Contribution ID: **64** Type: **Oral**

Asymptotic behaviour of solutions of the differential equation of the form $y^{(4)} = \alpha_0 p(t)\varphi(y)$ with rapidly varying nonlinearity in the case of $\lambda_0 = 1$.

1. Introduction.
In this paper we study the asymptotic behaviour of solutions of a fourth order differential equation of the form $y^{(4)} = \alpha_0 p(t)\varphi(y)$ (1). The purpose of this paper is to obtain the asymptotics $P_\omega(Y_0, \lambda_0)$ solutions of the differential equation (1) for the special case when $\lambda_0 = 1$.
2. Object of research.
Consider a differential equation of the form (1) where $\alpha_0 \in \{-1, 1\}$, $p : [a, \omega[\longrightarrow]0, +\infty[$ -is a continuous function, $-\infty < a < \omega \leq +\infty$, $\varphi : \Delta_{Y_0} \longrightarrow]0, +\infty[$ – a twice continuously differentiable function such that $\varphi'(y) \neq 0$ where $y \in \Delta_{Y_0}$, $\lim\limits_{\substack{y\to Y_0\\ y\in\Delta_{Y_0}}} \varphi(y) = \begin{cases} \text{or} & 0, \\ \text{or} & +\infty, \end{cases}$ $\lim\limits_{\substack{y\to Y_0\\ y\in\Delta_{Y_0}}} \frac{\varphi(y)\varphi''(y)}{\varphi'^2(y)} = 1$,
Y_0 is equal to either 0, or $\pm\infty$, Δ_{Y_0} -is a one-sided neighbourhood of Y_0.
3. Basic definitions and notations.
The solution y of the differential equation (1) is called $P_\omega(Y_0, \lambda_0)$-solution, where $-\infty \leq \lambda_0 \leq +\infty$, if it is defined on the segment $[t_0, \omega[\subset [a, \omega[$ and satisfies the following conditions $y(t) \in \Delta_{Y_0}$ at $t \in [t_0, \omega[$, $\lim\limits_{t\uparrow\omega} y(t) = Y_0$,
$\lim\limits_{t\uparrow\omega} y^{(k)}(t) = \left[\begin{array}{ll} \text{or} & 0, \\ \text{or} & \pm\infty, \end{array}\right.$ $(k = 1, 2, 3)$, $\lim\limits_{t\uparrow\omega} \frac{[y^{(3)}(t)]^2}{y^{(2)}(t)y^{(4)}(t)} = \lambda_0$.
Let us introduce additional auxiliary notations
$J_0(t) = \int\limits_{A_0}^{t} p_0^{\frac{1}{4}}(\tau)$, $q(t) = \frac{(\Phi^{-1}(\alpha_0 J_0(t)))'}{\alpha_0 J_3(t)}$, $H(t) = \frac{\Phi^{-1}(\alpha_0 J_0(t))\varphi'(\Phi^{-1}(\alpha_0 J_0(t)))}{\varphi(\Phi^{-1}(\alpha_0 J_0(t)))}$,
$J_1(t) = \int\limits_{A_1}^{t} p_0(\tau)\varphi(\Phi^{-1}(\alpha_0 J_0(\tau)))\, d\tau$, $J_2(t) = \int\limits_{A_2}^{t} J_1(\tau)\, d\tau$, $J_3(t) = \int\limits_{A_3}^{t} J_2(\tau)\, d\tau$, where the integration boundary A_i is either ω or constant and is defined so that the integral tends either to 0 or to $\pm\infty$.
4. Main results.
The following two theorems are valid for equation (1).
Theorem 1. For the existence $P\omega(Y_0, 1)$-solutions of differential equation (1) that the inequalities $\alpha_0\nu_2 > 0$, $\alpha_0\mu_0 J_0(t) < 0$, at, $t \in]a, \omega[$, $\alpha_0\nu_0 < 0$, or, $Y_0 = 0$, $\alpha_0\nu_0 > 0$, or, $Y_0 = \pm\infty$ (2),
