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Asymptotic behaviour of solutions of the differential equation of the form $y^{(4)} = \alpha_0 p(t) \varphi(y)$ with rapidly varying nonlinearity in the case of $\lambda_0 = 1$.

1. Introduction.

In this paper we study the asymptotic behaviour of solutions of a fourth order differential equation of the form $y^{(4)} = \alpha_0 p(t) \varphi(y)$ (1). The purpose of this paper is to obtain the asymptotics $P_\omega(Y_0, \lambda_0)$ solutions of the differential equation (1) for the special case when $\lambda_0 = 1$.

2. Object of research.

Consider a differential equation of the form (1) where $\alpha_0 \in \{-1, 1\}, p : [a, \omega[\longrightarrow]0, +\infty[$ -is a continuous function, $-\infty < a < \omega \leq +\infty, \varphi : \Delta_{Y_0} \longrightarrow]0, +\infty[$ – a twice continuously differentiable function such that $\varphi'(y) \neq 0$ where $y \in \Delta_{Y_0}$, $\lim_{\substack{y \to Y_0 \ y \in \Delta_{Y_0}}} \varphi(y) = \begin{cases} \text{or } 0, \\ \text{or } +\infty, \end{cases}$, $\lim_{\substack{y \to Y_0 \ y \in \Delta_{Y_0}}} \frac{\varphi(y)\varphi''(y)}{\varphi'^2(y)} = 1,$ $Y_0 \text{ is equal to either 0, or <math>\pm\infty$, Δ_{Y_0} -is a one-sided neighbourhood of Y_0 .

3. Basic definitions and notations.

The solution y of the differential equation (1) is called $P_{\omega}(Y_0, \lambda_0)$ -solution, where $-\infty \leq \lambda_0 \leq +\infty$, if it is defined on the segment $[t_0,\omega[\subset [a,\omega[$ and satisfies the following conditions $y(t)\in \Delta_{Y_0}$ at $t\in$ $[t_0, \omega[, \quad \lim_{t\uparrow\omega}y(t)=Y_0,$

 $\lim_{t \uparrow \omega} y^{(k)}(t) = \begin{bmatrix} \text{or } 0, \\ \text{or } \pm \infty, \end{bmatrix} (k = 1, 2, 3), \quad \lim_{t \uparrow \omega} \frac{[y^{(3)}(t)]^2}{y^{(2)}(t)y^{(4)}(t)} = \lambda_0.$ Let us introduce additional auxiliary notation

$$J_{0}(t) = \int_{A_{0}}^{t} p_{0}^{\frac{1}{4}}(\tau), \ q(t) = \frac{(\Phi^{-1}(\alpha_{0}J_{0}(t)))'}{\alpha_{0}J_{3}(t)}, \ H(t) = \frac{\Phi^{-1}(\alpha_{0}J_{0}(t))\varphi'(\Phi^{-1}(\alpha_{0}J_{0}(t)))}{\varphi(\Phi^{-1}(\alpha_{0}J_{0}(t)))},$$
$$J_{1}(t) = \int_{A_{1}}^{t} p_{0}(\tau)\varphi(\Phi^{-1}(\alpha_{0}J_{0}(\tau))) \ d\tau, \ J_{2}(t) = \int_{A_{2}}^{t} J_{1}(\tau) \ d\tau, \ J_{3}(t) = \int_{A_{2}}^{t} J_{2}(\tau) \ d\tau, \ \text{where the integration}$$

boundary A_i is either ω or constant and is defined so that the integral tends either to 0 or to $\pm\infty$.

4. Main results.

The following two theorems are valid for equation (1).

