

Towards the study of quantum properties of the Borromean rings complement in the Poincaré ball

Guided by physical needs, we deal with the rotationally isotropic Poincaré ball, when considering the complement of Borromean rings embedded in it [1]. We describe the geometry of the complement and realize the fundamental group as isometry subgroup in three dimensions. According to Penner, we construct the Teichmüller space of the decorated ideal octahedral surface related to the quotient space of the fundamental group action. Using the conformality of decoration, we define six moduli and the mapping class group generated by cyclic permutations of the ideal vertices. Intending to quantize the geometric area, we state the connection between the induced geometry and the sine-Gordon model. Due to such a correspondence we obtain the differential two-form in the cotangent bundle. Focusing on a starlike body formed by regular ideal squares in three orthogonal planes (discs), we analyze quantum fluctuations of the body area, using the canonical quantization of the sine-Gordon model resulting in the Mathieu differential equation. Since, according to general predictions in quantum geometry [2], the area quantum fluctuations arise at the boundaries of geometric regions, we associate them with the decorated cuspidal tails of the surface. Using the (single) quantum state generated by the Hamiltonian constraint, we still get an indication of the presence of a gap in the spectrum of the area [2].

Keywords: Borromean rings complement; fundamental group; decorated Teichmüller space; sine-Gordon equation; area quantization; Mathieu equation

1. A.A. Nazarenko and A.V. Nazarenko, Structures associated with the Borromean rings complement in the Poincaré ball, arXiv: 2404.02615 [math-ph]; accepted for publication in Ukr. J. Phys. 7 (2024)
2. A. Ashtekar and J. Lewandowski, Quantum theory of geometry: I. Area operators. Class. Quantum Grav. 14 (1997) A55; arXiv: gr-qc/9602046

Primary authors: NAZARENKO, Andriy (Bogolyubov Institute for Theoretical Physics of NAS of Ukraine); NAZARENKO, Anton (Faculty of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv)

Presenter: NAZARENKO, Anton (Faculty of Mechanics and Mathematics, Taras Shevchenko National University of Kyiv)

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