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Oleksandr Bukalov

ON THE SOLUTION OF THE PROBLEM OF THE COSMOLOGICAL CONSTANT

Center for Physical and Space Research of the International Institute of Socionics,
Yu. Illenka str., 12, Kiev-050, 04050, Ukraine
e-mail: boukalov@gmail.com

Abstract.
Calculation of the vacuum energy density in quantum field theory gives a value10^{122} times higher than the observed one, and many proposed approaches have not solved this problem and have not calculated its real value. However, the application of the microscopic theory of superconductivity to the description of the physical vacuum on the Planck scale made it possible to solve the problem of the cosmological constant and obtain a formula for the observed vacuum density or dark energy. Its numerical value is $6.09\cdot10^{-30}$g/cm^3, and it is in complete agreement with observations, since the experimental value is $(6.03\pm0.13)\cdot10^{-30}$g/cm^3 (J. Prat, C. Hogan, C. Chang, J. Frieman, 2022).
The cosmological model with superconductivity (CMS), proposed by the author, also implies a description of the earliest stage of the Universe evolution preceding the inflation stage. It describes the formation of the inflaton field as a special condensate of primordial fermions with the Planck mass, followed by the inflationary expansion of the early Universe. The current expansion of the Universe and its evolution are described as an ongoing second-order phase transition, and the flow of physical cosmological time is a consequence of processes occurring on Planck scales. The value of the Hubble parameter H_0=69.76 km·s^{-1}·Mpc^{-1} calculated in CMS corresponds to the average value for most values of this parameter obtained by different methods. CMS also describes black holes as a quantum condensate of primary fermions with Planck mass.
Key words: vacuum energy density, superconductivity, cosmological constant, cosmology, cosmological model with superconductivity, inflation, phase transitions, passage of time, string theory, shadow matter, fine structure constant, Hubble parameter, Kaluza-Klein theory.

In memory of prof. P. I. Fomin
and prof. Yu. A. Sitenko

1. Introduction

The problem of the cosmological constant [1] or the dark energy problem is the one of the most important and difficult in modern cosmology. A number of authors have proposed many options for solving this problem, but almost all of them do not provide an opportunity to accurately calculate the value of the density of dark energy.

In addition, as is known, standard estimates in the framework of quantum field theory give vacuum energy density values that are 10^{122} times higher than the observed one: $\rho_v \approx \int_0^{E_P} E^3 dE \approx E_P^4 \approx (10^{19}\,\text{GeV})^4$, where $E_P = (\hbar c^5 / G_N)^{1/2}$ is the Planck energy.

In the works of P. I. Fomin [7, 8] it is shown that quasi-closed worlds of Planck dimensions can form regular structures [7], a crystal-like lattice with cells of the Planck order, acting as “space atoms”: $L_P = (G_N \hbar / c^3)^{1/2}$.

Since these worlds are quasi-closed objects, their effective mass is close to zero. At the same time, they must interact with each other by quadrupole gravitational forces. The additional energy contributions of vacuum condensates to the vacuum energy density are automatically compensated due to the corresponding deformation of space atoms.

Because of this, P. I. Fomin in 1989 proposed to consider the physical vacuum as a four-dimensional gravitational quantum crystal with energy bands that correspond to generations of observed fermions.

Indeed, the connection between quantum solid state physics and elementary particle physics is fairly obvious. Moreover, they use not only the same mathematical apparatus, but also in modern solid-state physics there are analogues of a number of elementary particles, and many processes describing the evolution of the Universe, including inflation, are modeled in metamaterials.

According to P. I. Fomin, all particles in the Universe as vacuum excitations are completely similar to quasi-particles living in a crystal, however, the space of a crystal is completely empty for them, although from an external point of view it has a regular ordered structure.

Let's take the next step in describing such a structure of the physical vacuum.

2. Structure of the vacuum on the Planck scale and superconductivity

Let us consider such a crystal-like structure not just as an analogue of a solid body, but as a structure similar to the structure of a metal, in which there are free primary fermions [4, 9]. Such fermions arise naturally in multidimensional Kaluza–Klein theories.

These fermions can interact with a spatial crystal-like lattice, and under certain conditions can pair through phonon interaction, similar to the Bardeen-Cooper-Schrieffer (BCS) mechanism [9] for electrons in a metal with the formation of a Bose condensate.

