The Biggest Structures in the Universe and the Cosmological Principle

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Natural assumption:

The laws of physics should be the same wherever in the Universe we are

⇓

Cosmological principle:

the Universe is isotropic and homogeneous at large enough scales

scale of homogeneity

↓

Starting from this scale the distribution of inhomogeneities (e.g. galaxies and groups of galaxies) should be statistically homogeneous

Scale of homogeneity ?

A number of attempts to estimate this value in literature:

End of XX-th century: 150-300 Mpc

Recent fractal dimension analysis: $\leq$400 Mpc ←

Effective scales of observation/simulation 1.5-2 Gpc

However, quite recently a number of much larger structures, with sizes up to 3 Gpc, were found.

!

The largest cosmic structures:

Sloan Great Wall. $l \approx 423\,\mathrm{Mpc} \approx 1.38\times10^{9}$ ly

$1\,\mathrm{pc} \approx 3\times10^{18}\,\mathrm{cm} \approx 3$ ly

Giant filament consisting of a number of superclusters

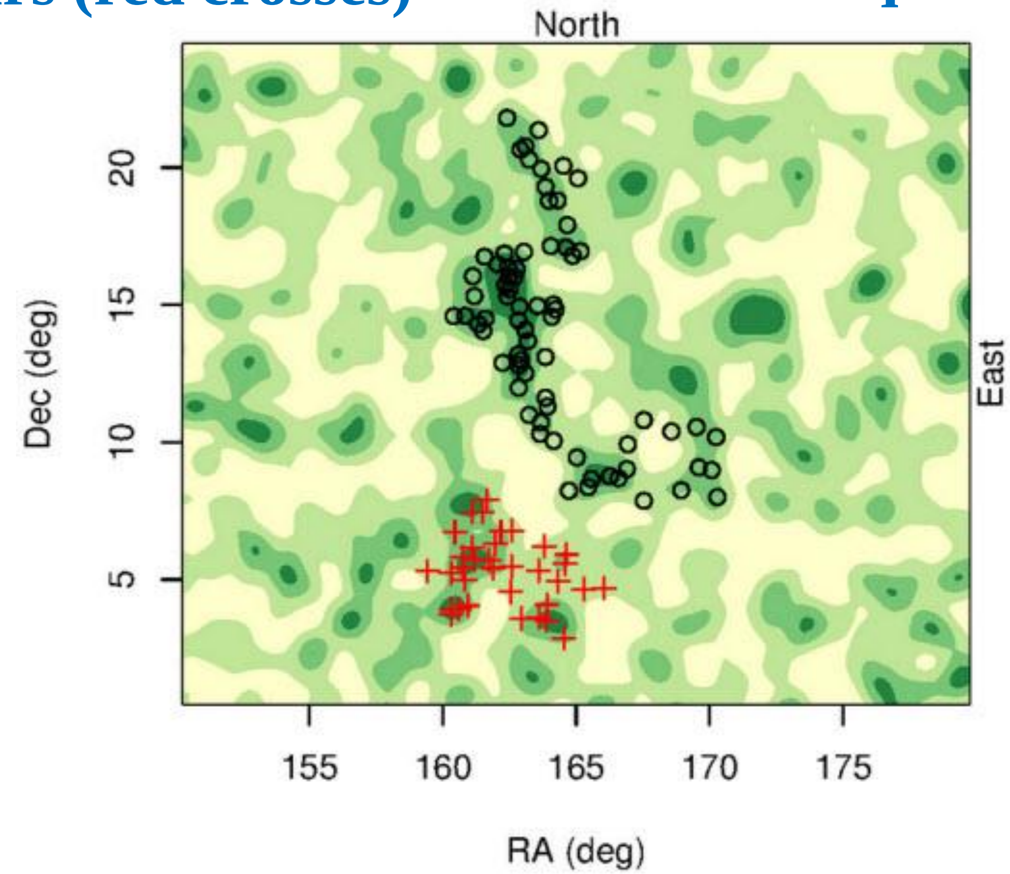
Two groups of Large Quasar Groups:

Clowes-Campusano LQG $l \approx 613\,\mathrm{Mpc} \approx 2\times 10^9$ ly

and Huge LQG $l \approx 1226\,\mathrm{Mpc} \approx 4\times 10^9$ ly

34 quasars (red crosses)

73 quasars (black rings)



Quasars: active nuclei of galaxies (e.g. super BH)

