

Asymptotic step-like solutions of the Burgers equation with variable coefficients and singular perturbation

The talk deals with the singular perturbed Burgers equation with variable coefficients

$$\varepsilon u_{xx} = a(x, t, \varepsilon)u_t + b(x, t, \varepsilon)uu_x, \quad (1)$$

where $a(x, t, \varepsilon)$, $b(x, t, \varepsilon)$ are given as asymptotic series in a small parameter ε . Equation (1) is straightforward generalization of the Burgers equation

$$u_{xx} = u_t + \nu uu_x \quad (2)$$

that has been known since 1906 and has attracted the attention of researchers, because in one-dimensional case the equation is a simple form of Navier–Stokes equation. At the end of the 30-th years of the past century J.M.Burgers studied this equation as the simplest model unifying typical nonlinearity and viscosity ν which describes phenomenon of hydrodynamic turbulence. Model (2) is one of the simplest describing nonlinear effects, in particular, appearance and evolution of shock waves.

We consider asymptotic solution of equation (1) that is a discontinuous step-like function in limits case as $\varepsilon \rightarrow 0$. Such solution is called an asymptotic step-like solution. We present a general algorithm of constructing these solutions and prove the theorems on the accuracy with which the main term and the first approximation satisfy equation (1). These results are demonstrated on an example, for which the first asymptotic step-like approximation is explicitly found. The asymptotic solution is global, and has a form of the shock wave type function. There are also given graphs of these approximate solutions for certain numerical parameters.

The talk presents results published in [1].

[1]. Samoilenko, V., Samoilenko, Yu., Zappale, E., Asymptotic step-like solutions of the singularly perturbed Burgers equation. *Phys Fluids*, 35(6), 067106 (2023).
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