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Collapse mitigation in a socioeconomic system under a systemic shock

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ABSTRACT

Systemic shocks inevitably lead to negative socioeconomic outcomes. The COVID-19 pandemic and the war in Ukraine are the prominent examples of such systemic shocks. Shock-initiated spreading processes often have a domino effect on both the social and economic levels. The war in Ukraine, despite its devastating effect on the Ukraine's society and economy, has not led to the full collapse, against all odds. In this work, we make an attempt to provide at least a qualitative illustration of the mechanisms governing the dynamics of a socioeconomic system in the state of collapse from the viewpoint of statistical physics. Surprisingly, we uncover common principles that allow the overall collapsing scenario to be mitigated, with the system's dynamics stabilized.

We consider a response of a socioeconomic system to a systemic shock in a group of economic agents with limited economic resource. To this end, we exploit a simple two-level model of active and passive economic agents with mutual negative feedback between the number of active agents and collective resource acquisition [1, 2]. In this case, economic resource is associated with the average amount of money or income per economic agent and formally corresponds to the effective market temperature, with the income distribution of economic agents obeying the Boltzmann–Gibbs statistics [3, 4]. The coupling between the spreading process and resource in such a system is supposed to be of activation type, with the transition rate between the passive and active populations governed by the activation mechanism (Arrhenius-like law). A characteristic level of resource consumption is associated with activation energy (e.g., corresponding to the minimum level of resource consumption in our particular case).

We show that the phase portrait of the system features a collapse phase, in addition to the shock-free and post-shock phases. The shock intensified by the increasing resource deficit can ultimately drive the system to a collapse at nonzero activation energy because of limited resource—the effect opposite to thermal explosion. In this case, the system can no longer stabilize and return to the stable shock-free state or a poorer post-shock state. We demonstrate that there exists a certain critical point at which the system collapses at any initial conditions. Moreover, social regulations in the case of low economic resource can have a negative effect and provoke the system's collapse. On the other hand, there are simple external measures that can protect the system against the collapse, which make the focus of our investigation. We demonstrate that the system's collapse can partially be mitigated by external subsidies meaning constant resource inflow from some external source or by means of debt interpreted as a negative resource.

It is interesting that a two-level model considered here formally describes the dynamics of cooling of a system of agents due to shock-induced transitions between two discrete inner states of agents. In this case, the crisis state of the financial market can be associated with a Bose condensate-like state at low market temperature [5]. A more complex multi-level system of interacting agents as well as different interacting social groups can also be considered [6].

THE MODEL

To illustrate the *collapse effect* in a socio-economic system undergoing a systemic shock (e.g., epidemic) and possible mitigation strategies we consider the simplest model [1, 2]. The system is described in terms of mean concentration (number density) $\boldsymbol{s}$ of *active* economic agents (susceptible), $1-s$ being the mean number density of nonactive (*passive* or infected) agents, and mean amount of economic resource $\boldsymbol{\rho}$ of this group:

$$\partial_t s = -\beta s(1-s) + \gamma(\rho)(1-s), \tag{1}$$

$$\partial_t \rho = Gs - \Gamma\rho + \Lambda, \tag{2}$$

$$\gamma(\rho) = \gamma_0 \exp(-E/\rho). \tag{3}$$

$\boldsymbol{\Gamma\rho}$ describes the collective expenses or taxes. Roughly speaking, the expenses are assumed to be proportional to earnings. Thus, the coefficient Γ represents the resource consumption rate.

$\boldsymbol{Gs}$ describes the resource acquisition by working (active) agents. The resource acquisition rate G formalizes the resource amount acquired by them per unit time.

The parameter $\boldsymbol{\Lambda}$ represents a resource source (constant resource inflow into the system from some external reservoir) or a resource sink (constant resource outflow from the system).

$\Lambda < 0$, **resource flows out** from the system, e.g., in the form of **infrastructure expenses**, depreciation, rent, interest payments, or other fixed expenses that do not depend on the agent's state.

$\Lambda > 0$, **resource is fed into** the system (e.g., in the form of **subsidies**) from some external source, e.g., a central bank or central government.

