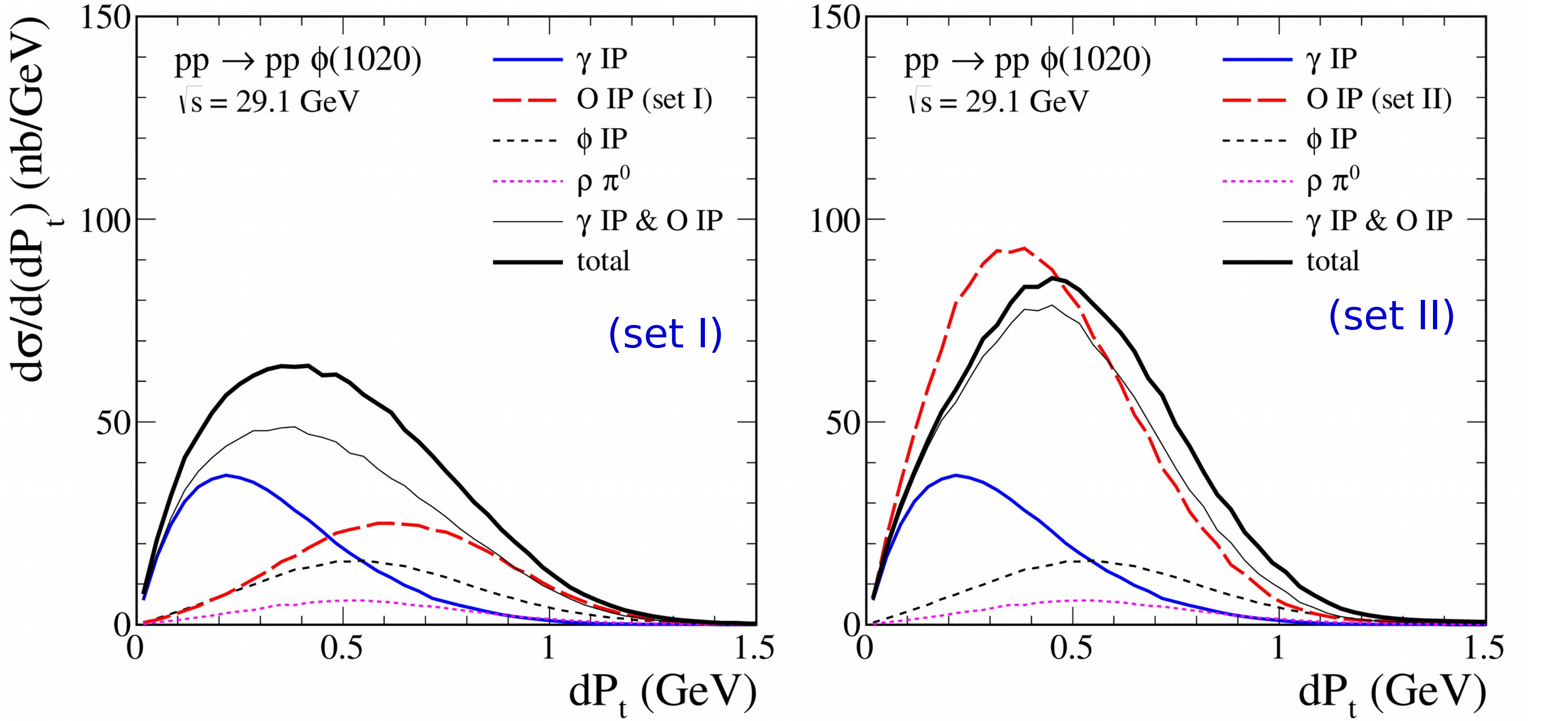
(educated guess, TOTEM)

$pp \to pp\phi$



- WA102 data from PLB489 (2000) 29
- Other mechanisms were also considered.
  The WA102 data support the existence of Odderon exchange !
- The ϕ_{pp} distribution allow us to determine the respective coupling constants $a_{I\!P\mathbb{O}\phi}$ and $b_{I\!P\mathbb{O}\phi}$ (here we show our preliminary results, set I and set II)
- Strong interference of γ IP and O IP