Great GRB Wall (Hercules-Corona-Borealis Great Wall)

$$l \approx 3066\,\mathrm{Mpc} \approx 10 \times 10^{9}\ \mathrm{ly}$$

A region of the sky seen in the data set mapping of gamma-ray bursts (GRBs) that has been found to have an unusually higher concentration of similarly distanced GRBs than the expected average distribution

Burst of gamma-rays during supernova collapse or the merger of two neutron stars into a new BH

The discovery of such superstructures raises a number of questions:

- **Do these structures contradict the cosmological principle?**
- **Does the upper limit on the superstructure size exist?**
- **Can we predict this limit?**

The answers to these questions are interrelated:

if the upper limit exists and the superstructures are within this limit, then their existence does not contradict the cosmological principle. The homogeneity scale is simply greater than this limit.

Upper limit on the superstructure size ?

Gravitational interaction is responsible for the structure formation

Newtonian mechanics : gravitating masses in Euclidian space

↑ Static empty background

Poisson eq. $\Delta\Phi = \frac{4\pi G_N}{c^2}\rho \equiv \frac{\kappa}{2}\rho c^2, \quad \rho -$ **rest-mass density**

$$\rho = \sum_n m_n \delta\left(\vec{R} - \vec{R}_n\right)$$

$$\Phi\left(R\right) \sim \sum_n \frac{m_n}{\left|\vec{R} - \vec{R}_n\right|}$$

The Newtonian potential does not contain any characteristic scale which could point to the upper limit of forming structures !

Cosmology: non-empty dynamical background !

Background FLRW metric:

$$ds^2 = a^2\left(d\eta^2 - \delta_{\alpha\beta}dx^\alpha dx^\beta\right)$$

scale factor $a(\eta)$ — conformal time — comoving spatial coordinate

physical distance: $R = ar$

Background matter (averaged CDM: $\bar{p} = 0$):

energy density $\bar{\varepsilon} = \bar{\rho}c^2 = \dfrac{\bar{\rho}_c c^2}{a^3}$, $\quad \bar{\rho}_c = \text{const}$

Non-empty background !

comoving rest mass density

Cosmological background is perturbed by the inhomogeneities (e.g. in the form of galaxies)

$$ds^2 = a^2\left[\left(1+2\Phi\right)d\eta^2 - \left(1-2\Phi\right)\delta_{\alpha\beta}dx^\alpha dx^\beta\right]$$

gravitational potential $\Phi << 1$ **weak-field limit**

Perturbed Einstein equations:

$$\delta G_\alpha^\beta = \kappa^2 \delta T_\alpha^\beta, \qquad \kappa^2 \equiv \frac{8\pi G_N}{c^4}$$

metrics fluctuations **matter fluctuations**

We consider CDM as a set of point-like inhomogeneities (e.g. galaxies, groups and clusters of galaxies)

Energy-momentum tensor (EMT) of inhomogeneities (e.g. Landau&Lifshitz):

$$T^{ik} = \sum_n \frac{m_n c^2}{(-g)^{1/2}[\eta]} \frac{dx_n^i}{d\eta} \frac{dx_n^k}{d\eta} \frac{1}{ds_n / d\eta} \delta(\mathbf{r} - \mathbf{r}_n)$$

$$\tilde{v}_n^\alpha \equiv \frac{dx_n^\alpha}{d\eta} = \frac{a}{c}\frac{dx_n^\alpha}{dt} = \frac{a v_n^\alpha}{c} = \frac{v_{phn}^\alpha}{c}, \quad \alpha = 1,2,3 -$$ **comoving peculiar velocity**

$$\rho_c = \sum_n m_n \delta(\mathbf{r} - \mathbf{r}_n) \equiv \sum_n \rho_{cn} -$$ **comoving rest-mass density**

Weak field approximation: $\Phi, \tilde{v}_n^\alpha \ll 1$. **However,** $\delta\rho_c / \bar{\rho}_c$ **can be** $\gg 1$

$$\delta T^{0}_{0(\mathrm{CDM})} \approx \frac{\delta\rho_{\mathrm{c}} c^2}{a^3} + \boxed{\frac{3\bar{\rho}_{\mathrm{c}} c^2}{a^3}\Phi}$$

Due to explicit dependence on g_{ik} and non-zero $\bar{\rho}$

00-component of the perturbed Einstein equation:

$$\Delta_c \Phi - \frac{a^2}{\lambda^2}\Phi = \frac{\kappa^2 c^2}{2a}\delta\rho_c$$

Helmholtz (not Poisson!) equation

!