Thus, pairing of fermions can occur near the Fermi surface of this spatial "quasicrystal". In this case, the maximum oscillation frequency of a crystal-like lattice, as an analogue of the Debye frequency, is close to the Planck frequency: $\omega_D \approx \omega_P$.

In [4, 9] we described the process of formation of dark energy as a condensate of primary fermions, by analogy with the theory of superconductivity by Bardeen-Cooper-Schrieffer (BCS). It gives a better understanding of the dark energy nature.

Let us consider the degenerate almost ideal Fermi gas with attraction between the particles, which are the primary fermions with a mass close to the Planck mass: $M \approx M_P$.

3. The energy spectrum of the superfluid gas

It is well known, that even in the presence of an arbitrarily weak attraction between the particles, the ground state of the system is unstable respect to the restructuring, changing whole system and lowering its energy [3, 5]. This instability arises from the Cooper effect, i.e. aspiration to the formation of bound states of fermions pairs that are in the p-space near the Fermi surface and have momenta equal in direction and antiparallel spins. For consideration of this problem, following to

[5], we introduce the Bogolyubov transformation of the operators, which bring together the operators of the particles with opposite momenta and spins:

$$\hat{b}_{\mathrm{p}^-} = u_p \hat{a}_{\mathrm{p}^-} + v_p \hat{a}^+_{-\mathrm{p},+},$$
$$\hat{b}_{\mathrm{p}^+} = u_p \hat{a}_{\mathrm{p}^+} - v_p \hat{a}^+_{-\mathrm{p},-}, \quad (1)$$

The indexes + and - refer to the two values of the spin projection. With gas isotropy the coefficients u_p, v_p can depend only on the absolute value of the momentum p. The operators comply with the creation and annihilation of quasiparticles on condition:

$$\hat{b}_{\mathrm{p}\alpha}\hat{b}^+_{\mathrm{p}\alpha} + \hat{b}^+_{\mathrm{p}\alpha}\hat{b}_{\mathrm{p}\alpha} = 1, \quad (2)$$

where the index α numbers the two values of the spin projection. Other pairs of operators are anticommutative. Therefore, the transform coefficients are imposed a condition:

$$u_p^2 + v_p^2 = 1. \quad (3)$$

The transformation inverse to $\hat{b}_{\mathrm{p}^-}, \hat{b}_{\mathrm{p}^+}$ takes the form

$$\hat{a}_{\mathrm{p}^-} = u_p \hat{b}_{\mathrm{p}^+} + v_p \hat{b}^+_{-\mathrm{p},-},$$
$$\hat{a}_{\mathrm{p}^+} = u_p \hat{b}_{\mathrm{p}^-} - v_p \hat{b}^+_{-\mathrm{p},+}. \quad (4)$$

Due to the primary role of the interaction between pairs of particles with opposite momenta and spins only write the Hamiltonian with the members, in which $\mathrm{p}_1 = -\mathrm{p}_2 \equiv \mathrm{p},\ \mathrm{p}_1' = -\mathrm{p}_2' \equiv \mathrm{p}'$:

$$\hat{H} = \sum_{\mathrm{p}\alpha} \frac{p^2}{2m} \hat{a}^+_{\mathrm{p}\alpha}\hat{a}_{\mathrm{p}\alpha} - \frac{g}{V} \sum_{\mathrm{pp}'} \hat{a}^+_{\mathrm{p}'+}\hat{a}^+_{-\mathrm{p}',-}\hat{a}_{-\mathrm{p},-}\hat{a}_{\mathrm{p}+}, \quad (5)$$

where $g = 4\pi\hbar^2 |b| / m$ is a "coupling constant", $b < 0$ is the scattering length.

Following to Pitaevskii L.P. & Lifshitz E.M. [5], we get the value of the energy gap:

$$\Delta_0 = \tilde{\varepsilon}\exp\left(-\frac{2\pi^2\hbar^3}{gmp_F}\right) = \tilde{\varepsilon}\exp\left(-\frac{\pi\hbar}{2p_F|b|}\right). \qquad (6)$$

or

$$\Delta_0 = \tilde{\varepsilon}\exp(-2/g\chi_F), \qquad (7)$$

where $\chi_F = mp_F/\pi^2\hbar^3$ is the energy density of the particle states on the Fermi surface, p_F is a momentum of a fermion at the Fermi surface, $g = 4\pi\hbar^2|b|/m$ is a "coupling constant", $b < 0$ is the scattering length.

