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Two-dimensional central extensions of some superconformal loop Lie algebra generalization and compatibly bi-Hamiltonian $(2|N+1)$-dimensional systems on functional supermanifolds

For $N \in \{1,2,3\}$ there is considered the semi-direct sum $\tilde{\mathcal{G}} \ltimes \tilde{\mathcal{G}}^*_{reg}$ of the loop Lie algebra $\tilde{\mathcal{G}}$, consisting of the even left superconformal vector fields on a supercircle $\mathbb{S}^{1|N}$ in the form $\tilde{a} := a\partial/\partial x + \frac{1}{2}\sum_{i=1}^{N}(D_{\vartheta_i}a)D_{\vartheta_i}$, where $a := a(x,\vartheta;\lambda) \in C^\infty(\mathbb{S}^{1|N} \times (\mathbb{D}^1_+ \cup \mathbb{D}^1_-);\Lambda_0)$ is holomorphic in the "spectral" parameter $\lambda \in \mathbb{D}^1_+ \cup \mathbb{D}^1_- \subset \mathbb{C}$, $\mathbb{D}^1_+, \mathbb{D}^1_-$ are the interior and exterior regions of the unit centrally located disk $\mathbb{D}^1 \subset \mathbb{C}$ respectively, $a(x,\vartheta;\infty) = 0$, $(x,\vartheta) \in \mathbb{S}^{1|N} \simeq \mathbb{S}^1 \times \Lambda_1^N$, $\Lambda := \Lambda_0 \oplus \Lambda_1$ is a commutative Banach superalgebra over the field $\mathbb{C} \subset \Lambda_0$, $\partial/\partial x$ is a partial derivative by the commuting variable x, $\vartheta := (\vartheta_1, \vartheta_2, \ldots, \vartheta_N)$, $\partial/\partial\vartheta_i$ is a left partial derivative by the anticommuting variable $\vartheta_i \in \Lambda_1$, $D_{\vartheta_i} := \partial/\partial\vartheta_i + \vartheta_i\partial/\partial x$, $i = \overline{1,N}$, and its regular dual space $\tilde{\mathcal{G}}^*_{reg}$ with respect to the parity $(\tilde{a},\tilde{l})_0 = \mathrm{res}_{\lambda\in\mathbb{C}} \int_{\mathbb{S}^{1|N}} dx d\vartheta_1 \ldots d\vartheta_N\,(al)$, where $\mathrm{res}_{\lambda\in\mathbb{C}}$ denotes the coefficient at λ^{-1} in the corresponding Laurent series, $\tilde{l} \in \tilde{\mathcal{G}}^*_{reg}$ is a right superdifferential 1-form on $\mathbb{S}^{1|N}$ such as $\tilde{l} := (dx - \sum_{i=1}^{N}(d\vartheta_i)\vartheta_i)l(x,\vartheta;\lambda) \in \tilde{\mathcal{G}}^*_{reg}$, $l := l(x,\vartheta;\lambda) \in C^\infty(\mathbb{S}^{1|N} \times (\mathbb{D}^1_+ \cup \mathbb{D}^1_-);\Lambda_s)$ is holomorphic in the "spectral" parameter $\lambda \in \mathbb{D}^1_+ \cup \mathbb{D}^1_-$, $l(x,\vartheta;\infty) = 0$, $s = 1$ if N is an odd natural number and $s = 0$ if N is an even one. The loop Lie algebra $\tilde{\mathcal{G}}$ is splitting into the direct sum $\tilde{\mathcal{G}} = \tilde{\mathcal{G}}_+ \oplus \tilde{\mathcal{G}}_-$ of its Lie subalgebras for which $\tilde{\mathcal{G}}^*_{+,reg} \simeq \tilde{\mathcal{G}}_-$, $\tilde{\mathcal{G}}^*_{-,reg} \simeq \tilde{\mathcal{G}}_+$, where $a(x,\vartheta;\infty) = 0$ for any $\tilde{a} \in \tilde{\mathcal{G}}_-$. On $\tilde{\mathcal{G}} \ltimes \tilde{\mathcal{G}}^*_{reg}$ one determines the commutator $[\tilde{a} \ltimes \tilde{l}, \tilde{b} \ltimes \tilde{m}] := [\tilde{a},\tilde{b}] \ltimes (ad^*_{\tilde{a}}\tilde{m} - ad^*_{\tilde{b}}\tilde{l})$ for any $\tilde{a},\tilde{b} \in \tilde{\mathcal{G}}$ and $\tilde{l},\tilde{m} \in \tilde{\mathcal{G}}^*_{reg}$, where $[\tilde{a},\tilde{b}] := \tilde{c}$, $\tilde{c} \in \tilde{\mathcal{G}}$, $c := a(\partial b/\partial x) - b(\partial a/\partial x) + \frac{1}{2}\sum_{i=1}^{N}(D_{\vartheta_i}a)(D_{\vartheta_i}b)$, ad^* is the coadjoint action of $\tilde{\mathcal{G}}$ with respect to the parity $(.,.)_0$, as well as the symmetric bilinear form $(\tilde{a} \ltimes \tilde{l}, \tilde{b} \ltimes \tilde{m})_0 = (\tilde{a},\tilde{m})_0 + (\tilde{b},\tilde{l})_0$. One constructs the central extensions $\hat{\mathfrak{G}} := \tilde{\mathfrak{G}} \oplus \mathbb{C}^2$ of the Lie algebra $\tilde{\mathfrak{G}} := \prod_{z\in\mathbb{S}^1}(\tilde{\mathcal{G}} \ltimes \tilde{\mathcal{G}}^*_{reg})$ by the superanalogs of the Ovsienko-Roger 2-cocycle such as $\omega_2(\tilde{a} \ltimes \tilde{l}, \tilde{b} \ltimes \tilde{m}) := (\omega_2^1(\tilde{a} \ltimes \tilde{l}, \tilde{b} \ltimes \tilde{m}), \omega_2^2(\tilde{a} \ltimes \tilde{l}, \tilde{b} \ltimes \tilde{m}))$, where $\omega_2^1(\tilde{a} \ltimes \tilde{l}, \tilde{b} \ltimes \tilde{m}) = \mathrm{res} \int_{\mathbb{S}^1} dz \int_{\mathbb{S}^{1|N}} dx d^N\vartheta\,(a(\mathcal{P}b))$, $\omega_2^2(\tilde{a} \ltimes \tilde{l}, \tilde{b} \ltimes \tilde{m}) = \int_{\mathbb{S}^1} dz\,((a,\partial m/\partial z)_0 - (b,(\partial l/\partial z)_0)$, $(\tilde{a} \ltimes \tilde{l}), (\tilde{b} \ltimes \tilde{m}) \in \tilde{\mathfrak{G}}$, $z \in \mathbb{S}^1$, and $\mathcal{P} = D_{\vartheta_1}\partial^2/\partial^2 x$ when $N = 1$, $\mathcal{P} = D_{\vartheta_1}D_{\vartheta_2}\partial/\partial x$ when $N = 2$, $\mathcal{P} = D_{\vartheta_1}D_{\vartheta_2}D_{\vartheta_3}$ when $N = 3$.

