In this talk, I will discuss defect conformal field theory (DCFT), with a focus on results from [1]. DCFT provides a powerful framework for studying quantum and statistical systems in the presence of localized objects, such as impurities, boundaries, or defects. These structures naturally arise across diverse areas of physics – from condensed matter to string theory – and can dramatically alter the behavior of the bulk system.

To illustrate this point, I will briefly review the Kondo effect, where a single magnetic impurity in a sea of conduction electrons leads to a nontrivial infrared (IR) fixed point and modifies the resistivity at low temperatures. This classic problem can be recast in terms of conformal boundary conditions, highlighting how impurities can drive strongly coupled physics even in otherwise free theories.

I will then only discuss one result from [1], concerning the study of localized magnetic field lines in the critical $O(N)$ model. Working in coordinates $x = (y, z, \mathbf{x})$ the defects are extended along the y-direction and are separated along the transverse z-direction by a distance r:

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi^I)^2 + \frac{\lambda_b \Lambda_T^{4-d}}{4} (\phi^I \phi^I)^2 \right)$$
$$+ h_{1,b} a_D^{\frac{d-4}{2}} \int dy \; \hat{n} \cdot \hat{\phi}(y, z = 0, \mathbf{x} = 0) + h_{2,b} a_D^{\frac{d-4}{2}} \int dy \; \hat{m} \cdot \hat{\phi}(y, z = r, \mathbf{x} = 0) \,. \tag{0.1}$$

We denote these defects as $\mathcal{D}(\hat{n})$ where the unit vector $\hat{n} \in S^{N-1}$ specifies the orientation of the defect in the $O(N)$ directions. I will show that the fusion product among the defect lines labeled by $\hat{n}, \hat{m} \in S^{N-1}$ takes the following intuitive form

$$\mathcal{D}(\hat{n}) \circ \mathcal{D}(\hat{m}) = \mathcal{D} \left(\frac{\hat{n} + \hat{m}}{\sqrt{2(1 + \hat{n} \cdot \hat{m})}} \right) . \tag{0.2}$$

I will also present the Casimir energy using the ϵ expansion:

$$\mathcal{E}(\hat{n}, \hat{m}) = -(\hat{n} \cdot \hat{m}) \frac{N+8}{4\pi} - \frac{\epsilon}{4\pi} \left((\hat{n} \cdot \hat{m}) \frac{N^2 - 3N - 22}{2(N+8)} - \frac{(1 + 2(\hat{n} \cdot \hat{m})^2)\, \pi^2 (N+8)}{16} \right) . \tag{0.3}$$

These results provide an example of non-topological line defect fusion in an interacting CFT, and may have further implications for understanding impurity-driven phase transitions in critical systems.

References

[1] O. Diatlyk, H. Khanchandani, F. K. Popov and Y. Wang, JHEP **09**, 006 (2024) doi:10.1007/JHEP09(2024)006 [arXiv:2404.05815 [hep-th]].