The energy of elementary excitations with a change in the filling of quasiparticles is:

$$\begin{aligned} \varepsilon(p) &= \sqrt{\Delta^2 + \eta_p^2} \\ \eta_p &= v_F(p - p_F) \end{aligned}, \qquad (8)$$

where $v_F = p_F/m$. Thus, the energy of quasiparticles cannot be less than Δ. For $p = p_F$ $\varepsilon(p) = \Delta$.

Therefore, the excited states of the system are separated from the main energy gap, as well as the quasi-particles must appear in pairs, it is possible to write down the value of this gap as 2Δ. From $\varepsilon(p) \neq 0$ it follows that the Fermi gas has superfluidity. Thus from quasiparticles with energies $\varepsilon(p)$ a gas appears, which translationally moves as a single unit relative to the fluid with velocity v. Such gas from quasiparticle corresponds to the normal component of the superfluid. The rest of the liquid will behave like a superfluid component. The density of such superfluid liquid is equal to the sum of the normal and superfluid components: $\rho = \rho_n + \rho_s$.

The energy 2Δ is the energy of the Cooper pairs. It must be expended to break a pair. The value of the distance between the particles with correlated momenta, or the coherence length, is $\xi_0 = \pi v_F/\Delta_0 = \hbar e^{\frac{\pi\hbar}{2p_F|b|}}/p_F$.

From thermodynamics of superfluid Fermi gas it follows [5] that $\Delta = 0$, when $T_c = \gamma\Delta_0/\pi \approx 0.57\Delta_0$

$$\Delta = T_c \sqrt{\frac{8\pi^2}{7\zeta(3)}\left(1-\frac{T}{T_c}\right)} = 3.063 T_c \sqrt{1-\frac{T}{T_c}}\,. \qquad (9)$$

The difference between the basic levels of the superfluid and normal systems is [5]:

$$E_s - E_n = -V\frac{mp_F}{4\pi^2\hbar^3}\Delta_0^2\,. \qquad (10)$$

The sign "–" in (10) is the instability of the "normal" ground state in the case of attraction between gas particles. On one particle it falls $\sim \Delta^2 / \mu$.

Hence the difference of entropies is:

$$S_s - S_n = -V\frac{4mp_F T_c}{7\zeta(3)\hbar^3}\left(1-\frac{T}{T_c}\right). \qquad (11)$$

In the case $T \to T_c$ with regard to (9) the difference between the free energies [5] is equal to

$$F_s - F_n = -V\frac{2mp_F T_c^2}{7\zeta(3)\hbar^3}\left(1-\frac{T}{T_c}\right)^2. \qquad (12)$$

We apply the theory outlined above to describe the dark energy of the Universe and the calculation of its density. Let's transform (10) into the expression for the density:

$$-\Delta\rho = \frac{E_s - E_n}{V} = -\frac{mp_F}{4\pi^2\hbar^3}\Delta_0^2\,. \qquad (13)$$

The observed density of dark energy can be considered as the density of the binding energy of fermions. Therefore, considering it as the difference between the densities of the energies of the levels of the superfluid and normal systems, it is necessary to attribute this difference as negative, indicating instability of the normal ground state for an arbitrarily small attraction between fermions, according to (13). In fact, **this interaction of primordial fermions gives rise to the inflationary scalar field that causes the accelerated expansion of the universe that is observed**.

With $\Delta\rho = \rho_{DE} = \Lambda / 8\pi G_N = mp_F\Delta_0^2 / 4\pi^2\hbar^3$ we choose, for example, $v_F = \pi c / 8$, in order to the fermion velocity on the Fermi surface was lower than the speed of light. Then

$$\Delta_0 = \tilde{\varepsilon} e^{-\frac{\pi\hbar}{2p_F|b|}} = \frac{M_P e^{-\frac{\pi\hbar}{2p_F|b|}}}{4\pi} = \frac{M_P e^{-\frac{\pi\lambda_F}{2|b|}}}{4\pi} = \frac{M_P e^{-\frac{1}{\lambda_i}}}{4\pi}, \tag{14}$$

where M_P is the Planck mass.

