Contribution ID: 54

Type: Oral

Asymptotic analysis of spectral problems for nonlocal operators

Many mathematical biology and population dynamics models involve nonlocal diffusion corresponding to long-range interactions in a system. These models are typically described by evolution problems with convolution-type integral operators and their qualitative and quantitative properties can be obtained by studying of the corresponding spectral problems.

We consider spectral problems

 $-\frac{1}{\varepsilon^d} \int_{\Omega} J\left(\frac{x-y}{\varepsilon}\right) \kappa(x,y) \rho_{\varepsilon}(y) dy + a(x) \rho_{\varepsilon}(x) = \lambda_{\varepsilon} \rho_{\varepsilon}(x) \tag{1}$ in a bounded domain in $\Omega \subset \mathbb{R}^d$ where $I(z) \ge 0$ is a continuous function on

in a bounded domain in $\Omega \subset \mathbb{R}^d$, where $J(z) \geq 0$ is a continuous function on \mathbb{R}^d decaying sufficiently fast as $|z| \to \infty$, $\kappa \in C^2(\overline{\Omega} \times \overline{\Omega})$, $\kappa > 0$ and (the potential) $a \in C^2(\overline{\Omega})$; $\varepsilon > 0$ is a scaling parameter. We study the asymptotic behavior of eigenvalues and eigenfunctions of (1) in the limit of small parameter ε .

We focus on the self-ajoint case when J(z) = J(-z), $\kappa(x, y) = \kappa(y, x)$ and show that the principal eigenvalue of (1) exists for sufficiently small ε and converges to the minimum $m(x^*) = \min_{\overline{\Omega}} m(x)$, where

 $m(x) = a(x) - \kappa(x, x)$. More precise asymptotic description is obtained when m satisfies some nondegeneracy conditions at x^* . Namely, if the minimum is strict and the point x^* is an inner point of Ω then we suppose the positiveness of Hessian and via rescaling by $\varepsilon^{1/2}$ we derive a limit differential spectral problem of the form:

 $-\operatorname{div} A \nabla \rho + \partial_{ij}^2 m(x^*) z_i z_j \rho = \mu \rho \quad \text{in } \mathbb{R}^d.$ $\tag{2}$

We prove that $\lambda_{\varepsilon} = m(x^*) + \mu_k \varepsilon + \bar{o}(\varepsilon)$, where μ_k are eigenvalues of (2). The case $x^* \in \partial\Omega$ is more sophisticated and we consider the situation when Ω is a polyhedron and m(x) attains its strict minimum at x^* on a face of $\partial\Omega$. Without loss of generality, we assume that $x^* = 0$ and locally Ω is given by $x_1 > 0$ in a neighborhood of 0. Then the non-degeneracy condition reads: $\partial_{x_1} m(0) > 0$, $\langle \partial^2_{x'_i x'_j} m(0) \xi'_i \xi'_j > 0$ $\forall \xi' \in \mathbb{R}^{d-1} \setminus \{0\}$. Under these conditions, we establish the following asymptotic formula for the eigenvalues $\lambda_{\varepsilon} = m(0) + \Lambda_1 \varepsilon^{2/3} + (\beta + \mu_k) \varepsilon + \bar{o}(\varepsilon)$, where Λ_1 is the principal eigenvalue of the 1D problem $-\theta \phi''_0(t) + \alpha t \phi_0(t) = \Lambda_1 \phi_0(t)$ on \mathbb{R}_+ , $\phi_0(0) = 0$, μ_k are eigenvalues of a harmonic oscillator in \mathbb{R}^{d-1} . In this case, eigenfunctions have the asymptotic form $\rho_{\varepsilon}(x) = \phi_0(\varepsilon^{-2/3}x_1) v(\varepsilon^{-1/2}x') + \dots$, that reveals emergence of two fine scales $\varepsilon^{2/3}$ and $\varepsilon^{1/2}$.

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Session Classification: MATHEMATICS

Track Classification: MATHEMATICS