$\boldsymbol{\beta}$ is the transmission rate of the shock (epidemic). It is defined as a product of the contact rate and the probability that a contact of a passive (infected) individual with a susceptible individual results in transmission.

We consider a possibility of the agent–resource coupling mechanism to be of activation type, with the recovery rate γ governed by the Arrhenius-like law. We show that such a coupling can lead to the system collapse caused by a systemic shock.

$\boldsymbol{\gamma(\rho)}$ is recovery rate that is determined by the quality of provision with medical services and food, apart from the individual peculiarities of the given member of population. The quickest recovery depends on the cost of medical services and the bare subsistence level of consumption E, as well as on the availability of the economic resource ρ. Since the cost of services is fixed, the service is terminated if there is no sufficient resource ($\rho \ll E$). In other words, the parameter E serves as the height of some energy barrier (activation energy) peculiar to the given system. The recovery rate $\boldsymbol{\gamma(\rho)}$ can have an activation-type (Arrhenius-like) dependence, $\boldsymbol{\sim \exp(-E/\rho)}$, similar to the temperature dependence of common activation processes with activation energy E. In this case, the economic resource ρ formally plays the role of *effective market temperature* and the minimum level of resource consumption is associated with activation energy E. We bear on the fact that the equilibrium distribution of income is governed by the Boltzmann law at least for low- and middle-income classes [3, 4]. The spread of shock (e.g., epidemic) and the associated quarantine measures result in the reduction of the collective resource ρ. When resource is depleted, the quality of medical services drops and the recovery rate goes down. As a result, the number of active members in population decreases. This, in turn, leads to a further reduction of the collective resource, with the level of income needed for the basic survival being lower and lower. Such a scenario finally results in the ultimate collapse of the system—the effect opposite to thermal explosion.

THE COLLAPSE EFFECT

Figure 1 shows that in the simplest case of $\Lambda = 0$ the spread of shock (epidemic) results in the reduction of the collective resource ρ. When the resource is depleted, the quality of medical services drops and the recovery rate goes down. As a result, the number of active members in population decreases. This, in turn, leads to a further reduction of the collective resource, with the level of income needed for the basic survival being lower and lower. Such a scenario can finally result in the ultimate collapse of the system—the effect opposite to thermal explosion. Phase diagram for the coupled agent-resource system shows that generally the system can occupy three different states (phases): **(I) shock-free** (disease-free), **(II) post-shock** (endemic), and **(III) collapse**.

COLLAPSE MITIGATION STRATEGIES

Our system given by Eqs. (1) and (2) consists of two subsystems: the economic one described by resource ρ and the population one described by the number density s of active individuals. Accordingly, it can be influenced either **through** the resource subsystem (e.g., using certain **(A) financial instruments**) or through the population subsystem (e.g., introducing **(B) social regulations** like quarantine). Here we consider several illustrative examples of the collapse mitigation strategies based on our model.