Theorem 1. For the existence $P\omega(Y_0, 1)$ -solutions of differential equation (1) that the inequalities $\alpha_0\nu_2 > 1$ $0, \ \alpha_{0}\mu_{0}J_{0}(t) < 0, \text{at, } t \in]a, \ \omega[, \alpha_{0}\nu_{0} < 0, \text{or, } Y_{0} = 0, \ \alpha_{0}\nu_{0} > 0, \text{or, } Y_{0} = \pm\infty (2),$ and conditions $\frac{\alpha_{0}J_{3}(t)}{\Phi^{-1}(\alpha_{0}J_{0}(t))} \sim \frac{J_{1}'(t)}{J_{1}(t)} \sim \frac{J_{2}'(t)}{J_{2}(t)} \sim \frac{J_{3}'(t)}{J_{3}(t)} \sim \frac{(\Phi^{-1}(\alpha_{0}J_{0}(t)))'}{\Phi^{-1}(\alpha_{0}J_{0}(t))} \text{ at } t \uparrow \omega, \alpha_{0} \lim_{t \uparrow \omega} J_{0}(t) = Z_{0},$ $\lim_{t \uparrow \omega} \frac{\pi_{\omega}(t)(\Phi^{-1}(\alpha_{0}J_{0}(t)))'}{\Phi^{-1}(\alpha_{0}J_{0}(t)))} = \pm\infty, \quad \lim_{t \uparrow \omega} \frac{\pi_{\omega}(t)J_{0}'(t)}{J_{0}(t)} = \pm\infty (3).$ Moreover, for each such solution, the

asymptotic representations at $y(t) = \Phi^{-1}(\alpha_0 J_0(t)) \left[1 + \frac{o(1)}{H(t)}\right], y'(t) = \alpha_0 J_3(t) [1 + o(1)],$

$$y''(t) = \alpha_0 J_2(t) [1 + o(1)], \ y'''(t) = \alpha_0 J_1(t) [1 + o(1)] (4).$$

Theorem 2. Let $p_0: [a, \omega[\rightarrow]0, +\infty[$ - a continuously differentiable function and along with the

(2) - (3) conditions
$$\lim_{t\uparrow\omega} \frac{q'(t)J_2(t)|H(t)|^{\frac{1}{4}}}{J'_2(t)} = 0$$
, $\lim_{\substack{y\to Y_0\\y\in\Delta_{Y_0}}} \frac{\left(\frac{\varphi'(y)}{\varphi(y)}\right)}{\left(\frac{\varphi'(y)}{\varphi(y)}\right)^2} \left|\frac{y\varphi'(y)}{\varphi(y)}\right|^{\frac{1}{4}} = 0$ then the differential

equation (1) contains at $\alpha_0 \mu_0 = -1$ a two-parameter family of $P_\omega(Y_0, 1)$ -solutions which admit at $t \uparrow \omega$ asymptotic representations (4) and furthermore such first, second and third order derivatives of which satisfy

at
$$t \uparrow \omega$$
 the asymptotic relations $y'(t) = \alpha_0 J_3(t) \left[1 + \frac{o(1)}{|H(t)|^{\frac{3}{4}}} \right], \ y''(t) = \alpha_0 J_2(t) \left[1 + \frac{o(1)}{|H(t)|^{\frac{1}{2}}} \right], \ y'''(t) = \alpha_0 J_2(t) \left[1 + \frac{o(1)}{|H(t)|^{\frac{1}{2}}} \right]$

 $\alpha_0 J_1(t) \left[1 + \frac{o(1)}{|H(t)|^{\frac{1}{4}}} \right].$ The question of whether the differential equation (1) has $P_{\omega}(Y_0, \lambda_0)$ - solutions admitting of the solution of the sol ting at $t \uparrow \omega$ asymptotic representations (4) in the case when $\alpha_0 \mu_0 = 1$ is still open.

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Primary author:ГОЛУБЄВ, Сергій (Одеський національний університет імені І. І. Мечникова)Co-author:GOLUBEV, SergiyPresenter:ГОЛУБЄВ, Сергій (Одеський національний університет імені І. І. Мечникова)Session Classification:MATHEMATICS

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