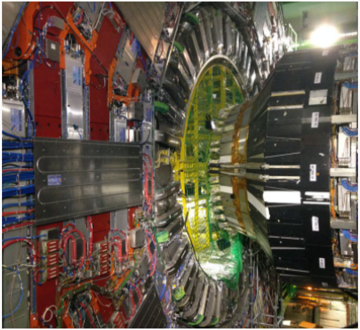
A question concerning phase transitions in a pion system of particles and antiparticles

Vladyslav Karpenko
D. Anchishkin, V. Gnatovskyy, D. Zhuravel

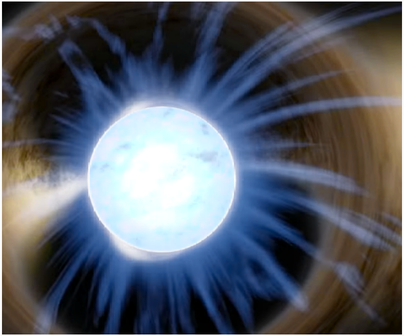
Bogolyubov Institute for Theoretical Physics
of the National Academy of Sciences of Ukraine

2025

Intro



Hadron collider



Neutron star

Helmholtz free energy density:

$$\phi(n, T) = \frac{F(N, V, T)}{V}, \qquad \mu = \frac{\partial \phi}{\partial n}, \qquad s = -\frac{\partial \phi}{\partial T}$$

Decomposition of the free energy:

$$\phi(n, T) = \phi_0(n, T) + \phi_{\mathrm{int}}(n, T)$$

Interaction and excess pressure

Interaction:

$$U(n, T) = \left[\frac{\partial \phi_{\rm int}(n, T)}{\partial n} \right]_T \, , \tag{1}$$

Excess pressure:

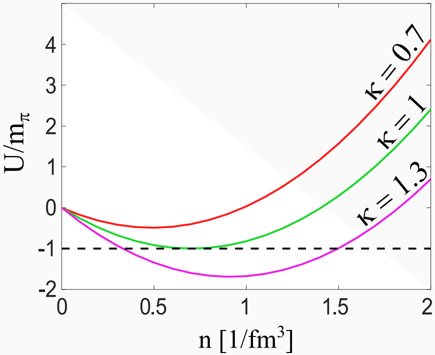
$$P_{\rm ex}(n, T) = n \, \left[\frac{\partial \phi_{\rm int}(n, T)}{\partial n} \right]_T - \, \phi_{\rm int}(n, T) \; . \tag{2}$$

Differential relation:

$$n \, \frac{\partial U(n, T)}{\partial n} \; = \; \frac{\partial P_{\rm ex}(n, T)}{\partial n} \, . \tag{3}$$

Parametrisation

$$U(n) = -\kappa\, A_c\, n + B\, n^2 \tag{4}$$



Basics

Bose particle distribution:

$$f\left(E,\mu\right) = \left[\exp\left(\frac{E-\mu}{T}\right) - 1\right]^{-1} \tag{5}$$

$$E(k,n) = \omega_k + U(n) \tag{6}$$

$$\omega_k = \sqrt{m^2 + k^2} \tag{7}$$

Chemical potential:

$$\mu^{(+)} = -\mu^{(-)} = \mu \tag{8}$$

EQ's (thermal)

In the mean-field approximation, the particle-antiparticle system can be described by the following equations:

$$n = \int \frac{d^3k}{(2\pi)^3} \left[f\big(E(k,n),\mu\big) + f\big(E(k,n),-\mu\big) \right] , \tag{9}$$

$$n_I = \int \frac{d^3k}{(2\pi)^3} \left[f\big(E(k,n),\mu\big) - f\big(E(k,n),-\mu\big) \right] , \tag{10}$$

Particle density

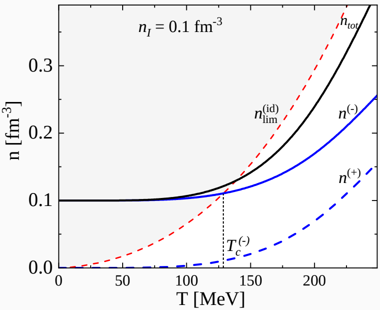


Рис. 1: $\kappa = 0.5$

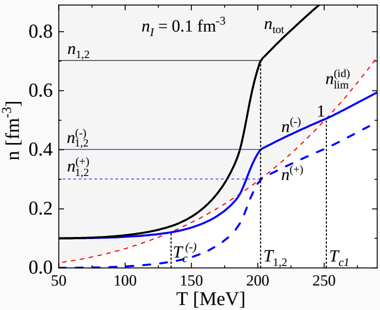


Рис. 2: $\kappa = 1$

- The blue solid line represents particles, the dashed - antiparticles
- The black solid line represents the total density
- The red dashed line represents the critical curve