$$\lambda \equiv \left[\frac{3\kappa}{2}\bar{\varepsilon}\right]^{-1/2} = \sqrt{\frac{2a^3}{3\kappa\bar{\rho}_c c^2}} = \sqrt{\frac{2c^2}{9H_0^2\Omega_M}\left(\frac{a}{a_0}\right)^3}, \quad \Omega_M \equiv \frac{\kappa\bar{\rho}_c c^4}{3H_0^2 a_0^3}$$

Note: we reproduce only the position-depended part of the equation.
For the late Universe, peculiar velocities negligibly contribute into Φ

Helmholtz equation → **Solution in the form of Yukawa potential**

$$\Phi = -G_N \sum_i \frac{m_i}{\left|\vec{R}-\vec{R}_i\right|} \exp\left(-\frac{\left|\vec{R}-\vec{R}_i\right|}{\lambda}\right)$$

λ **defines the range of the Yukawa interaction**

At present time: $\lambda_0 \approx 3700\ \text{Mpc}$ → **Bigger then Great GRBs Wall** (3066 Mpc)

Transition to the Newtonian limit: $\lambda \to \infty \left(\bar{\varepsilon} \to 0\right)$

Cosmological screening (i.e. finite λ) is the effect of the background: $\bar{\varepsilon} \neq 0$

The Yukawa interaction range λ and the horizons:

$$\lambda = \left[\frac{3\kappa}{2}\bar{\varepsilon}\right]^{-1/2} = \sqrt{\frac{2a^3}{3\kappa\bar{\rho}_c c^2}} = \frac{1}{\sqrt{3(1+q)}}\frac{c}{H} \quad \sim \quad a^{3/2}(\eta)$$

At the present time:

$$\lambda_0 \approx 3.7\times10^3 \text{ Mpc}$$

Hubble horizon (radius): (This horizon is not really a physical size)

$$c/H_0 \approx 4.1\times10^3 \,\text{Mpc} > \lambda_0$$

$$c/H = \lambda \text{ at the deceleration parameter } q = -\ddot{a}/\left(aH^2\right) = -2/3$$

$$\uparrow$$

$$a = 1.16\, a_0$$

Particle horizon: (This is the farthest distance that any photon can freely stream from the Big Bang – the size of the observable Universe)

$$l_p(t_0) = a(t_0)\int_0^{t_0}\frac{cd\tilde{t}}{a(\tilde{t})} \approx 14.26\times10^3 \text{ Mpc}$$

-radius of the observable Universe

Number of Yukawa regions:

$$l_p^3(t_0)/\lambda_0^3 \approx 60$$

Cosmological screening :

⇒ Yukawa-type exponential screening of the gravitational potential at distances $\left|\vec{R}-\vec{R}_i\right|>\lambda$

⇓

λ sets an upper limit on the size of greatest structures in the Universe **!**

Let us demonstrate it with the help of a simple numerical simulation

Cosmological mosaic of galactic superstructures cooked on a home computer

The scheme of the simulation:

1. we populate a simulation box with a number of randomly distributed point-like massive particles.

2. we investigate the growth of the mass density contrast in the framework of the cosmic screening approach.

3. we analyze formed structures and compare sizes of the largest ones with the value of the screening length λ_{f} at that moment.

initial	final

Scale factors

$$a_{\mathrm{i}} \qquad a_{\mathrm{f}}$$

Screening lengths

$$\lambda_{\mathrm{i}} = \lambda_0 \left(z_{\mathrm{i}} + 1\right)^{-3/2} \qquad \lambda_{\mathrm{f}} = \lambda_0 \left(z_{\mathrm{f}} + 1\right)^{-3/2}$$

Redshifts (our choice)

MD stage $z_{\mathrm{i}} = 300$ $\downarrow$ $\lambda_{\mathrm{i}} = 0.71\,\mathrm{Mpc}$

$z_{\mathrm{f}} = 2$ $\downarrow$ $\lambda_{\mathrm{f}} = 710\,\mathrm{Mpc}$ **MD stage**

Edge of simulation box

$$L_{\mathrm{i}} \qquad L_{\mathrm{f}}$$

Our requirements

$$\lambda_{\mathrm{i}} << \frac{L_{\mathrm{i}}}{N_1}$$

-- typical distance between particles

$\lambda_{\mathrm{f}} << L_{\mathrm{f}}$ **Let** $L_{\mathrm{f}} = 6\,\mathrm{Gpc}$

