BARYON FLUCTUATIONS SIGNATURES of the ONSET of DECONFINEMENT

MARK GORENSTEIN

Bogoluyubov Institute for Theoretical Physics, Kyiv

M.Gazdzicki, M.Gorenstein, and A. Rustamov, arXiv:2603.15819[nucl-th]

I acknowledge support from the National Research Foundation of Ukraine under grant No. 2025.07/0050.



Quarks

Gell-Man and Zweig (1963)

	u	d	s
e	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
b	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

1). $p(uud)$, $n(udd)$, $\Lambda(uds), \ldots$ $\pi^+(u\bar{d})$, $K^0(d\bar{s}), \ldots$

2). Boris Struminsky 1965: Color $\Delta(sss)$

3). SU(3)-color: 3 colors of quarks and 8 colors of gluons

4). 1974: $J/\psi(c\bar{c})$, 1977: $Y(b\bar{b})$, t

5). 1970-75 Gross, Politzher, Wilczek: asymptotic freedom (NP 2004)

$m_u = 2.3\ \mathrm{MeV}, \quad m_d = 4.8\ \mathrm{MeV}, \quad m_s = 95\ \mathrm{MeV}$

QGP Discovery, CERN 2000, Special Seminar

Title: **A New State of Matter: Results from the CERN Lead-Beam Programme at $\sqrt{s_{NN}} = 17.3$ GeV**
Time: Thursday 10 February at 09.30 hrs
Place: Council Chamber, bldg 503

1. **Ulrich Heinz** (CERN) Making Quark-Gluon Matter in Relativistic Nuclear Collisions
2. **Louis Kluberg (IN2 P3**) The J/ψ suppression pattern observed in Pb-Pb collisions ions: a signature for the production of a new state of matter.
3. **Johanna Stachel** (University of Heidelberg) Virtual and real photons radiated by the cooling and hadronizing fireball.
4. **Reinhard Stock** (University of Frankfurt) Hadron Signals of the Little Bang.
5. **Emanuele Quercigh** (CERN) Strange signals of a new state of matter from nuclear collisions at SPS.
6. **Luciano Maiani** (Director General, CERN) Summary

QGP after 26 years at 2026

1. No Nobel Prize for the QGP discovery.
2. Numerous attempts to explain "QGP-signatures" without QGP.
3. New data from BNL Brookhaven and LHC CERN.
4. No new confirmation of QGP existence?!
5. No doubt in QGP existence?!

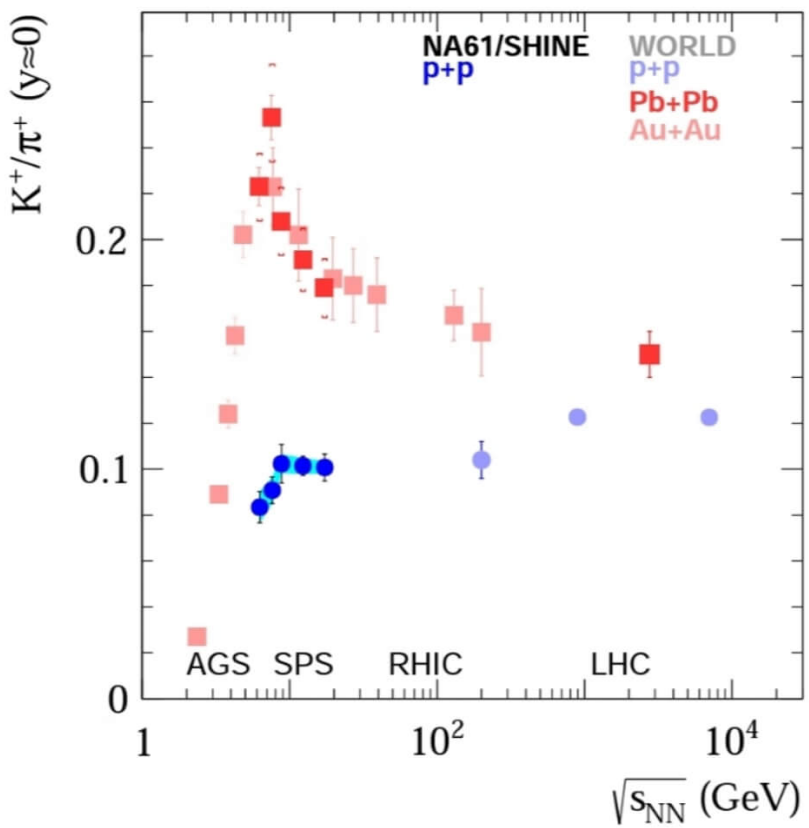
The situation with QGP today is similar to the situation with **atoms at the end of the 19th century.**