Closer look

We have problems here:

$$f = \left[\exp\left(\frac{\sqrt{m^2 + k^2} + U(n) - \mu}{T}\right) - 1\right]^{-1} \tag{11}$$

Critical curve

We assume that particles may be in a condensate, considering $U(n) - \mu = -m$ and define the critical curve as:

$$n_{\rm lim}^{\rm (id)}(T) = \int \frac{d^3k}{(2\pi)^3} f\left(\omega_k, \mu\right)\Big|_{\mu=m}, \qquad (12)$$

or (almost the same):

$$n_{\rm lim}^{\rm (id)}(T) = \int \frac{d^3k}{(2\pi)^3} f\left(E(k,n), \mu\right)\Big|_{\mu=m+U(n)}. \qquad (13)$$

EQ's (condensed)

Generalized equations:

$$n = n_{\rm cond}^{(-)}(T) + n_{\rm lim}^{(\rm id)}(T) + \int \frac{d^3k}{(2\pi)^3}\, f\big(E(k,n), -\mu\big)\,, \quad (14)$$

$$n_I = n_{\rm cond}^{(-)}(T) + n_{\rm lim}^{(\rm id)}(T) - \int \frac{d^3k}{(2\pi)^3}\, f\big(E(k,n), -\mu\big)\,. \quad (15)$$

The system can be reduced to a single equation for $n^{(+)}$:

$$n^{(+)} = \int \frac{d^3k}{(2\pi)^3}\, f\big(E(k,n), -\mu\big)\Big|_{\mu=U(n)+m} \quad (16)$$

where $E(k,n) = \omega_k + U\big(2n^{(+)} + n_I\big)$.

Particle density

Particle density

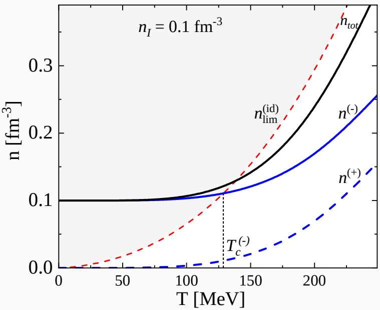


Рис. 3: $\kappa = 0.5$

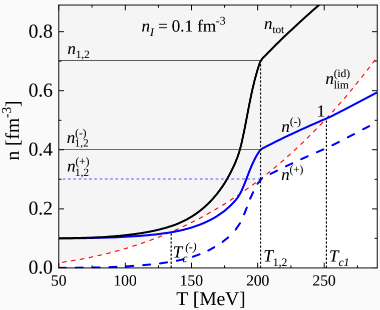


Рис. 4: $\kappa = 1$

- The blue solid line represents particles, the dashed - antiparticles
- The black solid line represents the total density
- The red dashed line represents the critical curve

Condensate and chemical potential

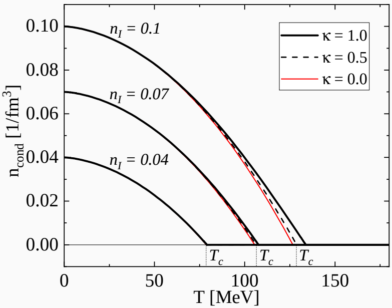


Рис. 5: Condenced phase density

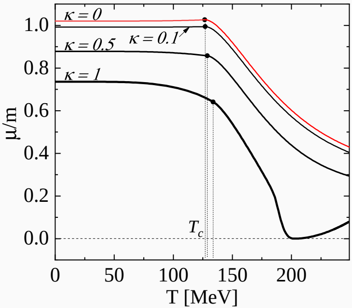


Рис. 6: Chemical potential

The marked points correspond to the critical temperature.

Energy

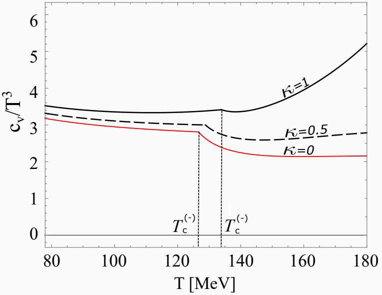


Рис. 7: Heat capacity

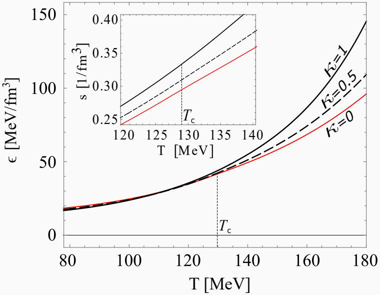


Рис. 8: Energy density

The obtained results are consistent with the fact that the Bose–Einstein condensation of an ideal gas is a third-order phase transition.

