

Extended symmetry analysis of (1+2)-dimensional fine Kolmogorov backward equation

Following [1], we discuss advances in the classical group analysis of the Kolmogorov backward equation with quadratic diffusivity

$$(1) \quad u_t + xu_y = x^2u_{xx}.$$

This equation belongs to the class of (1+2)-dimensional ultraparabolic linear equations denoted by $\bar{\mathcal{F}}$ in [1] and is distinguished within this class $\bar{\mathcal{F}}$ by its excellent symmetry properties. More specifically, modulo the point equivalence, it is a unique equation in the class $\bar{\mathcal{F}}$ whose essential Lie invariance algebra is five-dimensional and nonsolvable.

We compute the point symmetry pseudogroup of the equation (1) using the advanced version of the direct method and analyze its structure. In particular, we single out the essential subgroup of this pseudogroup and identify its independent discrete elements, which are two involutions alternating the signs of the space and dependent variables, respectively. We exhaustively classify all subalgebras of the corresponding essential Lie invariance algebra up to inner automorphisms and up to the action of the essential point-symmetry group. This allowed us to classify Lie reductions and Lie invariant solutions of the equation (1). We also discuss the generation of its solutions using point and linear generalized symmetries and carry out its peculiar generalized reductions. Consequently, we construct wide families of its solutions parameterized by an arbitrary finite number of arbitrary solutions of the (1+1)-dimensional linear heat equation or one or two arbitrary solutions of (1+1)-dimensional linear heat equations with inverse square potentials.

[1] Koval S.D. and Popovych R.O., Extended symmetry analysis of (1+2)-dimensional fine Kolmogorov backward equation, *Stud. Appl. Math.* **153** (2024), e12695, arXiv:2402.08822.

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