Contribution ID: **80** Type: **Oral**

Discrete symmetries of differential equations

Discrete symmetries of differential equations are not so well studied objects as continuous ones. We discuss the notion of discrete symmetries and the methods of their computation. Then we analyze common errors in such computations and show connection of these errors with those in solving other problems of classical group analysis of differential equations, including the classification of Lie reductions and the construction of Lie-invariant solutions.

The main illustrative example is given by the (1+1)-dimensional linear heat equation. We derive a nice representation for its point symmetry transformations and properly interpret them. This allows us to prove that the pseudogroup of these transformations has exactly two connected components. That is, the heat equation admits a single independent discrete symmetry, which can be chosen to be alternating the sign of the dependent variable. We introduce the notion of pseudo-discrete elements of a Lie group and show that alternating the sign of the space variable, which was for a long time misinterpreted as a discrete symmetry of the heat equation, is in fact a pseudo-discrete element of its essential point symmetry group. As a result, the classification of subalgebras of the essential Lie invariance algebra of the heat equation is enhanced.

The developed approach to point-symmetry groups whose elements have components that are linear fractional in some variables can directly be extended to many other linear and nonlinear differential equations. We also consider the Burgers equation because of its relation to the heat equation and prove that it admits no discrete point symmetries. Further examples are the Harry Dym equation and a nonlinear diffusion equation with a special power diffusion coefficient.

[1] Koval S.D. and Popovych R.O., Point and generalized symmetries of the heat equation revisited, J. Math. Anal. Appl. 527 (2023), 127430, arXiv:2208.11073.

Primary authors: POPOVYCH, Roman (Silesian University in Opava & Institute of Mathematics oif NAS of Ukraine); KOVAL, Serhii (Institute of Mathematics of NAS of Ukraine)

Presenter: KOVAL, Serhii (Institute of Mathematics of NAS of Ukraine)

Session Classification: MATHEMATICS

Track Classification: MATHEMATICS