At initial time particles can be considered as free

The effect of periodic boundary conditions is weakened and the simulation box can embrace the largest cosmic structures

$$\lambda_i << \frac{L_i}{N_1} \longrightarrow N_1 << \frac{L_f}{\lambda_0}\frac{a_0}{a_f}\left(\frac{a_0}{a_i}\right)^{1/2} \approx 84$$

Let $N_1 = 40$

Number of particles/cells along edge of the box

Total number of cells: $N = N_1^2 = 1600$

At the end of simulation the length of the cell edge:

$$L_{cell} = \frac{L_f}{N_1} = 150\,\text{Mpc}$$

Screening lengh: $\lambda_f \approx 710\ \text{Mpc} \approx 4\,L_{cell}$

Calculations:

A system of N_1^2 **identical point-like particles with mass density**

$$\rho \propto \sum_{n=1}^{N_1^2} \delta\left(x - x_n\right)\left(y - y_n\right)$$

positions of randomly distributed particles

Mass density contract $\delta = \frac{\rho}{\bar{\rho}} - 1$ **for arbitrary scale factor** a **:**

$$\delta = \frac{2}{N_1^2} \sum_{n=1}^{N_1^2} \left\{ \sum_{k_1}^{N_1} \tilde{f}_1 \left[\cos 2\pi k_1 \left(\tilde{x} - \tilde{x}_n\right) + \cos 2\pi k_1 \left(\tilde{y} - \tilde{y}_n\right) \right] \right.$$

$$\left. + 2 \sum_{k_1, k_2}^{N_1} \tilde{f}_{12} \cos 2\pi k_1 \left(\tilde{x} - \tilde{x}_n\right) \cos 2\pi k_2 \left(\tilde{y} - \tilde{y}_n\right) \right\}, \quad \tilde{x}, \tilde{y} \in [0,1]$$

Structure growth functions (exact solution!):

$$\tilde{f}_1 \equiv \frac{1 + \beta k_1^2 \tilde{a}}{1 + \beta k_1^2 \tilde{a}_\mathrm{i}}, \quad \tilde{f}_{12} \equiv \frac{1 + \beta\left(k_1^2 + k_2^2\right)\tilde{a}}{1 + \beta\left(k_1^2 + k_2^2\right)\tilde{a}_\mathrm{i}}, \quad \tilde{a} \equiv \frac{a}{a_0}, \; \tilde{a}_\mathrm{i} \equiv \frac{a_i}{a_0},$$

Averaged density contrast over the cell with coordinates $(\tilde{x}_\nu, \tilde{y}_\nu)$ of its center $\nu = 1, 2, \ldots, N_1^2$ (window functions):

$$\boxed{\delta_\nu} = \frac{2}{N_1^2}\sum_{n=1}^{N_1^2}\left\{\sum_{k_1}^{N_1}\tilde{f}_1\frac{N_1}{\pi k_1}\sin\left(\frac{\pi k_1}{N_1}\right)\left[\cos 2\pi k_1(\tilde{x}-\tilde{x}_n)+\cos 2\pi k_1(\tilde{y}-\tilde{y}_n)\right]\right.$$

$$\left.+2\sum_{k_1,k_2}^{N_1}\tilde{f}_{12}\frac{N_1}{\pi k_1}\sin\left(\frac{\pi k_1}{N_1}\right)\frac{N_1}{\pi k_2}\sin\left(\frac{\pi k_2}{N_1}\right)\cos 2\pi k_1(\tilde{x}-\tilde{x}_n)\cos 2\pi k_2(\tilde{y}-\tilde{y}_n)\right\}$$

The root-mean-square deviation (RMSD):

$$\mathrm{RMSD} = \frac{1}{N_1}\sqrt{\sum_{\nu=1}^{N_1^2}\left(\boxed{\delta_\nu}\right)^2}$$

Structure: $\boxed{\delta_\nu} \geq \mathrm{RMSD}$ **red cell**

Void: $\boxed{\delta_\nu} \leq -\mathrm{RMSD}$ **blue cell**

$$\tilde{a} = \tilde{a}_{\mathrm{f}} = 1/3, \quad N_1 = 40, \quad \beta \approx 1$$

Comments on simulations:

1. According to the symmetry of the simulation box and the splitting of this box into square cells, the projections of structures on the vertical and horizontal axes are considered as linear sizes of those structures. Hence, when we speak about the largest structures consisting, e.g., of four cells, we mean four **projective** (either on the vertical or horizontal axis) **cells**.

2. We check all structures consisting of 4 or more projective cells.

3. Due to periodic boundary conditions, structures can cross the boundaries of the box forming superstructures.

4. If cells are connected through edges, they evidently form the common structure. However, if they are connected via vertices, these vertices should be checked additionally.

