

Soliton- and peakon-like solutions to the modified Camassa-Holm equation with variable coefficients

We consider asymptotic soliton- and peakon-like solutions to the modified Camassa–Holm equation with variable coefficients and a singular perturbation

$$a(x,t,\varepsilon)u_t - \varepsilon^2 u_{xxt} + b(x,t,\varepsilon)u^2 u_x - 2\varepsilon^2 u_x u_{xxx} - \varepsilon^2 u u_{xxx} = 0. \quad (1)$$

Here ε is a small parameter and the coefficients $a(x, t, \varepsilon)$ and $b(x, t, \varepsilon)$ with $(x, t) \in \mathbf{R} \times [0; T]$ for some $T > 0$ can be presented as:

$$a(x, t, \varepsilon) = \sum_{k=0}^N \varepsilon^k a_k(x, t) + O(\varepsilon^{N+1}), \quad b(x, t, \varepsilon) = \sum_{k=0}^N \varepsilon^k b_k(x, t) + O(\varepsilon^{N+1}).$$

This equation is a generalization of the well-known modified Camassa–Holm equation [1, 2], which is an integrable system having both smooth and peaked soliton solutions, named peakons. We present an approach of constructing asymptotic solutions for equation (1) and we study their accuracy [3].

The results are illustrated by nontrivial examples of both asymptotic soliton- and peakon-like solutions, for which we compute the main and the first terms of their expansions. We also present graphics illustrating the obtained solutions for various values of small parameter. The proposed technique can be applied to construct wave-like solutions of different types for other equations [4].

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