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Approximation by interpolation trigonometric polynomials in Weyl-Nagy classes $W_{\beta,1}^r$

Let L_p , $1 \le p \le \infty$, and C be the spaces of 2π -periodic functions with standard norms $\|\cdot\|_{L_p}$ and $\|\cdot\|_C$, respectively. Further, let $W^r_{\beta,p}$, r > 0, $\beta \in \mathbb{R}$, $1 \le p \le \infty$, be classes of 2π -periodic functions f that can be represented in the form of convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(x-t) B_{r,\beta}(t) dt, \quad a_0 \in \mathbb{R}, \quad (1)$$

with Weyl–Nagy kernels of the form $B_{r,\beta}(t) = \sum_{k=1}^{\infty} k^{-r} \cos\left(kt - \frac{\beta\pi}{2}\right)$, of function φ satisfying the condition

$$\varphi \in B_p^0 = \big\{ \varphi \in L_p : \|\varphi\|_{L_p} \le 1, \int_{-\pi}^{\pi} \varphi(t) dt = 0 \big\}.$$

The classes $W_{\beta,p}^r$ are called the Weyl–Nagy classes, and the function φ in representation (1) is called the (r, β) -derivative of the function f in the Weyl–Nagy sense and denoted by f_{β}^r .

Let $f \in C$. By $\tilde{S}_{n-1}(f;x)$ we denote a trigonometric polynomial of degree n-1, that interpolates f(x) at the equidistant nodes $x_k^{(n-1)} = 2k\pi/(2n-1), k \in \mathbb{Z}$, i.e., such that

$$\tilde{S}_{n-1}(f; x_k^{(n-1)}) = f(x_k^{(n-1)}), \quad k \in \mathbb{Z}$$

Theorem 1. Let r > 2,

 $\beta \in \mathbb{R}, \; x \in \mathbb{R} \; \text{and} \; \; n \in \mathbb{N}.$ The following estimate is true

$$ca\tilde{l}E_{n}(W_{\beta,1}^{r};x) = \sup_{f \in W_{\beta,1}^{r}} \left| f(x) - \tilde{S}_{n-1}(f;x) \right| = \left| \sin \frac{(2n-1)x}{2} \right| n^{-r} \left(\frac{2}{\pi(1-e^{-r/n})} + \mathcal{O}(1)\delta_{r,n} \right),$$

where $\mathcal{O}(1)$ is a quantity uniformly bounded in all analyzed parameters,

$$\delta_{r,n} = \begin{cases} 1 + \frac{n}{r(r-2)}, & 2 < r \le n+1, \\ \frac{r}{n^2} e^{-r/n}, & n+1 \le r \le n^2, \\ e^{-r/n} & r \ge n^2. \end{cases}$$

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