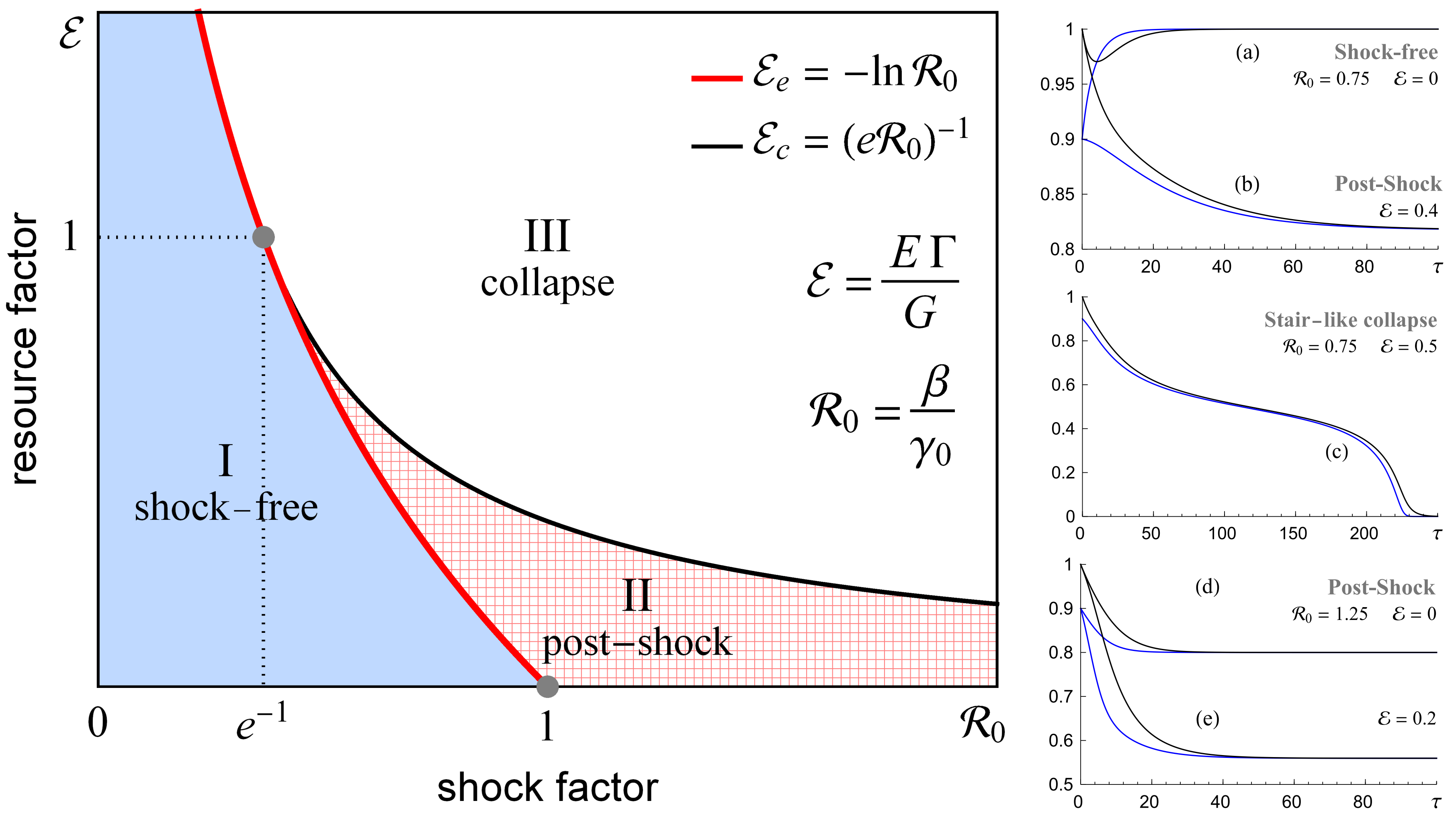

Figure 1: **Phase diagram and temporal dynamics of the system's transition from the initial state.** Here, $\tau = \gamma_0 t$ and $\varrho = \rho\Gamma/G$, with $\Gamma/\gamma_0 = 0.2$ and $\Lambda = 0$.

