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Representations of solutions of some nonlinear PDEs in the form of series in powers of the δ-function

Representations of solutions of some nonlinear PDEs in the form of series in powers of the δ-function

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Let K be an arbitrary integral domain with identity and let $K[x]$ be the ring of polynomials with coefficients in K. By a *copolynomial* we mean a K-linear mapping $T : K[x] \to K$. The module of copolynomials is denoted by $K[x]'$. If $T \in K[x]'$ and $p \in K[x]$, then the result of application $T \in K[x]'$ to $p \in K[x]$ is written as (T, p). The derivative T' of a copolynomial $T \in K[x]'$ is defined in the same way as in the classical theory of generalized functions: $(T', p) = -(T, p'), \quad p \in K[x]$. An important example of a copolynomial is the δ-function which is defined by $(\delta, p) = p(0),\ p \in K[x]$.

The *Cauchy-Stieltjes transform* of a copolynomial $T \in K[x]'$ is defined as the following formal Laurent series from the ring $\frac{1}{s}K[[\frac{1}{s}]]$: $C(T)(s) = \sum\limits_{k=0}^{\infty} \frac{(T,x^k)}{s^{k+1}}$. The mapping
$C : K[x]' \to \frac{1}{s}K[[\frac{1}{s}]]$ is an isomorphism of K-modules. The multiplication of copolynomials is defined through the multiplication of their Cauchy-Stieltjes transforms.
The theory of linear PDEs over the module $K[x]'[[t]]$ was studied in [1,2]. We prove the following existence and uniqueness theorem for the Cauchy problem for some nonlinear PDEs.

Theorem. *Let $K \supset \mathbb{Q}$, $a \in K$ and let $m_j \in \mathbb{N}_0$ $(j = 0, 1, 2, 3)$.*
Then the Cauchy problem
$\frac{\partial u}{\partial t} = au^{m_0} \left(\frac{\partial u}{\partial x}\right)^{m_1} \left(\frac{\partial^2 u}{\partial x^2}\right)^{m_2} \left(\frac{\partial^3 u}{\partial x^3}\right)^{m_3}$, $u(0, x) = \delta(x)$ *has a unique solution in $K[x]'[[t]]$. This solution is of the form*
$u(t, x) = \sum\limits_{k=0}^{\infty} u_k \delta^{nk+1} t^k$, *where $u_k \in K$ and $n = \sum\limits_{j=0}^{3} (j + 1)m_j - 1$. Moreover, for every $t \in K$ this series converges in the topology of $K[x]'$.*

As examples we consider a Cauchy problem for the Euler-Hopf equation $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$, for a Hamilton-Jacobi type equation $\frac{\partial u}{\partial t} = \left(\frac{\partial u}{\partial x}\right)^2$ and for the Harry Dym equation $\frac{\partial u}{\partial t} = u^3 \frac{\partial^3 u}{\partial x^3}$.

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[1] S.L. Gefter, A.L. Piven', Linear Partial Differential Equations in Module of Formal Generalized Functions over Commutative Ring, J. Math. Sci., \textbf{257} (2021), No.5, 579–596.
[2] S.L. Gefter, A.L. Piven', Linear Partial Differential Equations in Module of Copolynomials of Several Variables over a Commutative Ring, http://arxiv.org/abs/2407.04122

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