$pp \to pp\phi$



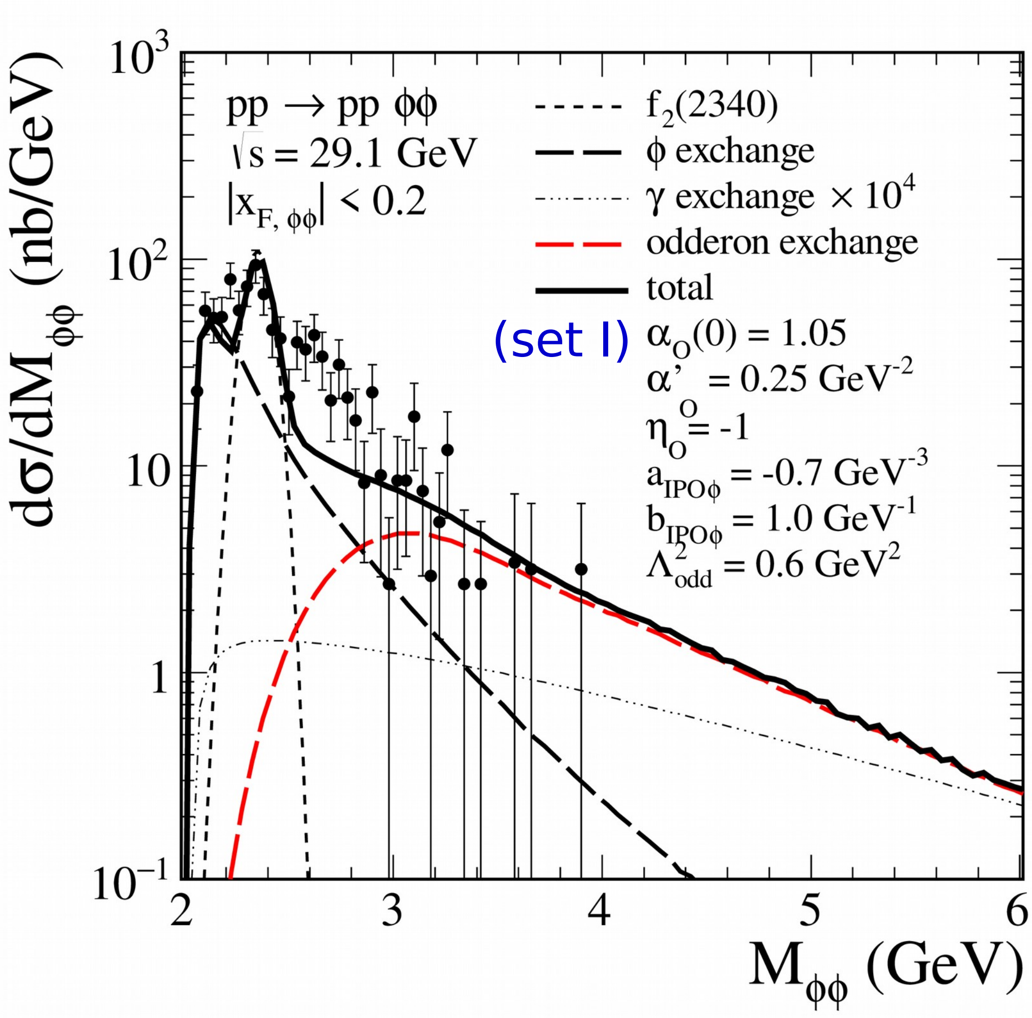
$$R = \frac{d\sigma/d(\mathrm{dP_t} \leqslant 0.2 \ \mathrm{GeV})}{d\sigma/d(\mathrm{dP_t} \geqslant 0.5 \ \mathrm{GeV})}$$