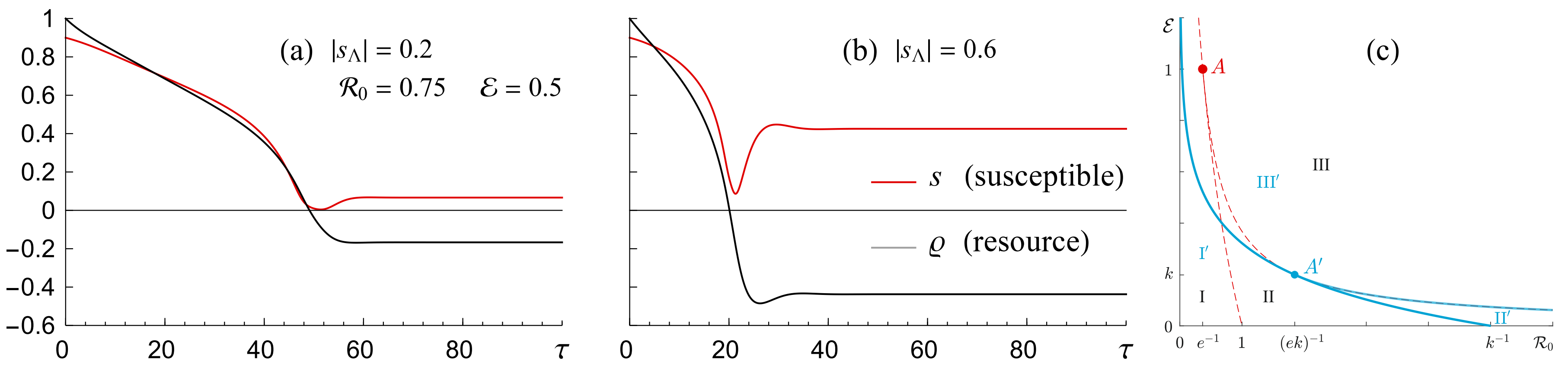

Figure 2: **Mitigation strategies.** Debt strategy and bottom bounce effect are shown on panels (a) and (b). Here, $\mathcal{E} = E/\rho_0$, $\varrho = \rho/\rho_0$, with $\rho_0 = (G - |\Lambda|)/\Gamma$, and $s_\Lambda = -|\Lambda|/G < 0$. Unambiguous effect of quarantine measures is shown on panel (c). Here, $0 < k < 1$ is a parameter describing the quarantine severity.

Mitigation via FINANCIAL INSTRUMENTS: Debt Strategy & Bottom Bounce Effect ($\Lambda < 0$)

We proceed from the standpoint adopted in the statistical mechanics of money implying that negative money can be associated with debt [3, 4]. Here, resource ρ is associated with average income per economic agent. By analogy to money, it can become negative if a group of economic agents described by Eqs. (1) and (2) starts to live in debt (on the average), borrowing resource (money) from an external reservoir. In terms of our equations, negative resource means that the term $\Gamma\rho$ changes its sign. This means that resource is no longer consumed but is collected in the form of debt. Thus, changing the taxation policy under the critical conditions ($\rho \to 0$) in the case of negative Λ can serve as a mitigating factor to the collapse scenario, with the system stabilization achieved by means of debt (negative resource). In this case, relation (3) is rewritten in terms of the resource's absolute value

$$\gamma(\rho) \mapsto \gamma(|\rho|) = \gamma_0 \exp(-E/|\rho|),$$

with asymptotic value $\gamma(\rho) = 0$ at $\rho = 0$. Figure 2 demonstrates an example of such a mitigated collapse scenario. The number density of active agents bounces from a horizontal axis close to $s = 0$ and stabilizes at some $s < |s_\Lambda|$, with resource passing through the zero point and stabilizing at a certain negative value. The parameter $|s_\Lambda| = |\Lambda|/G$ defines the minimum number of active agents required to secure external payments, e.g., to sustain infrastructure. The larger the parameter $|s_\Lambda|$, the greater is the debt required to finance external payments and the larger is the number of active agents.

Mitigation via SOCIAL REGULATIONS: Unambiguous Effect of Quarantine Strategy

Our model allows us to make a rough estimate of the possible outcomes of quarantine regulations. Quarantine measures are all aimed to reduce the transmission rate β. Suppose that the reduction of β is achieved by decreasing the local population density as a result of certain quarantine measures like social distancing, self-isolation, shortened workday, etc. These quarantine measures should also affect the resource acquisition rate G (if it depends on local population density as well) and, as a result, the average income per agent. In this case, the use of such quarantine measures formally means that the rate constants β and G are renormalized, namely, $\beta \to \beta'$ and $G \to G'$, with $\beta' < \beta$ and $G' < G$. If we roughly assume linear relation ($\beta' = k\beta$, $G' = kG$), the factor k would formally correspond to the quarantine severity factor.

The smaller the factor k, the stronger are the quarantine regulations. The phase diagram shift under this transformation (on the right panel) shows that quarantine can have an ambiguous effect. For socioeconomic systems with small initial resource or high level of minimum resource consumption, the above-discussed quarantine strategy can ultimately result in a collapse even if the system was initially in quite a controllable situation. On the contrary, quarantine always has a positive effect on systems with high initial resourse or low activation barrier, such that it can even suppress the epidemic. So, quarantine can have an ambiguous effect, depending on initial resource ρ_0 and activation resource (energy) E.