Onset of Deconfinement:

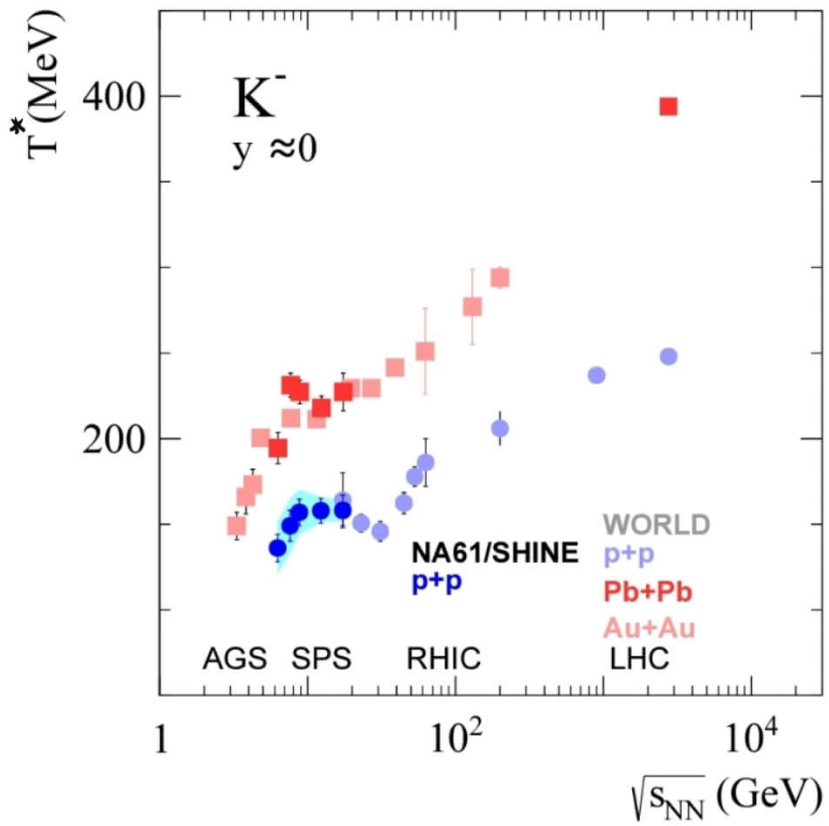
PREDICTIONS: Gazdzicki and Gorenstein (1998), CONFIRMATION: NA49 Collaboration