Phase diagrams with no attraction, $\kappa = 0$

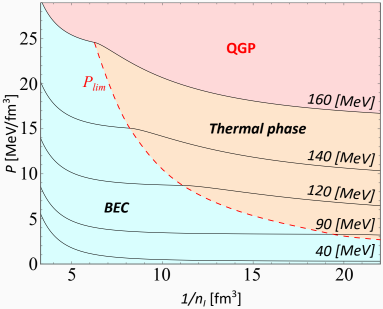


Рис. 9: Inverse density

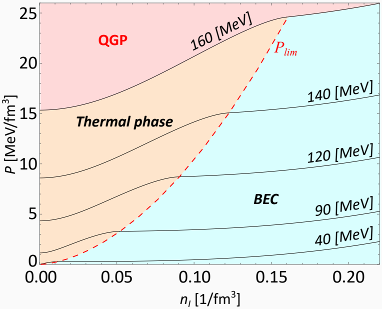


Рис. 10: Direct density

Phase diagrams with an attraction, $\kappa = 0.2$

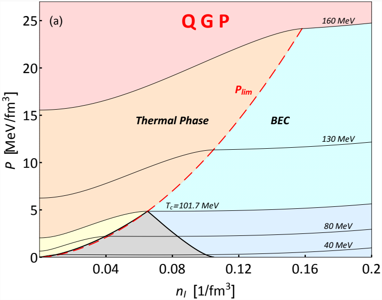


Рис. 11: The overall picture

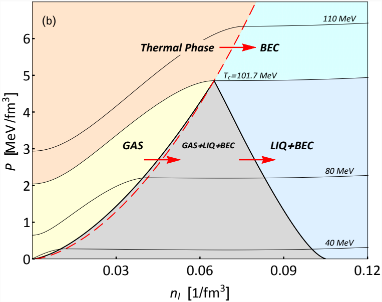


Рис. 12: In detail

- The possibility of Bose–Einstein condensation in a particle–antiparticle system with fixed isospin charge density is demonstrated
- Phase diagrams are constructed
- A generalization of Maxwell's construction rules for phase transitions is formulated in terms of charge density dependencies

Publications

1. *“Relativistic Selfinteracting Particle-Antiparticle System of Bosons”* Anchishkin, V. Gnatovskyy, D. Zhuravel, **V. Karpenko.** J. of Phys. and Electr. 28, No. 2, p.3-18 (2020) [DOI: https://doi.org/10.15421/332016]

2. *“Phase Diagram of the Selfinteracting Particle-Antiparticle Boson System”* D. Anchishkin, V. Gnatovskyy, D. Zhuravel, **V. Karpenko** J. of Phys. and Electr., 29, No. 1, p.5-14 (2021) [DOI: https://doi.org/10.15421/332101]

3. *“Self-interacting particle-antiparticle system of bosons”* D. Anchishkin, V. Gnatovskyy, D. Zhuravel, **V. Karpenko** Phys. Rev. C 105, 045205 (2022) [DOI: https://doi.org/10.1103/PhysRevC.105.045205]

4. *“Phase Transitions in the Interacting Relativistic Boson Systems”* D. Anchishkin, V. Gnatovskyy, D. Zhuravel, **V. Karpenko**, I. Mishustin and H. Stoecker. Universe 9, 411 (2023) [DOI: https://doi.org/10.3390/universe9090411]

5. *“Canonical Ensemble vs. Grand Canonical Ensemble in the Description of Multicomponent Bosonic Systems.”* D. Anchishkin, V. Gnatovskyy, D. Zhuravel, **V. Karpenko**, I. Mishustin and H. Stoecker Ukr. J. Phys 69, No. 1 (2024) [DOI: https://doi.org/10.15407/ujpe69.1.3]

gratitude

 Thank you for your attention

appendix (pressure)

The pressure:

$$p = \frac{1}{3}\int \frac{d^3k}{(2\pi)^3}\,\frac{\mathbf{k}^2}{\omega_k}\big[f\big(E(k,n),\mu\big) \,+\, f\big(E(k,n),-\mu\big)\big] \,+\, P(n)\,, \tag{17}$$

Where $P(n)$ is an excess pressure from the Skyrme parameterization:

$$P(n) \,=\, -\frac{A}{2}\,n^2 \,+\, \frac{2B}{3}\,n^3\,, \tag{18}$$