

On controllability problems for the heat equation controlled by the Dirichlet boundary condition

Consider the following control system:

$$w_t = w_{xx}, x \in (0, +\infty), t \in (0, T),$$

$$w(0, \cdot) = u, t \in (0, T),$$

$$w(\cdot, 0) = w^0, x \in (0, +\infty),$$

where $T > 0$ is a constant, w^0 is a given function, $u \in L^\infty(0, T)$ is a control. The control system is considered in Sobolev spaces.

An initial state w^0 of control system (1)-(3) is said to be null-controllable in a given time $T > 0$ if we can find a control $u \in L^\infty(0, T)$ such that the state of the solution to the control system at $t = T$ satisfies the condition $w(\cdot, T) = 0$. An initial state w^0 of control system (1)-(3) is said to be approximately controllable to a target state w^T in a given time $T > 0$ if for each neighbourhood of a target state w^T there exists a control $u \in L^\infty(0, T)$ such that the end state of the solution to the control system (at $t = T$) belongs to this neighbourhood of w^T .

We prove that any initial state of the control system (except the zero one) is not null-controllable in a given time $T > 0$.

We also prove that each initial state of the control system is approximately controllable to any target state in a given time $T > 0$.

The results are illustrated by examples.

The results on controllability of the heat equation controlled by the Dirichlet boundary condition was published in [1-3].

References

- [1] L. Fardigola and K. Khalina, Reachability and controllability problems for the heat equation on a half-axis, J. Math. Phys. Anal. Geom. 15 (2019), 57-78.
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