At $\Lambda = \Delta_0^2 / 4 = \tilde{\varepsilon}^2 e^{-2\frac{\pi\hbar}{2p|b|}} = \tilde{\varepsilon}^2 e^{-2/\lambda_i}$, where λ_i is the constant of fermions interaction, we estimate the value of λ_i. Since $\Lambda^{1/2} = \tilde{\varepsilon} / e^{1/\lambda_i} = M_P / e^{1/\lambda_i} C$, then assuming a natural cutoff parameter of maximum energy equal to $\tilde{\varepsilon} = M_P$, when $\lambda_i \cong \alpha_{em} = (137.0599)^{-1}$ and $C = 8\pi$, we obtain:

$$\rho_{DE} = \frac{1}{4\pi G_N \left(8\pi t_P e^{1/\lambda_i}\right)^2} = \frac{1}{256\pi^3 G_N^2} \frac{c^5}{\hbar e^{2\alpha_{em}^{-1}}}, \tag{15}$$

$\rho_{DE} = 6.09 \cdot 10^{-30}\,\text{g/m}^3$ in excellent agreement with the PLANK data [6].

J. Prat, C. Hogan, C. Chang, J. Frieman obtain **$\rho_\Lambda = (6.03 \pm 0.13) \cdot 10^{-30}$ g/cm^3 as "the most accurate constraint to date, with an absolute calibration of cosmological measurements based on CMB temperature"** [10].

Note that the smallness of the energy gap Δ_0 compared to the Planck energy makes the use of the nonrelativistic BCS approach completely justified and quite sufficient, even at Grand Unified energies.

Thus, in the modern era, at $z = 0$, the observed density of dark energy interaction parameter of primary fermions is very close to the electromagnetic fine structure constant α_{em} or equal to it. There are two possibilities: either the interaction of fermions has electromagnetic nature, or the equality $\lambda_i \cong \alpha_{em}$ points to the existence of "shadow" or "mirror" long-range interactions, like the electromagnetic ones (for example, "dark photons"), and a number of corresponding charges from the shadow sectors of matter q_i, q_j, q_k,…, some of them may be equal in magnitude with the electric charge. Then

$$\lambda_i = \frac{q_i^2}{\hbar c},\ \lambda_j = \frac{q_j^2}{\hbar c},\ \lambda_k = \frac{q_k^2}{\hbar c}\ldots. \tag{16}$$

The existence of shadow electric charges and their corresponding shadow electromagnetic fields can be described in extended versions of the Kaluza-Klein theory with additional microscopic dimensions.

The condensation of primordial fermions forms a scalar field, which is used in inflation theories.

If the parameters λ_i, λ_j, λ_k,… are similar to α_{em}, then we can estimate the dynamics of their change depending on the energy density in the early Universe. Let us consider the process of formation of modern values of dark energy in the hot early Universe. As we know from quantum electrodynamics, the value of the electromagnetic fine structure constant is a function of the four-momentum Q^2:

$$\alpha_i^{-1} = \alpha_{em}^{-1} - \frac{\beta}{3\pi}\ln\left(\frac{Q}{2m_e}\right)^2. \tag{17}$$

For $\lambda_i \approx \alpha_{em}$, m_x is equal to the electron mass m_e, and the effective dark energy density is:

$$\rho_{DE} = \frac{\Lambda}{8\pi G_N} = \frac{c^5}{256\pi^3 G_N^2 \hbar e^{2\left(\alpha_{em}^{-1} - \frac{\beta}{3\pi}\ln\left(\frac{E}{4m_e}\right)^2\right)}} = \frac{c^5}{256\pi^3 G_N^2 \hbar e^{2\alpha_{em}^{-1}}}\left(\frac{Q}{2m_e}\right)^{\frac{4\beta}{3\pi}}, \tag{18}$$

where $Q = kT/c$ is momentum of radiation quanta in the early Universe. In this particular case, ρ_{DE} reaches a minimum and becomes constant at $Qc = 2m_e c^2 = 1.022\,\text{MeV}$.