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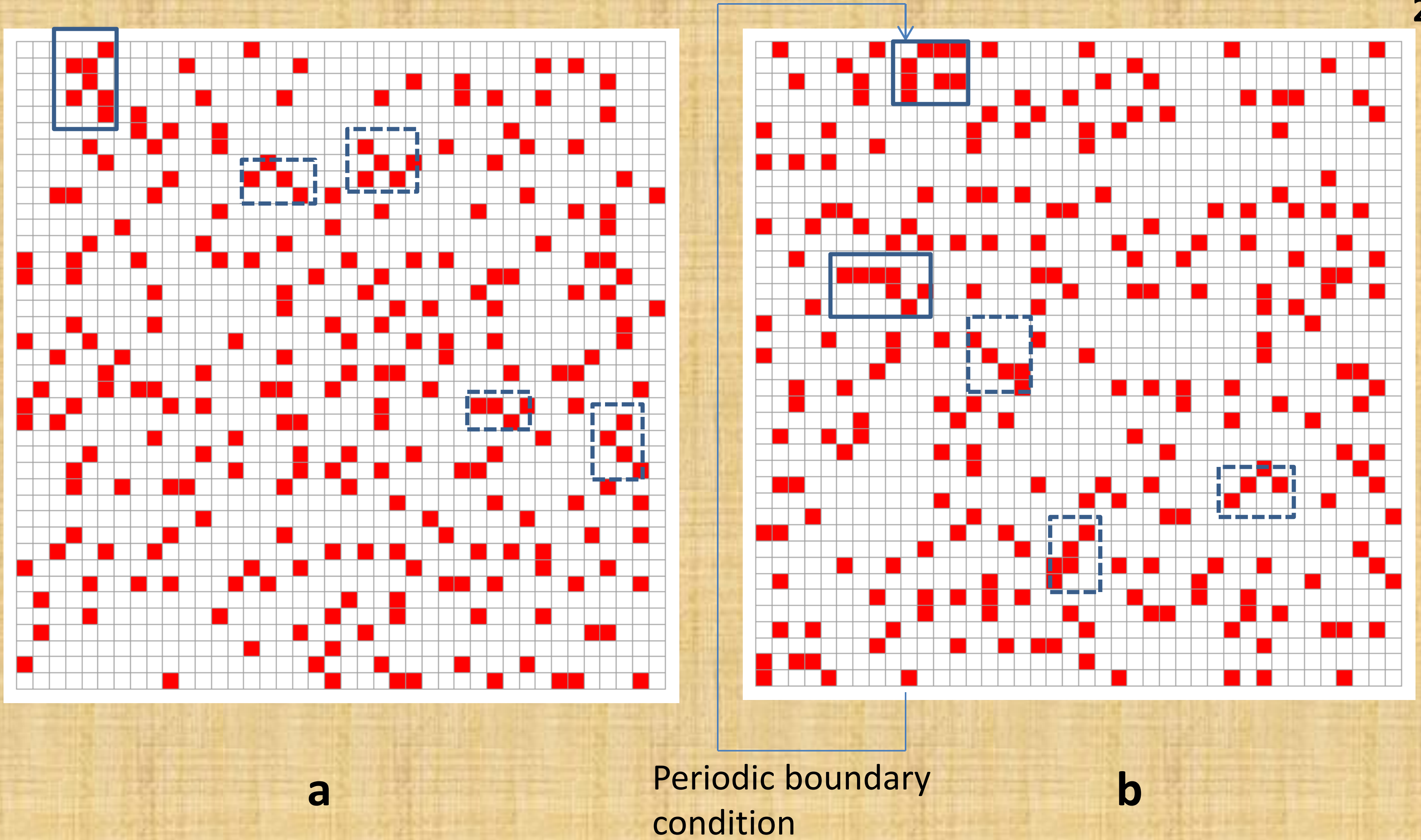
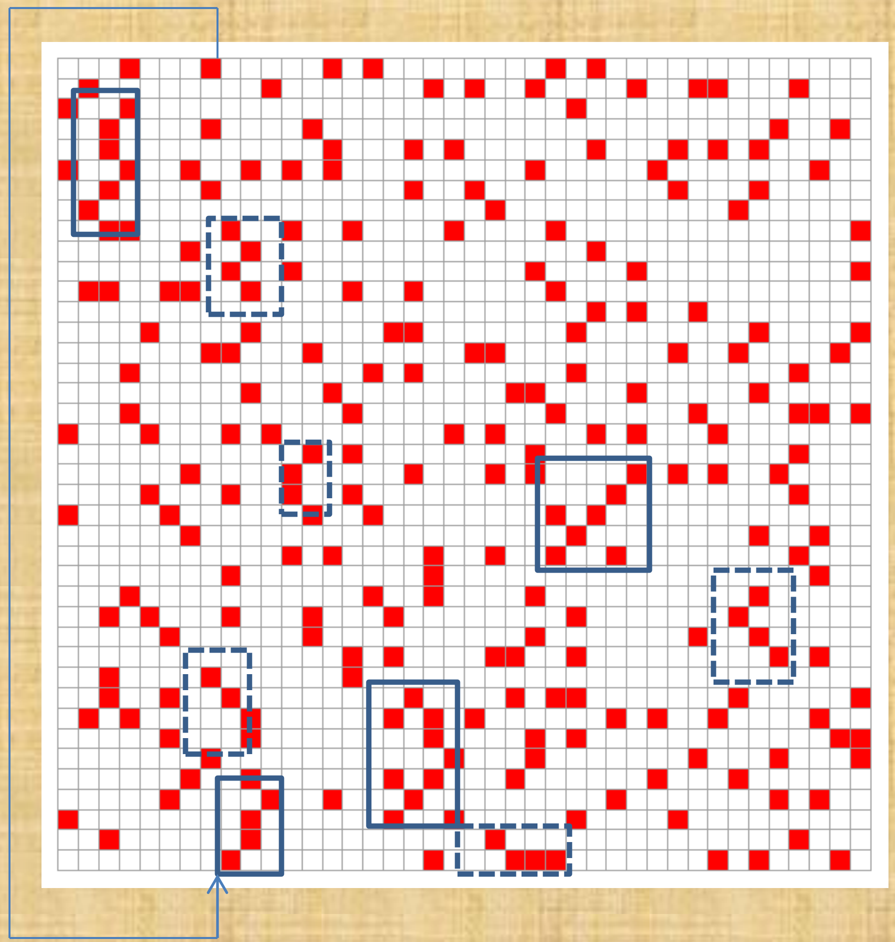
We build a cell centered on the corresponding vertex, calculate the window function for this cell and compare it with RMSD. If it passes this test (i.e. the window function exceeds RMSD), then the cells connected via this vertex form a common structure.