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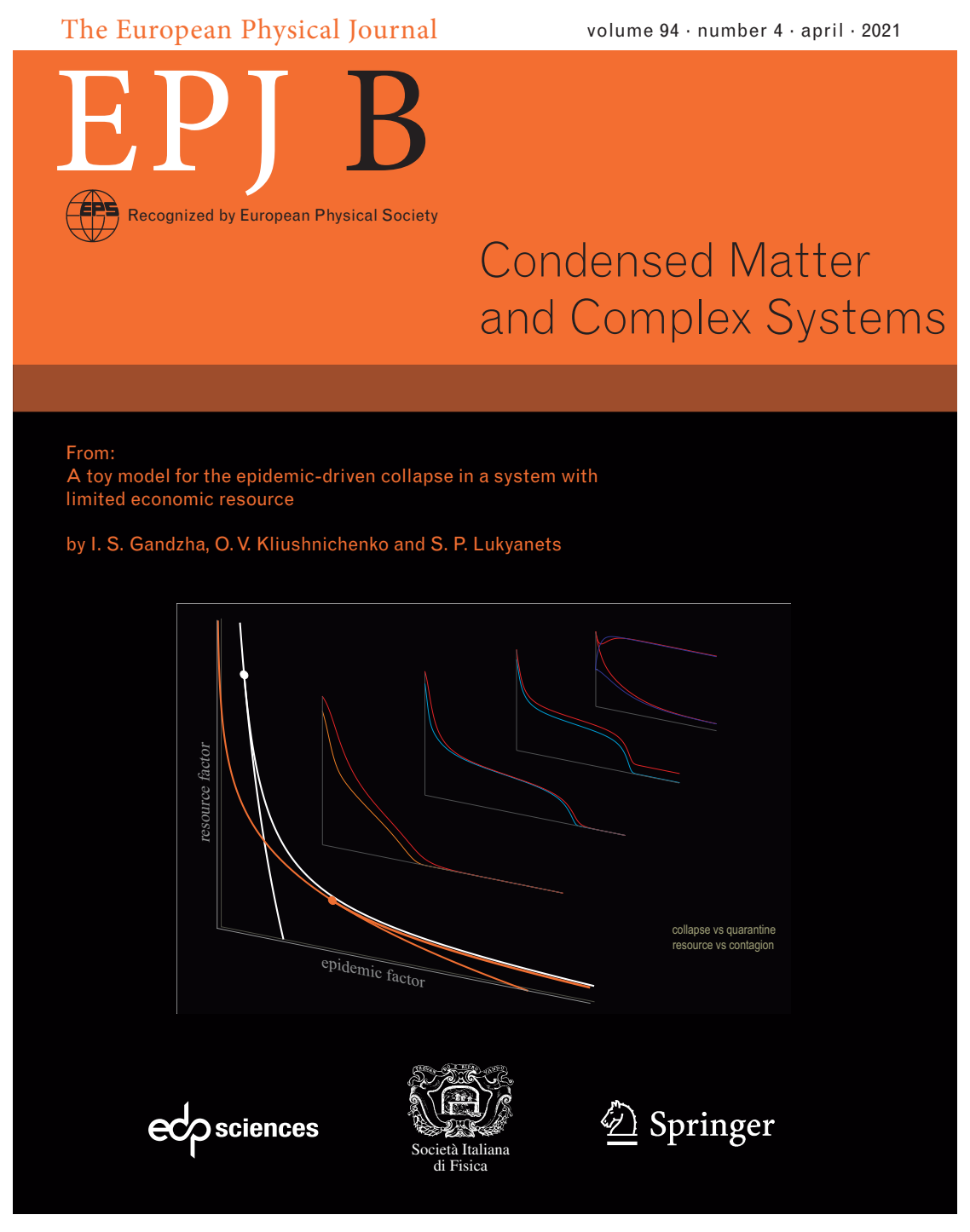
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