In the general case, the laws of change in the parameters λ_i, λ_j, λ_k,…, may not be related to the dynamics of changes in α_{em} and $m_x \neq m_e$.

Because dark energy is the only one of the components of the observable Universe, but it is comparable to other, so it is rightful to consider the energy density of the entire observable Universe as evolving dynamically changing difference of density of normal and superfluid fermion systems, i.e. being in a state of a phase transition with changing energy density. Then the density $\Delta\rho$ can be identified with the critical density of the Universe. When

$$\Delta\rho = \rho_c = \frac{3}{8\pi G_N} H_0^2 = \frac{m p_F}{4\pi^2 \hbar^3} \Delta_0^2 \qquad (19)$$

and $m = M_{Pl}$, choose $p_F = \pi M_P c / 4$, in order to the fermion velocity on the Fermi surface will be lower than the speed of light. Then the square of the dynamically changing energy gap determines the Hubble radius: $\Delta_0^2 = 6H_0^2$. That means that the time parameter t_H is a function of the occurring phase transition of type II, corresponding to the Universe evolution and the variable λ_j:

$$\Delta_j = \frac{\tilde{\varepsilon}}{e^{\frac{\pi\hbar}{2 p_F |b|}}} = \frac{M_{Pl}}{4\pi e^{\frac{\pi\hbar}{2 p_F |b|}}} = \frac{M_{Pl}}{4\pi e^{\frac{\pi \lambda_F}{2|b|}}} = \frac{M_{Pl}}{4\pi e^{\frac{1}{\lambda_j}}}. \qquad (20)$$

4. The Hubble parameter

From $t_H = H_0^{-1} = 1.4 \cdot 10^{10}$ years, $t_H = 8\pi t_{Pl} e^{\lambda_j^{-1}} = 8\pi t_{Pl} e^{\frac{\pi\lambda_{F_j}}{2|b|}}$, $\lambda_j^{-1} = \pi\lambda_{F_j} / 2|b| \approx$ $\approx 137.03599... = \alpha_{em}^{-1}$ at $z = 0$, where α_{em} is the fine structure constant. In this case, **the Hubble radius is determined by the distance between the interacting fermions, or the coherence length**, $\xi_0 \approx ct_H$.

The critical density corresponds to the Hubble parameter with a value $H_0 = 69.76\ \mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$

$$\rho_c = \frac{3}{8\pi G_N} H_0^2 = \frac{3}{8\pi G_N}\left(\frac{1}{8\pi t_{Pl} e^{\lambda_j^{-1}}}\right)^2 = \frac{3}{8\pi G_N}\left(\frac{1}{8\pi t_{Pl}} e^{\frac{\pi\lambda_F}{2|b|}}\right)^2 \approx 9.14 \cdot 10^{-30}\,\mathrm{g/sm^3}. \quad (21)$$

This value ρ_c is in good agreement with the observational results. It is noteworthy that the value of H_0 obtained in the **cosmological model with superconductivity** (CMS), proposed by us, is the arithmetic mean between the value obtained by PLANK (H_0 = 67.4 ± 0.5 km·s^{-1}·Mpc^{-1}) [6] and the local value obtained from Cepheid (H_0 = 73.04 ± 1.04 km·s^{-1}·Mpc^{-1}) [11], and within the limits of error it coincides with the values obtained by other methods:

H_0 = 69.8 ± 1.3 km·s^{-1}·Mpc^{-1} [12],
H_0 = 69.5 ± 1.7 km·s^{-1}·Mpc^{-1} [13],
H_0 = 69.0 ± 1.1 km·s^{-1}·Mpc^{-1} [14],
H_0 = 69.51+0.70 − 0.65 km·s^{-1}·Mpc^{-1} [15],
H_0 = 69.79 ± 0.99 km·s^{-1}·Mpc^{-1} [16],
H_0 = 69.82+ 0.63 − 0.76 km·s^{-1}·Mpc^{-1} [17],
H_0 = 69.74+1.60 − 1.56 km·s^{-1}·Mpc^{-1} [18],
H_0 = 69.88 ± 0.76 km·s^{-1}·Mpc^{-1} [19].