$$8 \text{ GeV} \leq \sqrt{s_{NN}} \leq 12 \text{ GeV}$$

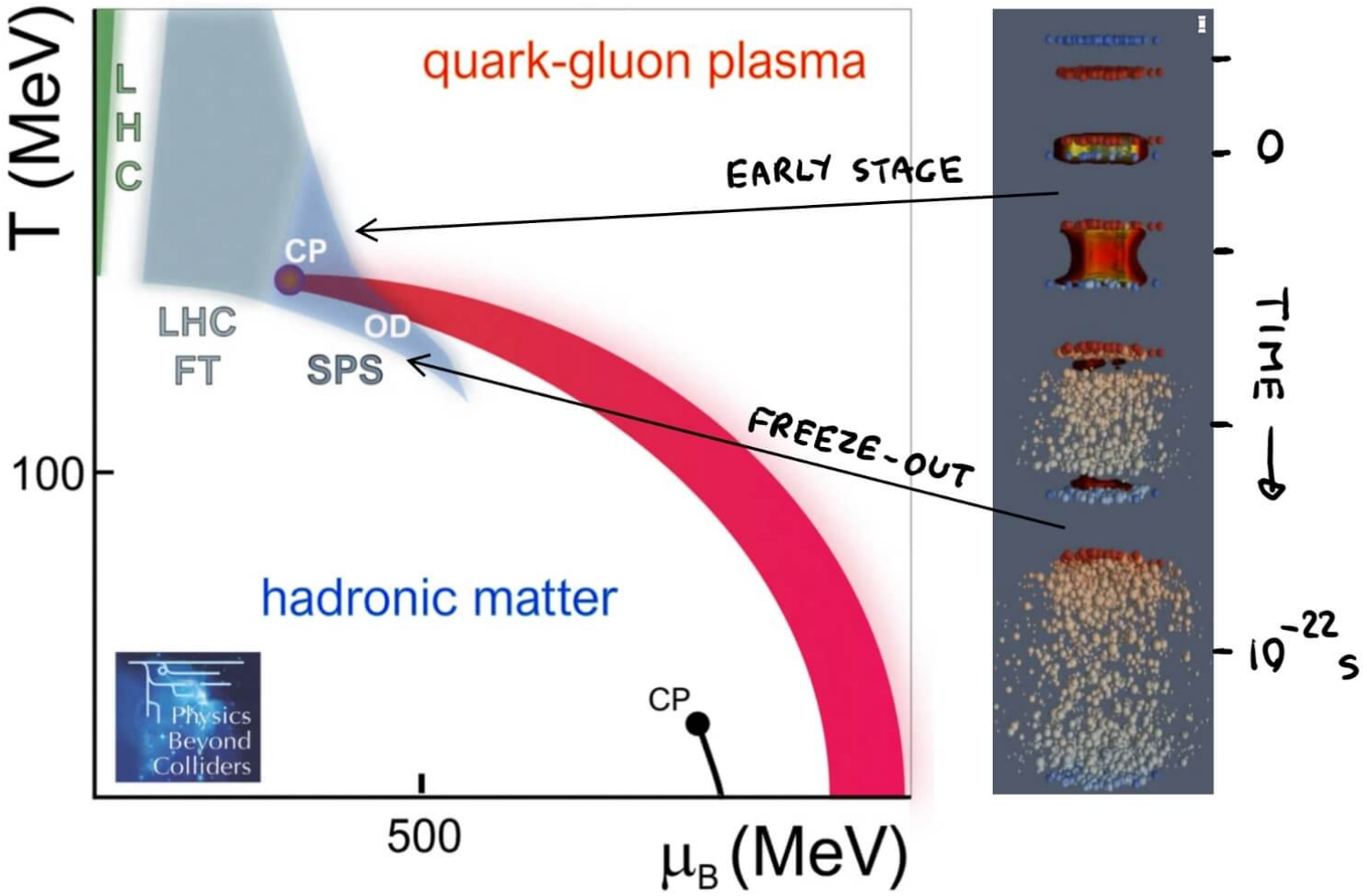
Horn



Step



Phase Diagram and QCD Critical Point



Electric Charge and Baryonic Number Fluctuations

HRG: $b_i = \pm 1, \quad q_i = \pm 1, \ \pm 2$
QGP: $b_i = \pm 1/3, \quad q_i = \pm 1/3, \ \pm 2/3$

M. Asakawa, U. W. Heinz, B. Muller, Fluctuation probes of quark deconfinement, Phys. Rev. Lett. 85 (2000) 2072–2075.
S. Jeon, V. Koch, Charged particle ratio fluctuation as a signal for QGP,450 Phys. Rev. Lett. 85 (2000) 2076–2079.

J. Parra, R. Poberezhniuk, V. Koch, C. Ratti, V. Vovchenko, Indications for Freeze-Out of Charge Fluctuations in the Quark-Gluon Plasma at the LHC, Phys. Rev. Lett. 135 (24) (2025) 242302.

