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Cubic-quintic interplay in the nonlinear Klein–Gordon model

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The nonlinear Klein–Gordon (nKG) model,
$\partial_{tt}\phi - c^2\partial_{xx}\phi + f(\phi) = 0$,
is a universal model for describing the propagation of nonlinear waves in various physical media. For example, its stationary version describes the macroscopic wave function of the condensed phase (i.e., the order parameter) in the Landau theory of phase transitions. Noteworthy is also an application of the nKG model to modelling the spatially localised oscillating excitations of biological structures such as DNA chains. Here, the unknown real function ϕ is a characteristic of the wave field, t is time, x is coordinate, c is the velocity parameter that deals with the speed of interaction propagation. The function f is a nonlinear function of the wave field that describes the nonlinear response of the medium.

In this work, we are interested in the interplay between the cubic and quintic terms of the nonlinear response function, which we present as a truncated polynomial (corresponding to a ϕ^6 field model),
$f(\phi) = \alpha_1\phi + \alpha_3\phi^3 + \alpha_5\phi^5$.
The real coefficient α_1 describes the linear response of the medium. The real coefficients α_3 and α_5 represent the cubic and quintic nonlinearities, respectively.

By reducing the nKG model to an extended cubic-quintic nonlinear Schrödinger equation in Hamiltonian form, we demonstrate that the quintic nonlinearity has a profound effect on the stability of wave packets to long-wave modulations. When there is no quintic nonlinearity (ϕ^4 field model), plain wave packets in such a system are known to be modulationally unstable for any carrier wave number in the case of negative coefficient at cubic nonlinearity. We show that such plain wave packets become modulationally stable for certain carrier wave numbers when the quintic nonlinearity becomes large enough. Such a stabilisation of the wave packet happens at certain critical ratio between the quintic and cubic coefficients of the nKG model.

This work proves that high-order nonlinear effects may play a decisive role in analysing physical phenomena in nonlinear models at certain conditions. This result may have practical implications for nonlinear media that exhibit the generation of higher harmonics and are characterised by a significant quintic nonlinearity (e.g, in polarisation). In particular, such conditions are met for ferroelectrics with first-order phase transition (e.g., $BaTiO_3$ crystals).

Publications:

1. Sedletsky Yu.V., Gandzha I.S. Hamiltonian form of extended cubic-quintic nonlinear Schrödinger equation in a nonlinear Klein-Gordon model. Phys. Rev. E 106, 064212 (2022). https://doi.org/10.1103/PhysRevE.106.064212
2. Sedletsky Yu.V., Gandzha I.S. Fifth-order nonlinear Schrödinger equation as Routhian reduction of the nonlinear Klein–Gordon model. Proc. R. Soc. A 479, 20230315 (2023). https://doi.org/10.1098/rspa.2023.0315

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