

## Extremal decomposition problem for points on an arbitrary ellipse

Let  $\mathbb{C}$  be the complex plane,  $\bar{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$  be its one point compactification. A function  $g_B(z, a)$  which is continuous in  $\bar{\mathbb{C}}$ , harmonic in  $B \setminus \{a\}$  apart from  $z$ , vanishes outside  $B$ , and in the neighborhood of  $a$  has the following asymptotic expansion  $g_B(z, a) = -\log|z-a| + \gamma + o(1)$ ,  $z \rightarrow a$ , is called the (classical) Green function of the domain  $B$  with respect to a point  $a$ . The inner radius  $r(B, a)$  of the domain  $B$  with respect to a point  $a$  is the quantity  $e^\gamma$ . By using the variational method G.M. Goluzin established that for functions  $f_k(z)$  which univalently map the disc  $|z| < 1$  onto mutually non-overlapping domains,  $k \in \{1, 2, 3\}$ , exact estimate holds  $\left| \prod_{k=1}^3 f'_k(0) \right| \leq \frac{64}{81\sqrt{3}} |(f_1(0) - f_2(0))(f_1(0) - f_3(0))(f_2(0) - f_3(0))|$ . Equality is attained only for functions  $f_k(z)$  which conformally and univalently map the disc  $|z| < 1$  onto the angles  $2\pi/3$  with vertex at point  $w = 0$  and bisectors of which pass through points  $f_k(0)$ ,  $|f_k(0)| = 1$ . E.V. Kostyuchenko proved that the maximum value of multiplication of inner radii for three simply connected non-overlapping domains in the disk is attained for three equal sectors. However, this statement remains valid for multiply connected domains  $D_k$ . We have considered an extremal problem on the maximum of product of the inner radii on a system of  $n$  mutually non-overlapping multiply connected domains  $D_k$  containing the points  $a_k$ ,  $k = 1, \dots, n$ , located on an arbitrary ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  for which  $d^2 - t^2 = 1$ .

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