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# Microscopic derivation of the generalixed Bohr Hamiltonian

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The Bohr's Hamiltonian is one of the main cornerstones of the nuclear structure theory. It was derived by Bohr [1], treating the nucleus as a liquid spherical drop with uniform density and sharp surface, performing quadrupole vibrations with small amplitude. During such oscillations at any moment of time the nucleus attains an ellipsoidal shape, retaining its volume constant due to small compressibility of the nuclear matter.
Following the microscopic theory [2], where the kinetic energy operator of the nucleus has been expressed in terms of the independent set of 3N collective variables, we constructed the collective Hamiltonian for nuclei with arbitrary deformation [3], which recovers the results of Bohr [1] at β « 1. First the classical kinetic energy T of N nucleons in the center-of-mass frame is expressed in terms of the Jacobi vectors $\mathbf{q}_i$, where $i = 1, 2, ..., n = N - 1$. Then we introduce the rotating frame with axes ξ, η, ζ directed along the principal axes of the nuclear tensor of inertia. Its orientation is determined by the Euler angles φ, ϑ, ψ. In this case the off-diagonal elements of the inertia tensor vanish, so that
$\sum_{i=1}^{n} a_{i\xi} a_{i\eta} = \sum_{i=1}^{n} a_{i\xi} a_{i\zeta} = \sum_{i=1}^{n} a_{i\eta} a_{i\zeta} = 0,$
where $a_{i\nu}$ denote the projections of of the Jacobi vectors on the axes frame ξ, η, ζ. We introduce an abstract Eucledian space with basis orthonormal vectors $\mathbf{e}_i$ and define there three vectors $\mathbf{A}_\nu$ with components $a_{i\nu}$ and lengths $a_\nu$. The above consraint can be treated as an orthogonality condition for these vectors, while their lengths $a_1,\ a_2,\ a_3$ serve as three collective coordinates, specifying size and shape of the nucleus. It is natural to determine the remaining 3n-6 internal coordinates as any rotational variables, which describe orientation of three vectors in the n-dimensional hyperspace. For this aim we take the generalized Euler angles θjk, introduced by Vilenkin.
Following Bohr's model [1], we demand that the vibrations and rotations only change a shape of the ellipsoidal nucleus keeping unchanged its volume, i.e., we demand that at arbitrary deformations the product of radii Rκ of the nuclear ellipsoid is related to the radius $R_0$ of the sphere with the same volume by $R_1 R_2 R_3 = R_{03}$ at $R_0 = const$. The radii, meeting this condition, we chose as [3] $R_\kappa = R_0 cal E_\kappa(\beta, \gamma)$, with $cal E_\kappa(\beta, \gamma) = \exp\left[\tilde{\beta}\cos\left(\gamma - \frac{2\pi}{3}\kappa\right)\right]$ and $\tilde{\beta} = (5/4\pi)^{1/2}\beta$. When β«1, the above expressions for Rκ coincide with well known ones and respectively our generalized parameters β, γ coincide with Bohr's variables. Expressing the kinetic energy T in the suggested collective coordinates and then quantizing it, we derived the Hamiltonian, depending on the exactly determined inertia functions $b_{\lambda\lambda'}(\beta, \gamma)$, in contrast to existing in literature Hamiltonians, written ad hoc with unknown inertia parameters.

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