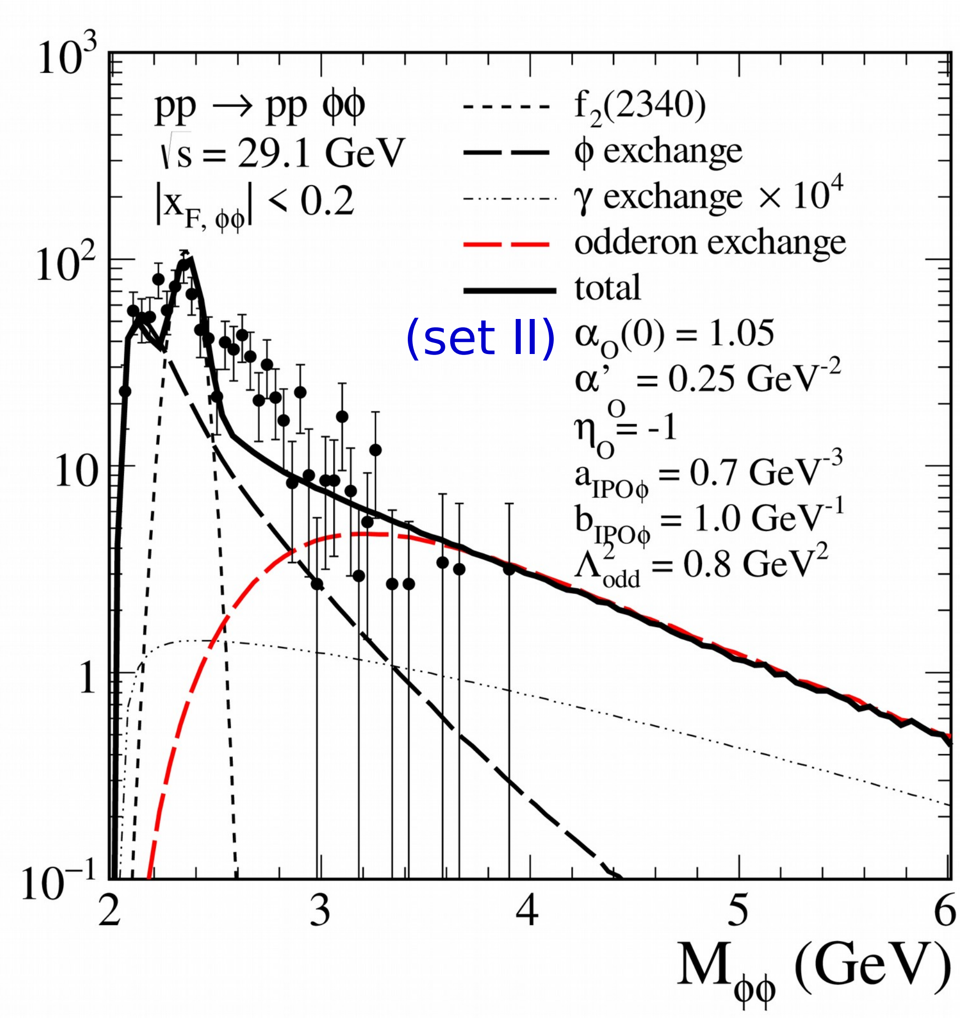
$R_{\mathrm{WA102}} = 0.18 \pm 0.07$ ← WA102 result, PLB489 (2000) 29

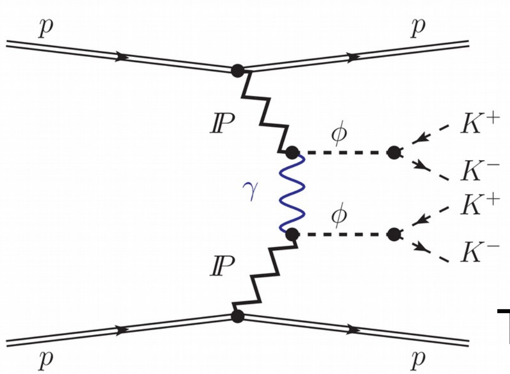
$R_{\mathrm{Set\ I}} = 0.33, \ R_{\mathrm{Set\ II}} = 0.24$ ←- our preliminary results

- We find that two couplings $a_{I\!P\mathbb{O}\phi}$ and $b_{I\!P\mathbb{O}\phi}$ are needed
- It would be very useful to measure the outgoing protons at the LHC