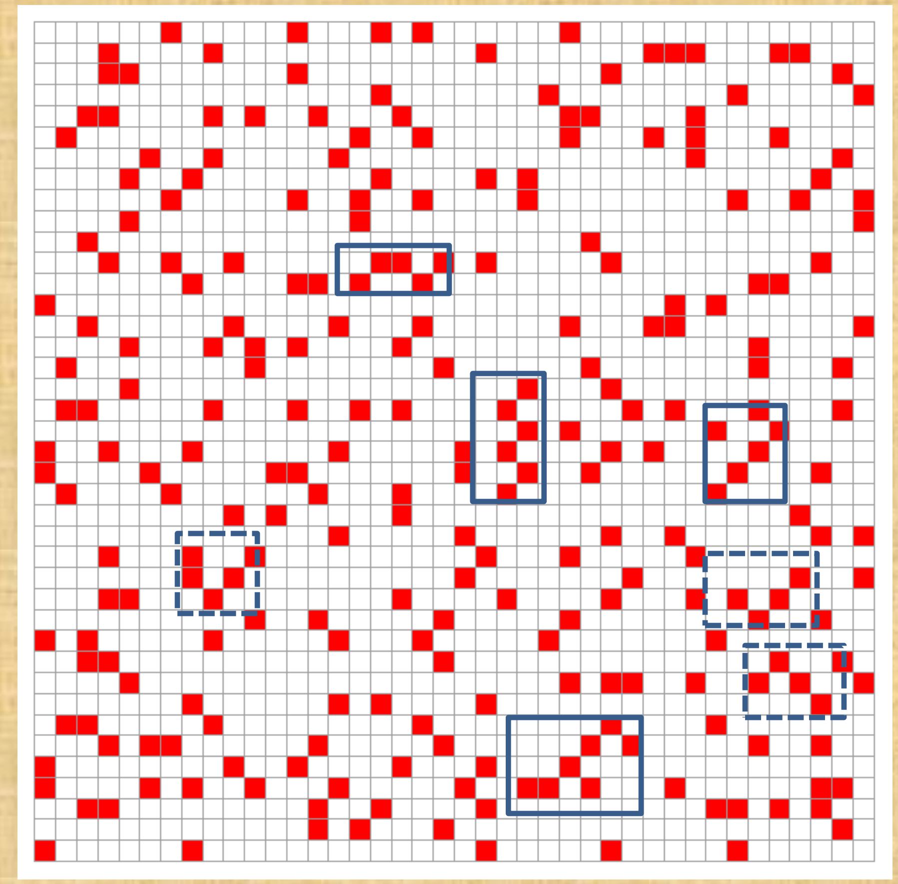


Dotted rectangles embrace formations with four projective cells.