It should be noted that the proximity of the dark energy density value of the critical density of matter and generally can be explained by the proximity or the equality of the interaction parameters in the modern era. Such equality can be explained by the approximation of the various parameters λ_i to a single value, similar to the behavior parameters in the era of Grand Unification: $\lambda_i \cong \lambda_j \cong \lambda_z \cong \lambda_{em}$. Thus, the observed dark energy and matter can be regarded as a set of quasi-particles with energy of communication of primary fermions. Therefore, the observed world can be seen as the difference between two energy levels of a fermion system, which density is close to the Planck density: $\rho_n = \rho_P \approx 3M_P^4 / 8\pi$, $\rho_s = \rho_n - \rho_c =$ $= 3(M_P^4 - M_P^2\Delta_j^2) / 8\pi$.

Thus, we can describe the observed critical density of the Universe as the difference between the densities of the superfluid and normal fermion systems, and this process is dynamic, providing the energy difference, which coincides with the energy of the observable Universe. Therefore, in the beginning we can start from the

Planck density, when $\rho_s = 0$, to $\rho_{P_n} - \rho_{s(t)} = \rho_{GUT}$ and then to $\rho_s \to \rho_P \left(1 - e^{2/\lambda_j}\right)$.

Let's note that the obtained energy density equations also naturally describe the exponential expansion of the early Universe.

Let's apply the developed theory to describe black holes. When $\Delta_g^2 = 6 / R_g^2$, $p_F = m_p \pi c / 4$

$$\rho_{BH} = \frac{3}{8\pi} \frac{c^2}{G_N R_g^2} = \frac{m p_F}{4\pi^2 \hbar^3} \Delta_g^2 . \qquad (22)$$

The value obtained by Δ_g^2 corresponds to the scalar curvature and is determined by the gravitational radius of the black hole.

Therefore, in the framework of the cosmological model with superconductivity (CMS), we can consider black holes as a gravitational condensate of primary fermions.

Thus, a **cosmological model with superconductivity (CMS)** arises, which makes it possible to give a new description of a whole series of cosmological processes.

5. Conclusion

From our proposed cosmological model with superconductivity (CMS), a number of consequences are derived:

1. Within the limits of the theory of superconductivity the real value of density of dark energy as density of energy interactions of a primary fermions condensate is received. These fermions do not give a contribution to observable energy density. The contribution to observable forms of energy is given only by the interaction energy of the primary fermions. This interaction generates an inflaton scalar field that causes the accelerated expansion of space.
2. Initial exponential expansion of the vacuum-like Universe within the limits of superconducting cosmology allows to provide the birth of the hot Universe and

solves the same problems which are solved by an inflationary cosmology.

3. Origin of cosmological time becomes clear: $t_U \approx t_H$ in the observable Universe time is a consequence of proceeding phase transition of II kind, which is similar to the phase transition, which has created the up-to-date vacuum energy density with change and fixing of a fine-structure constant $\alpha_j = \ln\left(t_H / \left(8\pi t_P\right)\right)$. This also solves the problem of time irreversibility. Therefore, the evolution of the entire observable Universe can be described as an ongoing second-order phase transition.
4. The closeness of the densities ρ_{DE}, ρ_M and ρ_c (coincidence problem) is due to the similarity or identity of the interaction constants: $\alpha_i \approx \alpha_j \approx \alpha_{em}$.
5. If the observer is at a point close to the end of the phase transition, he records the coincidence of a number of dynamic and static quantities, such as the Large Dirac numbers, etc., which also happens in reality.
6. Black holes can be described as gravitational condensates of primordial fermions.
7. The transition from the macroscopic classical dynamics of general relativity to the microscopic dynamics of fermions near the Fermi surface shows that the real structure and dynamics of space-time are described by coherent quantum processes. In particular, the evolutionary cosmological time parameter itself is determined by the dynamics of microscopic quantum processes on Planck scales.
8. The macroscopic nature of the observed space-time is provided by the factor $e^{\alpha^{-1}}$, which varies in the range from 1 to $3.26 \cdot 10^{59}$ and determines the scale of coherence of quantum processes in superconducting cosmology.
9. Instead of 10^{500} variants of worlds possible in superstring theory, CMS has a single version with several variables λ_i, λ_j, λ_k,… which run through all possible ranges during the evolution of the Universe.

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