Return to $pp \to pp\phi\phi$



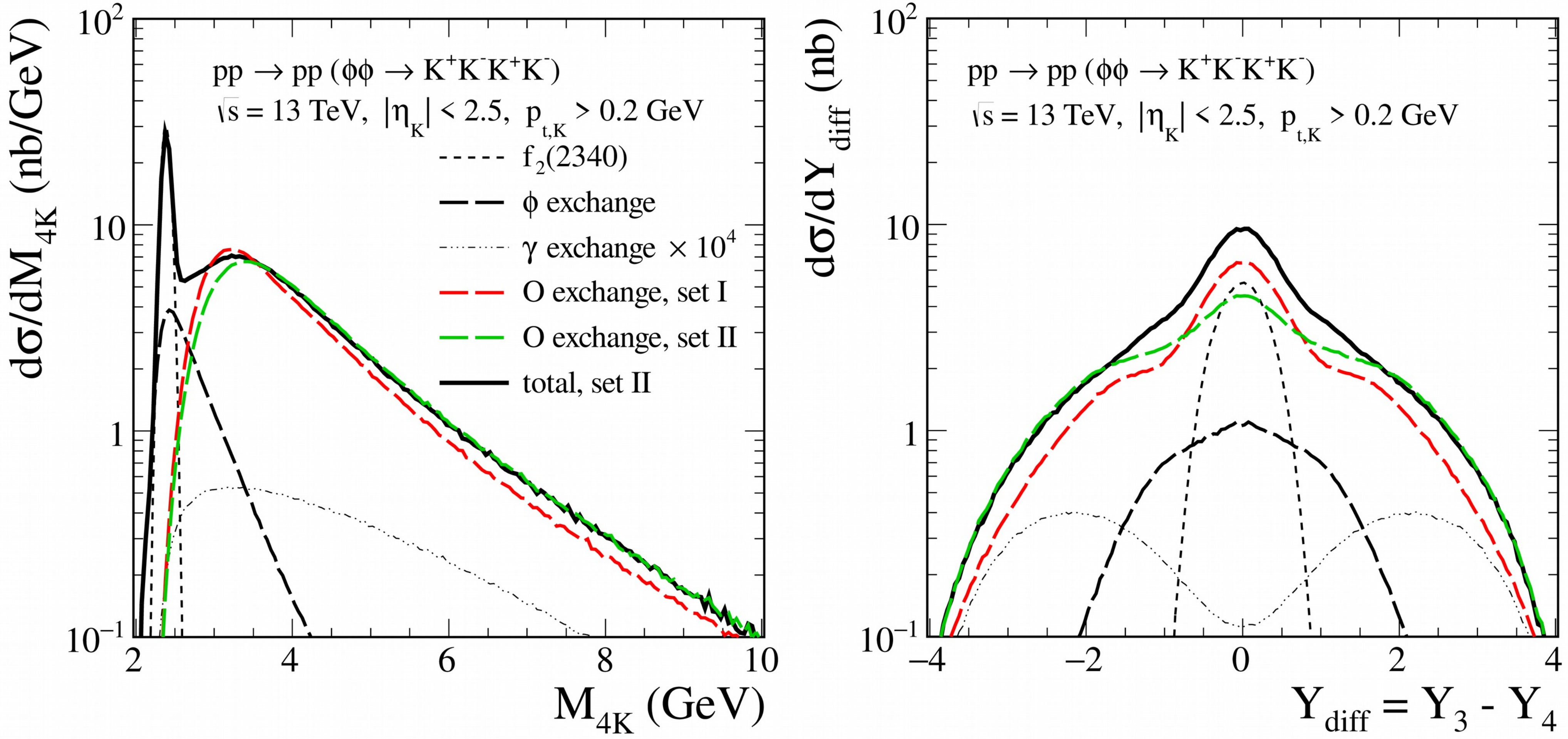




An upper limit for the Odderon exchange has been established based on the WA102 data

This process (γ exchange continuum) turned out to be small

$pp \to pp\ (\phi\phi \to K^+K^-K^+K^-)$



The small intercept of the ϕ reggeon exchange, $\alpha_\phi(0) = 0.1$ makes the ϕ-exchange contribution steeply falling with increasing M_{4K} and $|Y_{\rm diff}|$. Therefore, an Odderon with an intercept $\alpha(0) \sim 1.0$ should be clearly visible in these distributions.

Conclusions

- The tensor-Pomeron and vector-Odderon model was applied to many reactions.

