

Ising-Markov chains equivalence as an inverse problem of statistical physics

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An important problem in the statistical description of big data random sequences is identifying macroscopic characteristics that summarize the essential properties of the system while ignoring microscopic details. These characteristics are analogous to thermodynamic variables that describe the macroscopic state of a physical system composed of many interacting degrees of freedom. Markov chains provide a natural starting point for such an analysis. Classical Nth-order Markov chains allow one to describe correlations extending over long histories but suffer from an exponential growth of parameters with increasing memory depth. Additive Nth-order Markov chains offer a principled reduction of this complexity by decomposing the conditional probability of the next symbol into a superposition of contributions from different delay positions.

In Ref. [1], we established an exact correspondence of one- and two-step-wise MC with one-dimensional Ising models with finite-range interactions, thereby introducing the macroscopic temperature to MC. We demonstrated that information temperature could be defined through two independent approaches: (i) matching conditional probabilities, and (ii) solving an entropy-based inverse problem of statistical physics. In particular, see Ref. [2], it was understood that a complex system can be characterized by temperature, or in other words, temperature is a fairly informative parameter characterizing the complexity of a system.

The main result of the work [3] is the establishment of a correspondence between an additive multistep chain and a chain with a stepwise memory function. This correspondence allows the introduction of the concept of information temperature not only for stepwise but also for general additive Nth-order Markov chains.

The present work substantially extends that framework in several directions. First, in addition to the two previous methods for introducing the information temperature, in this work, we present a method based on joint probability. Second, the equivalence between Markov chains and Gibbs-Ising representations is formulated within a constructive inverse statistical mechanics framework, thereby making explicit the assumptions of positivity, compatibility, stationarity, and finite memory. Third, three complementary constructions of the information temperature are systematically compared and shown to be mutually consistent for the unbiased one-parameter step-wise Markov chain. Fourth, the entropy-based definition is reformulated as an inverse problem with an auxiliary energy scale, clarifying its relation to the true Ising energy. Finally, the analysis of the two-parameter inverse problem establishes the limits of this equivalence, demonstrating that the coincidence of the three temperature constructions is a special property of the one-parameter model rather than a generic feature of finite-memory symbolic processes.

The primary objective for the future is to understand the importance of introducing a macroscopic temperature parameter for characterizing Large Linguistic Models.

[1] O. V. Usatenko, S. S. Melnyk, G. M. Pritula, and V. A. Yampol'skii, Phys. Rev. E 106, 034127 (2022).

[2] O. V. Usatenko and G. M. Pritula, Chaos, Solitons and Fractals 202 (2026) 117452.

[3] O. V. Usatenko, G. M. Pritula, and S. S. Melnyk, Phys. Rev. E 114, 014116 (2026).

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