Contribution ID: 115

Type: Poster

Two-dimensional central extensions of some superconformal loop Lie algebra generalization and compatibly bi-Hamiltonian (2|N+1)-dimensional systems on functional supermanifolds

For $N \in \{1, 2, 3\}$ there is considered the semi-direct sum $\tilde{\mathcal{G}} \propto \tilde{\mathcal{G}}^*_{reg}$ of the loop Lie algebra $\tilde{\mathcal{G}}$, consisting of the even left superconformal vector fields on a supercircle $\mathbb{S}^{1|N}$ in the form $\tilde{a} := a\partial/\partial x + \frac{1}{2}\sum_{i=1}^{N}(D_{\vartheta_{i}}a)D_{\vartheta_{i}}$, where $a := a(x, \vartheta; \lambda) \in C^{\infty}(\mathbb{S}^{1|N} \times (\mathbb{D}^{1}_{+} \cup \mathbb{D}^{1}_{-}); \Lambda_{0})$ is holomorphic in the "spectral" parameter $\lambda \in$ $\mathbb{D}^1_+ \cup \mathbb{D}^1_- \subset \mathbb{C}, \mathbb{D}^1_+, \mathbb{D}^1_-$ are the interior and exterior regions of the unit centrally located disk $\mathbb{D}^1 \subset \mathbb{C}$ respectively, $a(x, \vartheta; \infty) = 0$, $(x, \vartheta) \in \mathbb{S}^{1|N} \simeq \mathbb{S}^1 \times \Lambda_1^N$, $\Lambda := \Lambda_0 \oplus \Lambda_1$ is a commutative Banach superalgebra over the field $\mathbb{C} \subset \Lambda_0, \partial/\partial x$ is a partial derivative by the commuting variable $x, \vartheta := (\vartheta_1, \vartheta_2, \dots, \vartheta_N)$, $\partial/\partial \vartheta_i$ is a left partial derivative by the anticommuting variable $\vartheta_i \in \Lambda_1$, $D_{\vartheta_i} := \partial/\partial \vartheta_i + \vartheta_i \partial/\partial x$, $i = \overline{1, N}$, and its regular dual space $\tilde{\mathcal{G}}_{reg}^*$ with respect to the parity $(\tilde{a}, \tilde{l})_0 = \operatorname{res}_{\lambda \in \mathbb{C}} \int_{\mathbb{S}^{1|N}} dx d\vartheta_1 \dots d\vartheta_N (al)$, where res_{$\lambda \in \mathbb{C}$} denotes the coefficient at λ^{-1} in the corresponding Laurent series, $\tilde{l} \in \tilde{\mathcal{G}}_{reg}^{*}$ is a right superdifferential 1-form on $\mathbb{S}^{1|N}$ such as $\tilde{l} := (dx - \sum_{i=1}^{N} (d\vartheta_i)\vartheta_i)l(x,\vartheta;\lambda) \in \tilde{\mathcal{G}}_{reg}^{*}, l := l(x,\vartheta;\lambda) \in C^{\infty}(\mathbb{S}^{1|N} \times (\mathbb{D}^{1}_{+} \cup \mathbb{D}^{1}_{-}); \Lambda_s)$ is holomorphic in the "spectral" parameter $\lambda \in \mathbb{D}^{1}_{+} \cup \mathbb{D}^{1}_{-}, l(x,\vartheta;\infty) = 0, s = 1$ if N is an odd natural number and s = 0 if N is an even one. The loop Lie algebra $\tilde{\mathcal{G}}$ is splitting into the direct sum $\tilde{\mathcal{G}} = \tilde{\mathcal{G}}_+ \oplus \tilde{\mathcal{G}}_-$ of its Lie subalgebras for which $\tilde{\mathcal{G}}^*_{+,reg} \simeq \tilde{\mathcal{G}}_-, \tilde{\mathcal{G}}^*_{-,reg} \simeq \tilde{\mathcal{G}}_+$, where $a(x,\vartheta;\infty) = 0$ for any $\tilde{a} \in \tilde{\mathcal{G}}_-$. On $\tilde{\mathcal{G}} \propto \tilde{\mathcal{G}}^*_{reg}$ one determines the commutator $[\tilde{a} \propto \tilde{l}, \tilde{b} \propto \tilde{m}] := [\tilde{a}, \tilde{b}] \propto (ad_{\tilde{a}}^*\tilde{m} - ad_{\tilde{b}}^*\tilde{l})$ for any $\tilde{a}, \tilde{b} \in \tilde{\mathcal{G}} \text{ and } \tilde{l}, \tilde{m} \in \tilde{\mathcal{G}}_{reg}^*, \text{ where } [\tilde{a}, \tilde{b}] := \tilde{c}, \tilde{c} \in \tilde{\mathcal{G}}, c := a(\partial b/\partial x) - b(\partial a/\partial x) + \frac{1}{2}\sum_{i=1}^{N} (D_{\vartheta_i}a)(D_{\vartheta_i}b),$ ad^* is the coadjoint action of $\tilde{\mathcal{G}}$ with respect to the parity $(.,.)_0$, as well as the symmetric bilinear form $(\tilde{a} \propto \tilde{l}, \tilde{b} \propto \tilde{m})_0 = (\tilde{a}, \tilde{m})_0 + (\tilde{b}, \tilde{l})_0$. One constructs the central extensions $\hat{\mathfrak{G}} := \hat{\mathfrak{G}} \oplus \mathbb{C}^2$ of the Lie algebra $\mathfrak{G} := \prod_{z \in \mathbb{S}^1} (\hat{\mathcal{G}} \propto \hat{\mathcal{G}}_{reg}^*)$ by the superanalogs of the Ovsienko-Roger 2-cocycle such as $\omega_2(\tilde{a} \propto l, b \propto \tilde{m}) :=$ $(\omega_2^1(\tilde{a} \propto \tilde{l}, \tilde{b} \propto \tilde{m}), \omega_2^2(\tilde{a} \propto \tilde{l}, \tilde{b} \propto \tilde{m})), \text{ where } \omega_2^1(\tilde{a} \propto \tilde{l}, \tilde{b} \propto \tilde{m}) = \text{ res } \int_{\mathbb{S}^1} dz \int_{\mathbb{S}^{1|N}} dx d^N \vartheta \left(a(\mathcal{P}b) \right),$
$$\begin{split} & \omega_2^2(\tilde{a} \propto \tilde{l}, \tilde{b} \propto \tilde{m}) = \int_{\mathbb{S}^1} dz \, ((a, \partial m/\partial z)_0 - (b, (\partial l/\partial z)_0), \, (\tilde{a} \propto \tilde{l}), (\tilde{b} \propto \tilde{m}) \in \tilde{\mathfrak{G}}, \, z \in \mathbb{S}^1, \, \text{and} \, \mathcal{P} = D_{\vartheta_1} \partial^2/\partial^2 x \text{ when } N = 1, \, \mathcal{P} = D_{\vartheta_1} D_{\vartheta_2} \partial/\partial x \text{ when } N = 2, \, \mathcal{P} = D_{\vartheta_1} D_{\vartheta_2} D_{\vartheta_3} \text{ when } N = 3. \end{split}$$