  All amplitudes are formulated in terms of effective vertices and propagators respecting the standard crossing and charge conjugation relations of QFT

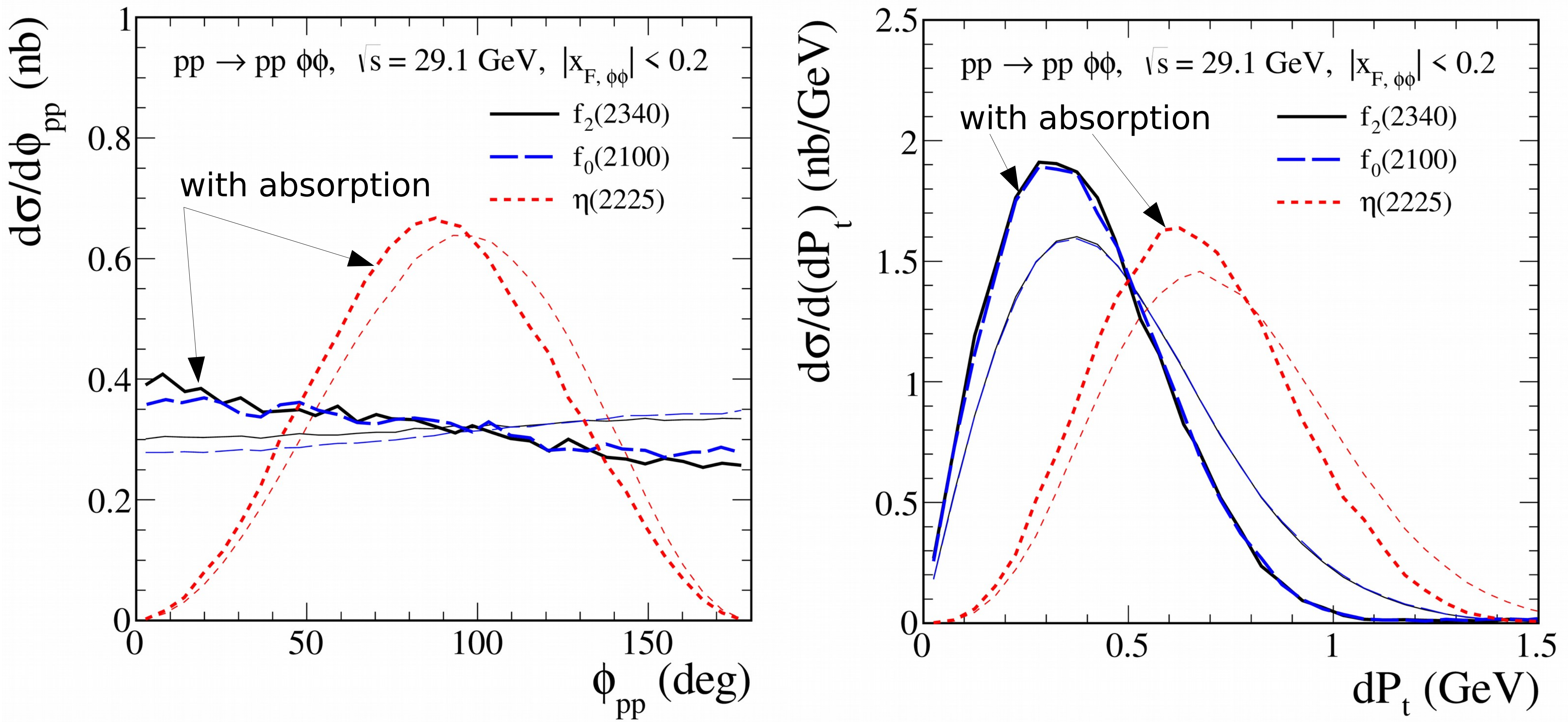
- The $pp \to pp\, K^+K^-K^+K^-$ reaction have been studied in the context of identifying Odderon exchange.
  We find from our model that the Odderon exchange contribution should be distinguishable from other contributions for large rapidity distance between the ϕ mesons and in the region of large four-kaon invariant masses (outside of the region of resonances)