and conditions $\frac{\alpha_0 J_3(t)}{\Phi^{-1}(\alpha_0 J_0(t))} \sim \frac{J_1'(t)}{J_1(t)} \sim \frac{J_2'(t)}{J_2(t)} \sim \frac{J_3'(t)}{J_3(t)} \sim \frac{(\Phi^{-1}(\alpha_0 J_0(t)))'}{\Phi^{-1}(\alpha_0 J_0(t))}$ at $t \uparrow \omega$, $\alpha_0 \lim_{t\uparrow\omega} J_0(t) = Z_0$,
$\lim_{t\uparrow\omega} \frac{\pi_\omega(t)(\Phi^{-1}(\alpha_0 J_0(t)))'}{\Phi^{-1}(\alpha_0 J_0(t)))} = \pm\infty$, $\lim_{t\uparrow\omega} \frac{\pi_\omega(t) J_0'(t)}{J_0(t)} = \pm\infty$ (3). Moreover, for each such solution, the asymptotic representations at $y(t) = \Phi^{-1}(\alpha_0 J_0(t)) \left[1 + \frac{o(1)}{H(t)}\right]$, $y'(t) = \alpha_0 J_3(t)[1 + o(1)]$,
$y''(t) = \alpha_0 J_2(t)[1 + o(1)]$, $y'''(t) = \alpha_0 J_1(t)[1 + o(1)]$ (4).
Theorem 2. Let $p_0 : [a, \omega[\to]0, +\infty[$ - a continuously differentiable function and along with the (2) - (3) conditions $\lim_{t\uparrow\omega} \frac{q'(t)J_2(t)|H(t)|^{\frac{1}{4}}}{J_2'(t)} = 0$, $\lim\limits_{\substack{y\to Y_0\\ y\in\Delta_{Y_0}}} \frac{\left(\frac{\varphi'(y)}{\varphi(y)}\right)'}{\left(\frac{\varphi'(y)}{\varphi(y)}\right)^2} \left|\frac{y\varphi'(y)}{\varphi(y)}\right|^{\frac{3}{4}} = 0$ then the differential equation (1) contains at $\alpha_0\mu_0 = -1$ a two-parameter family of $P_\omega(Y_0, 1)$-solutions which admit at $t \uparrow \omega$ asymptotic representations (4) and furthermore such first, second and third order derivatives of which satisfy at $t \uparrow \omega$ the asymptotic relations $y'(t) = \alpha_0 J_3(t)\left[1 + \frac{o(1)}{|H(t)|^{\frac{3}{4}}}\right]$, $y''(t) = \alpha_0 J_2(t)\left[1 + \frac{o(1)}{|H(t)|^{\frac{1}{2}}}\right]$, $y'''(t) = \alpha_0 J_1(t)\left[1 + \frac{o(1)}{|H(t)|^{\frac{1}{4}}}\right]$.The question of whether the differential equation (1) has $P_\omega(Y_0, \lambda_0)$- solutions admitting at $t \uparrow \omega$ asymptotic representations (4) in the case when $\alpha_0\mu_0 = 1$ is still open.
Bibliography: 1. V.M. Evtukhov, A.M. Samoilenko. \ Conditions of existence of vanishing solutions of real nonautonomous systems of quasilinear differential equations at a special point// Ukr. Mat. Mag. - 2010. - 62, № 1. - p. 52-80. 2. Maric V. Regular variation and differential equations. Lecture Notes in Math.,1726(2000).-127p. 3. A. G. Chernikova, Asymptotic images of solutions of differential equations with rapidly changing nonlinearities, PhD thesis, Odesa (2019)

Primary author: ГОЛУБЄВ, Сергій (Одеський національний університет імені І. І. Мечникова)

Co-author: GOLUBEV, Sergiy

Presenter: ГОЛУБЄВ, Сергій (Одеський національний університет імені І. І. Мечникова)

Session Classification: MATHEMATICS

Track Classification: MATHEMATICS