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Asymptotic analysis of spectral problems for nonlocal operators

Many mathematical biology and population dynamics models involve nonlocal diffusion corresponding to long-range interactions in a system. These models are typically described by evolution problems with convolution-type integral operators and their qualitative and quantitative properties can be obtained by studying of the corresponding spectral problems.

We consider spectral problems
$-\frac{1}{\varepsilon^d}\int_\Omega J\left(\frac{x-y}{\varepsilon}\right)\kappa(x,y)\rho_\varepsilon(y)dy + a(x)\rho_\varepsilon(x) = \lambda_\varepsilon\rho_\varepsilon(x)$ (1)
in a bounded domain in $\Omega\subset\mathbb{R}^d$, where $J(z)\geq 0$ is a continuous function on $\mathbb{R}^d$ decaying sufficiently fast as $|z|\to\infty$, $\kappa\in C^2(\overline{\Omega}\times\overline{\Omega})$, $\kappa>0$ and (the potential) $a\in C^2(\overline{\Omega})$; $\varepsilon>0$ is a scaling parameter. We study the asymptotic behavior of eigenvalues and eigenfunctions of (1) in the limit of small parameter ε.

We focus on the self-ajoint case when $J(z) = J(-z)$, $\kappa(x,y) = \kappa(y,x)$ and show that the principal eigenvalue of (1) exists for sufficiently small ε and converges to the minimum $m(x^*) = \min_{\overline{\Omega}} m(x)$, where $m(x) = a(x) - \kappa(x,x)$. More precise asymptotic description is obtained when m satisfies some non-degeneracy conditions at x^*. Namely, if the minimum is strict and the point x^* is an inner point of Ω then we suppose the positiveness of Hessian and via rescaling by $\varepsilon^{1/2}$ we derive a limit differential spectral problem of the form:
$-\mathrm{div}A\nabla\rho + \partial^2_{ij}m(x^*)z_iz_j\rho = \mu\rho \quad \text{in } \mathbb{R}^d.$ (2)
We prove that $\lambda_\varepsilon = m(x^*) + \mu_k\varepsilon + \bar{o}(\varepsilon)$, where μ_k are eigenvalues of (2). The case $x^*\in\partial\Omega$ is more sophisticated and we consider the situation when Ω is a polyhedron and $m(x)$ attains its strict minimum at x^* on a face of $\partial\Omega$. Without loss of generality, we assume that $x^* = 0$ and locally Ω is given by $x_1>0$ in a neighborhood of 0. Then the non-degeneracy condition reads: $\partial_{x_1}m(0)>0$, \ $\partial^2_{x'_ix'_j}m(0)\,\xi'_i\xi'_j>0$ $\forall\xi'\in\mathbb{R}^{d-1}\setminus\{0\}$. Under these conditions, we establish the following asymptotic formula for the eigenvalues $\lambda_\varepsilon = m(0) + \Lambda_1\varepsilon^{2/3} + (\beta+\mu_k)\varepsilon + \bar{o}(\varepsilon)$, where Λ_1 is the principal eigenvalue of the 1D problem $-\theta\phi_0''(t) + \alpha t\phi_0(t) = \Lambda_1\phi_0(t)$ on $\mathbb{R}_+$, $\phi_0(0)=0$, μ_k are eigenvalues of a harmonic oscillator in $\mathbb{R}^{d-1}$. In this case, eigenfunctions have the asymptotic form $\rho_\varepsilon(x) = \phi_0(\varepsilon^{-2/3}x_1)\,v(\varepsilon^{-1/2}x') + \ldots$, that reveals emergence of two fine scales $\varepsilon^{2/3}$ and $\varepsilon^{1/2}$.

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