- The $\phi\phi$ invariant mass distribution has a rich structure (continuum, resonances, interference effects) which strongly depends on kinematical cuts.
  $\phi\phi$ seems to be a favorable channel for the search for tensor glueball

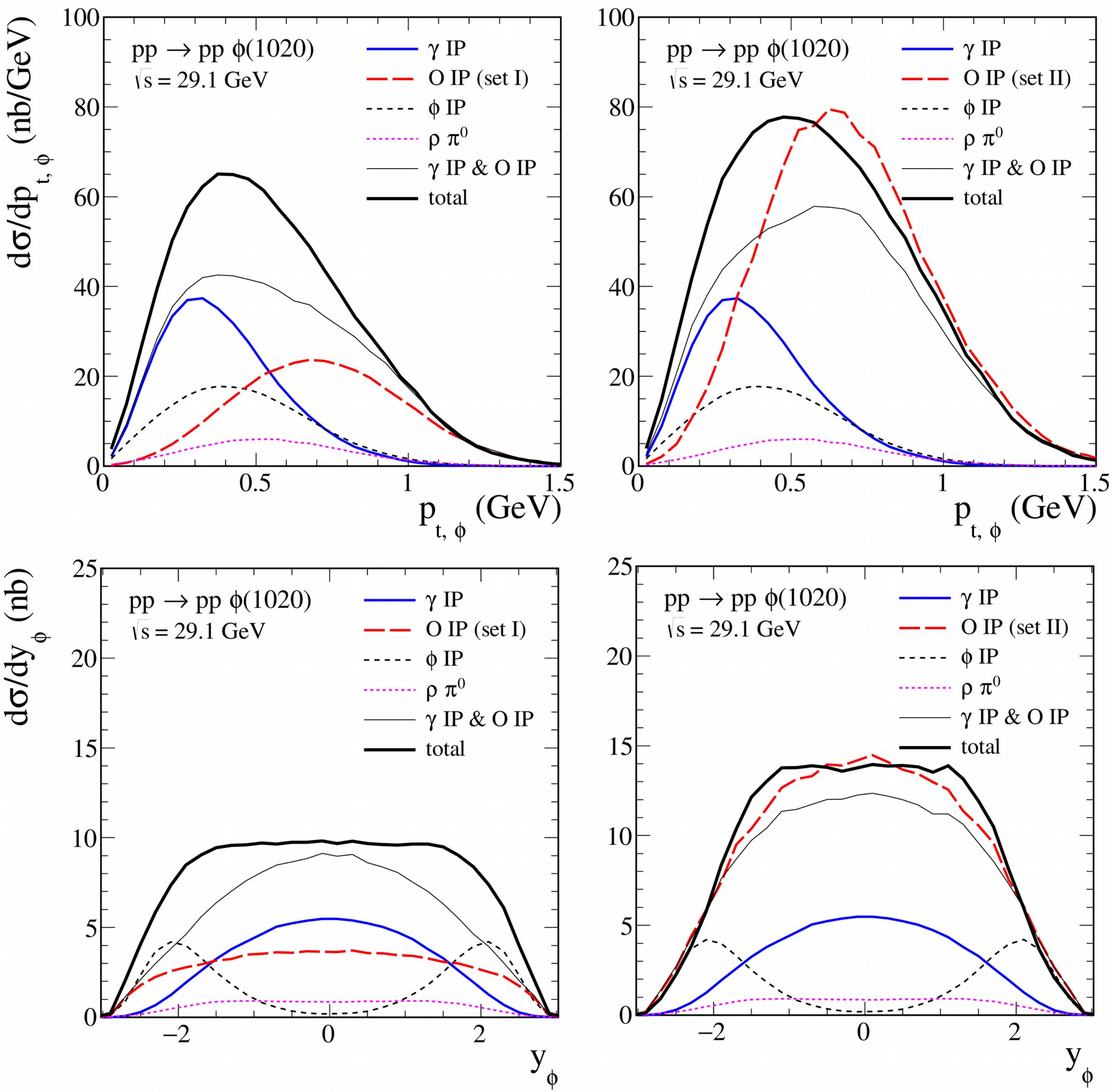
- Comparison with 'exclusive' data expected from LHCb, ALICE, CMS+TOTEM, ATLAS+ALFA, and STAR experiments should provide further information on CEP and should be very valuable for clarifying the status of the Odderon

Thank you for your attention!

