

Application of Chebyshev polynomials for optimal differentiation and summation.

In our research, we study the problems of numerical differentiation and summation of univariate functions. Many works are devoted to these problems, among which we highlight [1-4].

As is known, in assessing the effectiveness of approximate methods, their optimality plays an important role. At the same time, in NDS, until recently the optimality of methods was understood only in the sense of their accuracy. The point is that the optimal accuracy can be achieved by using different amounts of discrete information. Therefore, it makes sense to also study information complexity of NDS. In other words, it is very important to research NDS methods that achieve optimal accuracy by using the minimal possible amount of discrete input data. To solve NDS problems, we propose an approach based on the truncation method. The essence of this method is to replace the infinite Fourier series with a finite sum. It is only necessary to properly select the order of this sum, which plays the role of a regularization parameter here. Moreover, the proposed approach allows us to construct algorithms that achieve the optimal order of accuracy by using the minimum amount of discrete information in the form of perturbed values of the Fourier-Chebyshev coefficients. In addition, the use of Chebyshev polynomials makes it possible to construct quadrature formulas for specially selected nodes, the number of which is the minimum possible. In other words, the proposed approach ensures the stability of approximations, leads to a reduction in computational costs without loss of accuracy, and does not require cumbersome computational procedures. Moreover, of interest to researchers is the question of determining the conditions under which the numerical summation problem is well-posed. The authors have found an answer to this question for the classes of functions under consideration.

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