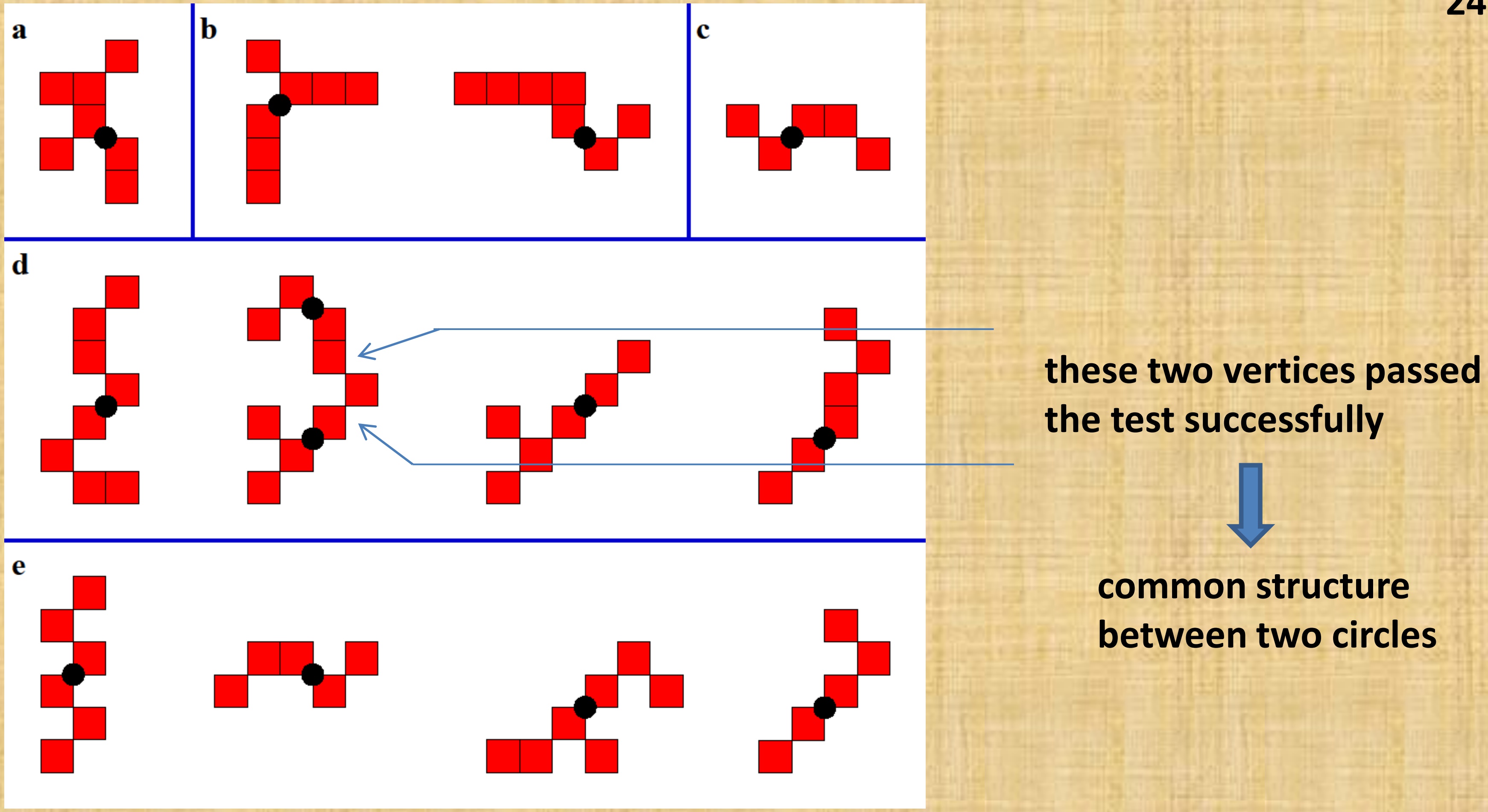
Bold rectangles embrace formations with more than four projective cells.