Since the Lie algebra $\tilde{\mathfrak{G}}$ permits the standard splitting $\tilde{\mathfrak{G}} := \tilde{\mathfrak{G}}_+ \oplus \tilde{\mathfrak{G}}_-$ into a direct sum of its Lie subalgebras $\tilde{\mathfrak{G}}_+ := \prod_{z \in \mathbb{S}^1} (\tilde{\mathcal{G}}_+ \propto \tilde{\mathcal{G}}^*_{-,reg})$ and $\tilde{\mathfrak{G}}_- := \prod_{z \in \mathbb{S}^1} (\tilde{\mathcal{G}}_- \propto \tilde{\mathcal{G}}^*_{+,reg}))$, on its dual space $\tilde{\mathfrak{G}}^*$ with respect to the symmetric bilinear form $\langle ., . \rangle_0 := \int_{\mathbb{S}^1} dz \, (., .)_0$ one can introduce for any smooth by Frechet functionals $\mu, \nu \in \mathcal{D}(\tilde{\mathfrak{G}}^*)$ the \mathcal{R} -deformed Lie-Poisson bracket $\{\mu, \nu\}_{\mathcal{R}} (\tilde{a} \propto \tilde{l}) = \langle \tilde{a} \propto \tilde{l}, [R \nabla_r \mu(\tilde{a} \propto \tilde{l}), \nabla_l \nu(\tilde{a} \propto \tilde{l})] + [\nabla_r \mu(\tilde{a} \propto \tilde{l}), R \nabla_l \nu(\tilde{a} \propto \tilde{l})] \rangle_0 + \langle e, \omega_2 (R \nabla_r \mu(\tilde{a} \propto \tilde{l}), \nabla_l \nu(\tilde{a} \propto \tilde{l})) + \omega_2 (\nabla_r \mu(\tilde{a} \propto \tilde{l}), R \nabla_l \nu(\tilde{a} \propto \tilde{l})) \rangle_{\mathcal{N}}$ where $e = (e_1, e_2) \in \mathbb{C}^2$, the brackets $\langle ., . \rangle$ denote the scalar product on \mathbb{C}^2 , $\mathcal{R} = (P_+ - P_-)/2$, P_+ and P_- are projectors on $\tilde{\mathfrak{G}}_+$ and $\tilde{\mathfrak{G}}_-$ respectively, $\nabla_l h(\tilde{a} \propto \tilde{l}) := (\nabla_l h_{\tilde{l}} \propto \nabla_l h_{\tilde{a}}) \in \tilde{\mathfrak{G}}$ and $\nabla_r h(\tilde{a} \propto \tilde{l})$ is $(\tilde{a} \propto \tilde{l}) \in \tilde{\mathfrak{G}^*}$, which due to the Adler-Kostant-Symes theory generates the hierarchy of Hamiltonian flows on $\tilde{\mathfrak{G}^* \simeq \tilde{\mathfrak{G}}$ in the form $\partial(\tilde{a} \propto \tilde{l})/\partial t_p = \{\tilde{a} \propto \tilde{l}, h^{(p)}(\tilde{a} \propto \tilde{l})\}_{\mathcal{R}}, p \in \mathbb{Z}_+$, where $h^{(p)}(\tilde{a} \propto \tilde{l}) = \lambda^p h(\tilde{a} \propto \tilde{l})$, for the Casimir invariant $h \in I(\hat{\mathfrak{G}^*)$. The reductions of this hierarchy on polynomial type coadjoint orbits of the Lie algebra $\hat{\mathfrak{G}}$ are shown to lead to hierarchies of compatibly bi-Hamiltonian (2|N+1)-dimensional systems on functional supermanifolds.

Primary author: HENTOSH, Oksana (Pidstryhach IAPMM, NAS of Ukraine)

Presenter: HENTOSH, Oksana (Pidstryhach IAPMM, NAS of Ukraine)

Session Classification: MATHEMATICS

Track Classification: MATHEMATICS