Contribution ID: **24** Type: **Oral**

Well-posedness of the Cauchy problem for the two-component peakon system

We consider the Cauchy problem for the following two-component peakon system with cubic nonlinearity:
$\partial_t m = \partial_x[m(u - \partial_x u)(v + \partial_x v)]$,

$\partial_t n = \partial_x[n(u - \partial_x u)(v + \partial_x v)]$,

$m = u - \partial_x^2 u, \quad n = v - \partial_x^2 v$,

where $m = m(t, x)$, $u = u(t, x)$, $v = v(t, x)$ and $t, x \in \mathbb{R}$. We assume that the initial data $u_0(x) = u(0, x)$ and $v_0(x) = v(0, x)$ belong to the space $C^{k+2}(\mathbb{R}) \cap W^{k+2,1}(\mathbb{R})$ with $k \in \mathbb{N} \cup \{0\}$.

Considered peakon system was introduced by Song, Qu and Qiao in [3] as a generalization of the celebrated Fokas-Olver-Rosenau-Qiao (FORQ) equation: taking $u = v$, one obtains the FORQ equation, which has the form
$\partial_t m = \partial_x \left[m \left(u^2 - (\partial_x u)^2\right)\right], \quad m = u - \partial_x^2 u.$

Our major goal is to investigate the local existence, uniqueness and continuous dependence on the initial data of the solution. Revisiting the method of characteristics, developed for addressing the FORQ equation in [1], we establish existence and uniqueness of the solution (u, v) of the Cauchy problem for the two-component system in the class $C\left([-T, T], C^{k+2}(\mathbb{R}) \cap W^{k+2,1}(\mathbb{R})\right)$ with $k \in \mathbb{N} \cup \{0\}$ and some $T > 0$ [2]. Notice that the class of regularity corresponded to $k = 0$ is lower than that previously considered in the works for the FORQ equation. The most challenging part of the analysis consists in proving uniqueness in the case $k = 0$. To this end we must demonstrate that a solution satisfies a specific conservation law involving m, n and the characteristic, which, in turn, entails studying the two-component system in a weak sense.

Also we prove the Lipschitz continuity of the data-to-solution map for (m, n) within the space $C^k(\mathbb{R}) \cap W^{k,1}(\mathbb{R})$ with $k \in \mathbb{N} \cup \{0\}$ [2]. To the best of our knowledge, this result was not reported before in the related works for the FORQ equation, wherein the Lipschitz property is established for (u, v) in $W^{1,1}$ under assumption of existence of weak solution (u, v) in $L^\infty\left([-T, T], W^{2,1}(\mathbb{R})\right)$, see [4].

[1] Y. Gao and J-G. Liu.The modified Camassa-Holm equation in Lagrangian coordinates. Discr. Cont. Dyn. Syst. Ser. B., 23(6):2545–2592, 2018.

[2] K.H. Karlsen, Ya. Rybalko. On the well-posedness of the Cauchy problem for the two-component peakon system in $C^k \cap W^{k,1}$. Z. Angew. Math. Phys., 75:113, 2024.

[3] J.F. Song, C.Z. Qu, Z.J. Qiao. A new integrable two-component system with cubic nonlinearity. J. Math. Phys., 52:013503, 2011.

[4] Q. Zhang. Global wellposedness of cubic Camassa-Holm equations. Nonlinear Anal., 133:61–73, 2016.

Primary author: RYBALKO, Yan (B.Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine)

Presenter: RYBALKO, Yan (B.Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine)

Session Classification: MATHEMATICS

Track Classification: MATHEMATICS