d



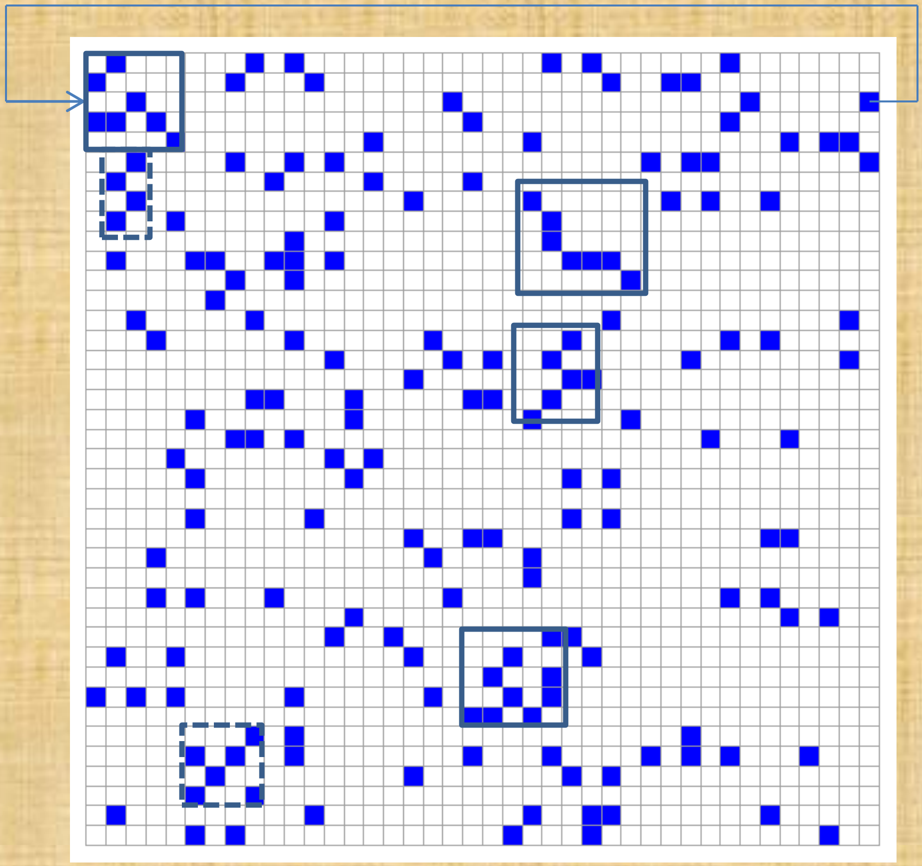
e



All "suspicious" formations consisting of more than four projective cells for simulations a,b,d,e, respectively. Black circles indicate the vertices that failed the test.

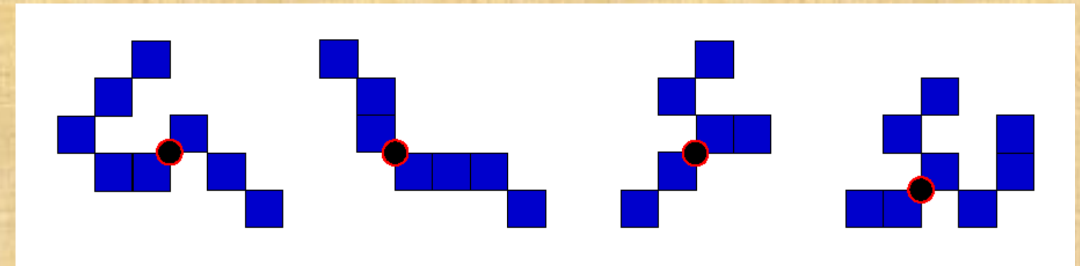
There are no common structure consisting of more than four projective cells !

Simulation of void distribution



There are no common voids consisting of more than four projective cells

!



Conclusions:

1. The sizes of the greatest superstructures or supervoids in the Universe do not exceed the cosmic screening length λ which determines the range of Yukawa gravitational interaction. At the present time $\lambda_0 \approx 3.7\ \text{Gpc}$.

2. The value λ_0 is bigger than the largest known structure in the Universe (Great GRBs Wall): $\lambda_0 \approx 3700\ \text{Mpc} > 3066\ \text{Mpc}$.

3. The scale of homogeneity must be greater than λ. It reconciles the cosmological principle, the cornerstone of modern cosmology, with the existence of superstructures.

THANK YOU!