$pp \to pp\phi\phi$



The distribution in $\mathrm{dP_t}$ and in ϕ_{pp} for the central exclusive $\phi\phi$ production at $\sqrt{s} = 29.1$ GeV and $|x_{F,\phi\phi}| \leqslant 0.2$. The results for scalar, pseudoscalar and tensor resonances without (the thin lines) and with (the thick lines) absorptive corrections are shown. Because here we are interested only in the shape of the distributions we normalised the differential distributions arbitrarily to 1 nb for both cases, with and without absorption corrections.



pp → pp φ(1020)
√s = 29.1 GeV
γ IP
O IP (set I)
φ IP
ρ π0
γ IP & O IP
total
dσ/dp_{t, φ} (nb/GeV)
p_{t, φ} (GeV)
O IP (set II)
dσ/dy_φ (nb)
y_φ

IP IP M couplings

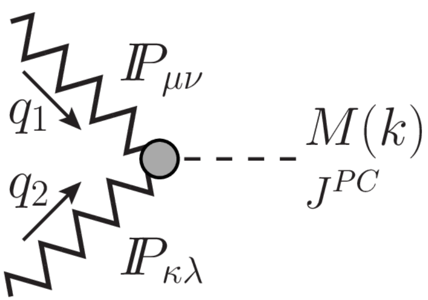
P. L., O. Nachtmann, A. Szczurek,
Annals Phys. 344 (2014) 301;
Phys. Rev. D93 (2016) 054015

l – orbital angular momentum
S – total spin, we have $S \in \{0, 1, 2, 3, 4\}$
J – total angular momentum (spin of the produced meson)
P – parity of meson
and Bose symmetry requires $l - S$ to be even

In table we list the values of *J* and *P* of mesons which can be produced in annihilation of two "real tensor pomerons".

For each value of *l*, *S*, *J*, and *P* we can construct a covariant Lagrangian density coupling *L*' the field operator for the meson *M* to the pomeron fields and then we can obtain the "bare" vertices corresponding to the *l* and *S*.

The lowest (*l*,*S*) term for a scalar meson $J^{PC} = 0^{++}$ is (0,0) while for a tensor meson $J^{PC} = 2^{++}$ is (0,2).

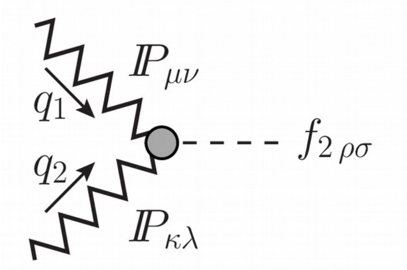


l	S	$\lvert l - S\rvert \leqslant J \leqslant l + S$	$P = (-1)^l$
0	0	0	+
	2	2	
	4	4	
1	1	0, 1, 2	−
	3	2, 3, 4	
2	0	2	+
	2	0,1,2,3,4	
	4	2,3,4,5,6	
3	1	2,3,4	−
	3	0,1,2,3,4,5,6	
4	0	4	+
	2	2,3,4,5,6	
	4	0,1,2,3,4,5,6,7,8	
5	1	4.5.6	−
	3	2,3,4,5,6,7,8	
6	0	6	+
	2	4,5,6,7,8	
	4	2,3,4,5,6,7,8,9,10	

There, *l* is related to the number of derivatives in *L*' thus giving an indication of the angular momentum barrier in the production of *M*.

$I\!P$-$I\!P$-f_2 couplings

In order to write the corresponding formulae of vertices in a compact and convenient form we find it useful to define the tensor $R_{\mu\nu\kappa\lambda} = \frac{1}{2} g_{\mu\kappa} g_{\nu\lambda} + \frac{1}{2} g_{\mu\lambda} g_{\nu\kappa} - \frac{1}{4} g_{\mu\nu} g_{\kappa\lambda}$



$$i\Gamma^{(I\!P I\!P f_2)(1)}_{\mu\nu,\kappa\lambda,\rho\sigma} = 2i\, g^{(1)}_{I\!P I\!P f_2} M_0\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1}\, R_{\rho\sigma\rho_1\sigma_1}\, g^{\nu_1\alpha_1}\, g^{\lambda_1\rho_1}\, g^{\sigma_1\mu_1}$$

