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Approximation by interpolation trigonometric polynomials in Weyl-Nagy classes $W^r_{\beta,1}$

Let L_p, $1 \le p \le \infty$, and C be the spaces of 2π-periodic functions with standard norms $\|\cdot\|_{L_p}$ and $\|\cdot\|_C$, respectively. Further, let $W^r_{\beta,p},\ r>0,\ \beta\in\mathbb{R},\ 1\le p\le\infty$, be classes of 2π-periodic functions f that can be represented in the form of convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int\limits_{-\pi}^{\pi} \varphi(x-t)B_{r,\beta}(t)dt, \quad a_0\in\mathbb{R}, \quad (1)$$

with Weyl–Nagy kernels of the form $B_{r,\beta}(t) = \sum\limits_{k=1}^{\infty} k^{-r}\cos\left(kt - \frac{\beta\pi}{2}\right)$, of function φ satisfying the condition

$$\varphi \in B^0_p = \left\{\varphi\in L_p : \|\varphi\|_{L_p}\le 1,\ \int\limits_{-\pi}^{\pi}\varphi(t)dt = 0\right\}.$$

The classes $W^r_{\beta,p}$ are called the Weyl–Nagy classes, and the function φ in representation (1) is called the (r,β)-derivative of the function f in the Weyl–Nagy sense and denoted by f^r_β.

Let $f\in C$. By $\tilde{S}_{n-1}(f;x)$ we denote a trigonometric polynomial of degree $n-1$, that interpolates $f(x)$ at the equidistant nodes $x_k^{(n-1)} = 2k\pi/(2n-1)$, $k\in\mathbb{Z}$, i.e., such that

$$\tilde{S}_{n-1}(f;x_k^{(n-1)}) = f(x_k^{(n-1)}), \quad k\in\mathbb{Z}.$$

Theorem 1. Let $r>2$,
$\beta\in\mathbb{R},\ x\in\mathbb{R}$ and $n\in\mathbb{N}$. The following estimate is true

$$ca\tilde{l}E_n(W^r_{\beta,1};x) = \sup_{f\in W^r_{\beta,1}}\left|f(x)-\tilde{S}_{n-1}(f;x)\right| = \left|\sin\frac{(2n-1)x}{2}\right| n^{-r}\left(\frac{2}{\pi(1-e^{-r/n})} + \mathcal{O}(1)\delta_{r,n}\right),$$

where $\mathcal{O}(1)$ is a quantity uniformly bounded in all analyzed parameters,

$$\delta_{r,n} = \begin{cases} 1+\dfrac{n}{r(r-2)}, & 2<r\le n+1,\\ \dfrac{r}{n^2}e^{-r/n}, & n+1\le r\le n^2,\\ e^{-r/n} & r\ge n^2.\end{cases}$$

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