Since the Lie algebra $\tilde{\mathfrak{G}}$ permits the standard splitting $\tilde{\mathfrak{G}} := \tilde{\mathfrak{G}}_+ \oplus \tilde{\mathfrak{G}}_-$ into a direct sum of its Lie subalgebras $\tilde{\mathfrak{G}}_+ := \prod_{z\in\mathbb{S}^1}(\tilde{\mathcal{G}}_+ \ltimes \tilde{\mathcal{G}}^*_{-,reg})$ and $\tilde{\mathfrak{G}}_- := \prod_{z\in\mathbb{S}^1}(\tilde{\mathcal{G}}_- \ltimes \tilde{\mathcal{G}}^*_{+,reg}))$, on its dual space $\tilde{\mathfrak{G}}^*$ with respect to the symmetric bilinear form $\langle .,. \rangle_0 := \int_{\mathbb{S}^1} dz\,(.,.)_0$ one can introduce for any smooth by Frechet functionals $\mu,\nu \in \mathcal{D}(\tilde{\mathfrak{G}}^*)$ the $\mathcal{R}$-deformed Lie-Poisson bracket $\{\mu,\nu\}_{\mathcal{R}}(\tilde{a} \ltimes \tilde{l}) = \langle \tilde{a} \ltimes \tilde{l}, [R\nabla_r\mu(\tilde{a} \ltimes \tilde{l}), \nabla_l\nu(\tilde{a} \ltimes \tilde{l})] + [\nabla_r\mu(\tilde{a} \ltimes \tilde{l}), R\nabla_l\nu(\tilde{a} \ltimes \tilde{l})]\rangle_0 + < e, \omega_2(R\nabla_r\mu(\tilde{a} \ltimes \tilde{l}), \nabla_l\nu(\tilde{a} \ltimes \tilde{l})) + \omega_2(\nabla_r\mu(\tilde{a} \ltimes \tilde{l}), R\nabla_l\nu(\tilde{a} \ltimes \tilde{l})) >$, where $e = (e_1, e_2) \in \mathbb{C}^2$, the brackets $< .,. >$ denote the scalar product on $\mathbb{C}^2$, $\mathcal{R} = (P_+ - P_-)/2$, P_+ and P_- are projectors on $\tilde{\mathfrak{G}}_+$ and $\tilde{\mathfrak{G}}_-$ respectively, $\nabla_l h(\tilde{a} \ltimes \tilde{l}) := (\nabla_l h_{\tilde{l}} \ltimes \nabla_l h_{\tilde{a}}) \in \tilde{\mathfrak{G}}$ and $\nabla_r h(\tilde{a} \ltimes \tilde{l}) := (\nabla_r h_{\tilde{l}} \ltimes \nabla_r h_{\tilde{a}}) \in \tilde{\mathfrak{G}}$ are left and right gradients of an arbitrary smooth functional $h \in \mathcal{D}(\tilde{\mathfrak{G}}^*)$ at $(\tilde{a} \ltimes \tilde{l}) \in \tilde{\mathfrak{G}}^*$, which due to the Adler-Kostant-Symes theory generates the hierarchy of Hamiltonian flows on $\tilde{\mathfrak{G}}^* \simeq \tilde{\mathfrak{G}}$ in the form $\partial(\tilde{a} \ltimes \tilde{l})/\partial t_p = \{\tilde{a} \ltimes \tilde{l}, h^{(p)}(\tilde{a} \ltimes \tilde{l})\}_{\mathcal{R}}$, $p \in \mathbb{Z}_+$, where $h^{(p)}(\tilde{a} \ltimes \tilde{l}) = \lambda^p h(\tilde{a} \ltimes \tilde{l})$, for the Casimir invariant $h \in I(\hat{\mathfrak{G}}^*)$. The reductions of this hierarchy on polynomial type coadjoint orbits of the Lie algebra $\hat{\mathfrak{G}}$ are shown to lead to hierarchies of compatibly bi-Hamiltonian $(2|N+1)$-dimensional systems on functional supermanifolds.

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