

A model of a conflict society with separate support for individual clusters

We study the mathematical model of an abstract society $calS = \{a_i\}_{i=1}^m$, $1 < m < \infty$ (a_i - players, opponents or their association) in the form of a complex dynamical system with a conflict interaction between its elements. The states of society S are described by stochastic vectors of players energy $\mathbf{p}^t = (p_1^t, \dots, p_i^t, \dots, p_m^t)$, $t = 0, 1, \dots$, which evolve according to the law of conflict interaction. Such models were studied in [1], [2], [3], for example.

Our new step is that we divide the set $calS$ into two clusters of individuals $calS = calS_J \cup calS_I$, where $J = \{j_1, \dots, j_n\}$ denotes a subset of such indices that individuals a_j receive external support at each step $t > 0$ in the form of an additive shift: $p_j^t \rightarrow p_j^t + b$, $b > 0$ and for $I = \{i_1, \dots, i_{m-n}\}$ individuals a_i remain without external support.

Thus, the dynamic system is given by difference equations:

$$p_i^{t+1} = \frac{p_i^t(1-r_i^t)}{z^t}, \quad p_j^{t+1} = \frac{(p_j^t+b_j)(1-r_j^t)}{z^t}, \quad i \in I, \quad j \in J, \quad t = 0, 1, \dots, \text{where } b_j = b > 0, \quad z^t = \sum_k (p_k^t + b \cdot \mathbf{1}_J(k))(1 -$$

$r_k^t)$, $\mathbf{1}_J(k)$ — indicator function of a subset J . Using of value $r_i^t = \frac{\sum_{k \neq i} p_k^t}{m-1} = \frac{1-p_i^t}{m-1}$ corresponds to the repulsive interaction of each individual a_i with the rest of society $calS_{a_i}^\perp$ in the mean field sense. Denominator z^t provides stochastic normalization.

The main results concern systems with three elements (players). In this case, a description of all equilibrium states is given and their stability is investigated with depending on the parameter of external influence. Besides basins of attraction for point attractors are partially described and illustrated.

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