

Geometric properties and exact solutions of dispersionless Nizhnik equation

The dispersionless (potential symmetric) Nizhnik equation has interesting algebraic and geometric properties. We computed the point- and contact-symmetry pseudogroups of this equation using an original megaideal-based version of the algebraic method. Note that this approach was used for the first time to find the contact-symmetry pseudogroup of a differential equation. By the same method, we also constructed the point-symmetry pseudogroups of the nonlinear Lax representation of the dispersionless Nizhnik equation and of the dispersionless counterpart of the symmetric Nizhnik system. We checked whether certain subalgebras of the maximal Lie invariance algebra \mathfrak{g} of the dispersionless Nizhnik equation and of its contact invariance algebra define point and contact transformations stabilizing these algebras. In addition, we described all the third-order partial differential equations in three independent variables that possess the algebra \mathfrak{g} as invariance algebra. We found geometric properties that single out the dispersionless Nizhnik equation from the entire set of third-order partial differential equations with three independent variables. They include the maximal Lie invariance algebra and three conservation laws with simplest characteristics.

We also classified one- and two-dimensional subalgebras of the algebra \mathfrak{g} and codimension-one and -two Lie reductions of the dispersionless Nizhnik equation. Lie and point symmetries of reduced equations were comprehensively studied, including the analysis of which of them correspond to hidden symmetries of the original equation. We constructed wide families of new exact invariant solutions of the dispersionless Nizhnik equation in closed form in terms of elementary, Lambert and hypergeometric functions as well as in parametric or implicit form. Multiplicative separation of variables was used to present an example of finding non-Lie solutions of the dispersionless Nizhnik equation that generalize invariant solutions.

1. Boyko V.M., Popovych R.O. and Vinnichenko O.O. Point- and contact-symmetry pseudogroups of dispersionless Nizhnik equation. *Commun. Nonlinear Sci. Numer. Simul.* **132** (2024), 107915, arXiv:2211.09759.
2. Vinnichenko O.O., Boyko V.M. and Popovych R.O. Lie reductions and exact solutions of dispersionless Nizhnik equation. *Anal. Math. Phys.* **14** (2024), 82, arXiv:2308.03744.

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