$$i\Gamma^{(I\!P I\!P f_2)(2)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0}\, g^{(2)}_{I\!P I\!P f_2} \Big((q_1\cdot q_2)\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} - q_{1\rho_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} - q_1^{\mu_1}\, q_{2\sigma_1}\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\mu_1}{}^{\alpha} + q_{1\rho_1}\, q_{2\sigma_1}\, R_{\mu\nu\kappa\lambda} \Big) R_{\rho\sigma}{}^{\rho_1\sigma_1}$$

$$i\Gamma^{(I\!P I\!P f_2)(3)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0}\, g^{(3)}_{I\!P I\!P f_2} \Big((q_1\cdot q_2)\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} + q_{1\rho_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\alpha}\, R_{\kappa\lambda\sigma_1}{}^{\alpha} + q_1^{\mu_1}\, q_{2\sigma_1}\, R_{\mu\nu\rho_1\alpha}\, R_{\kappa\lambda\mu_1}{}^{\alpha} + q_{1\rho_1}\, q_{2\sigma_1}\, R_{\mu\nu\kappa\lambda} \Big) R_{\rho\sigma}{}^{\rho_1\sigma_1}$$

$$i\Gamma^{(I\!P I\!P f_2)(4)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{i}{M_0}\, g^{(4)}_{I\!P I\!P f_2} \Big(q_1^{\alpha_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1} + q_2^{\alpha_1}\, q_1^{\mu_1}\, R_{\mu\nu\alpha_1\lambda_1}\, R_{\kappa\lambda\mu_1\nu_1} \Big) R^{\nu_1\lambda_1}{}_{\rho\sigma}$$

$$i\Gamma^{(I\!P I\!P f_2)(5)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0^3}\, g^{(5)}_{I\!P I\!P f_2} \Big(q_1^{\mu_1}\, q_2^{\nu_1}\, R_{\mu\nu\nu_1\alpha}\, R_{\kappa\lambda\mu_1}{}^{\alpha} + q_1^{\nu_1}\, q_2^{\mu_1}\, R_{\mu\nu\mu_1\alpha}\, R_{\kappa\lambda\nu_1}{}^{\alpha} - 2(q_1\cdot q_2)\, R_{\mu\nu\kappa\lambda} \Big) q_{1\alpha_1}\, q_{2\lambda_1}\, R^{\alpha_1\lambda_1}{}_{\rho\sigma}$$

$$i\Gamma^{(I\!P I\!P f_2)(6)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = \frac{i}{M_0^3}\, g^{(6)}_{I\!P I\!P f_2} \Big(q_1^{\alpha_1}\, q_1^{\lambda_1}\, q_2^{\mu_1}\, q_{2\rho_1}\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1} + q_2^{\alpha_1}\, q_2^{\lambda_1}\, q_1^{\mu_1}\, q_{1\rho_1}\, R_{\mu\nu\alpha_1\lambda_1}\, R_{\kappa\lambda\mu_1\nu_1} \Big) R^{\nu_1\rho_1}{}_{\rho\sigma}$$

$$i\Gamma^{(I\!P I\!P f_2)(7)}_{\mu\nu,\kappa\lambda,\rho\sigma}(q_1,q_2) = -\frac{2i}{M_0^5}\, g^{(7)}_{I\!P I\!P f_2}\, q_1^{\rho_1}\, q_1^{\alpha_1}\, q_1^{\lambda_1}\, q_2^{\sigma_1}\, q_2^{\mu_1}\, q_2^{\nu_1}\, R_{\mu\nu\mu_1\nu_1}\, R_{\kappa\lambda\alpha_1\lambda_1}\, R_{\rho\sigma\rho_1\sigma_1}$$

We can associate the couplings j = 1, ..., 7 with (*l*,*S*) values:
(0,2), (2,0) – (2,2), (2,0) + (2,2), (2,4), (4,2), (4,4), (6,4), respectively.

see P. L., O. Nachtmann, A. Szczurek, Phys. Rev. D93 (2016) 054015