Cumulants

The probability distribution $P(N)$.
The average value $\sum NP(N) \equiv E[N]$.
The raw moments $r_n[N] = E[N^n]$, and central moments μ_n are
$r_n \equiv E[N^n], \quad \mu_n[N] = E[(N - E[N])^n]$.
Characteristic function

$$\chi_N(q) = \sum_N \exp(iqN)p(N) \ , \tag{1}$$

$$\ln(\chi_N(q)) = \sum_{k\geq 1} \kappa_n \frac{(iq)^k}{k!} \tag{2}$$

The first four cumulants are:
$\kappa_1[N] = E[N]$ (mean),
$\kappa_2[N] = E[(N - E[N])^2]$ (variance),
$\kappa_3[N] = E[(N - E[N])^3]$
$\kappa_4[N] = E[(N - E[N])^4] - 3E[(N - E[N])^2]^2$
Frequently used:
scaled variance $\omega = \kappa_2[N]/\kappa_1[N]$, **skewness** $S\sigma \equiv \kappa_3[N]/\kappa_2[N]$
and **kurtosis** $\kappa\sigma^2 \equiv \kappa_4[N]/\kappa_2[N]$.

Poisson distribution for Baryons and Quarks

$$P(N) = \frac{\langle N \rangle^N \exp(-\langle N \rangle)}{N!} \tag{3}$$

For baryons $N_b = N$ and

$$\frac{\kappa_2[N_b]}{\kappa_1[N_b]} = 1 \,, \quad \frac{\kappa_3[N_b]}{\kappa_2[N_b]} = 1 \,, \quad \frac{\kappa_4[N_b]}{\kappa_2[N_b]} = 1 \,. \tag{4}$$

For quarks $N_b \approx N/3$ and

$$\frac{\kappa_2[N_b]}{\kappa_1[N_b]} \approx \frac{1}{3} \,, \quad \frac{\kappa_3[N_b]}{\kappa_2[N_b]} \approx \frac{1}{3} \,, \quad \frac{\kappa_4[N_b]}{\kappa_2[N_b]} \approx \frac{1}{9} \,. \tag{5}$$

QGP Equation of State

$$\frac{P}{T^4} = \sum_{f=u,d,s} \left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4 \right] , \qquad (6)$$

where the chemical potentials of u, d, and s quarks are related to baryon and strange chemical potentials μ_B and μ_S as

$$\mu_u = \frac{1}{3}\mu_B , \quad \mu_d = \frac{1}{3}\mu_B , \quad \mu_s = \frac{1}{3}\mu_B - \mu_S = 0 . \qquad (7)$$

$\mu_S = \mu_B/3$ to grant zero net-strangeness and $\mu_Q \approx 0$.

The cumulants in the QGP

The cumulants $\kappa_n[B]$ of the net-baryon number distribution are obtained from derivatives of the pressure,

$$\kappa_n[B]_Q = VT^3 \left[\frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}\right]_{\mu_S,T}, \qquad n = 1,2,3,4,.. \tag{8}$$

This gives

$$\kappa_1[B]_Q = VT^3 \left[\frac{2}{9}\frac{\mu_B}{T} + \frac{2}{81\pi^2}\left(\frac{\mu_B}{T}\right)^3\right], \tag{9}$$

$$\kappa_2[B]_Q = VT^3 \left[\frac{2}{9} + \frac{2}{27}\left(\frac{\mu_B}{\pi T}\right)^2 + \frac{1}{9}\right], \tag{10}$$

$$\kappa_3[B]_Q = VT^3 \left(\frac{4}{27\pi^2}\frac{\mu_B}{T}\right), \tag{11}$$

$$\kappa_4[B]_Q = VT^3 \left(\frac{4}{27\pi^2} + \frac{2}{27\pi^2}\right). \tag{12}$$

The cumulants' ratios

The relevant cumulants' ratios are:

$$\frac{\kappa_2[B]_Q}{\kappa_1[B]_Q} = \frac{1}{\pi z}\,\frac{3/2 + z^2/3}{1 + z^2/9}\ , \tag{13}$$

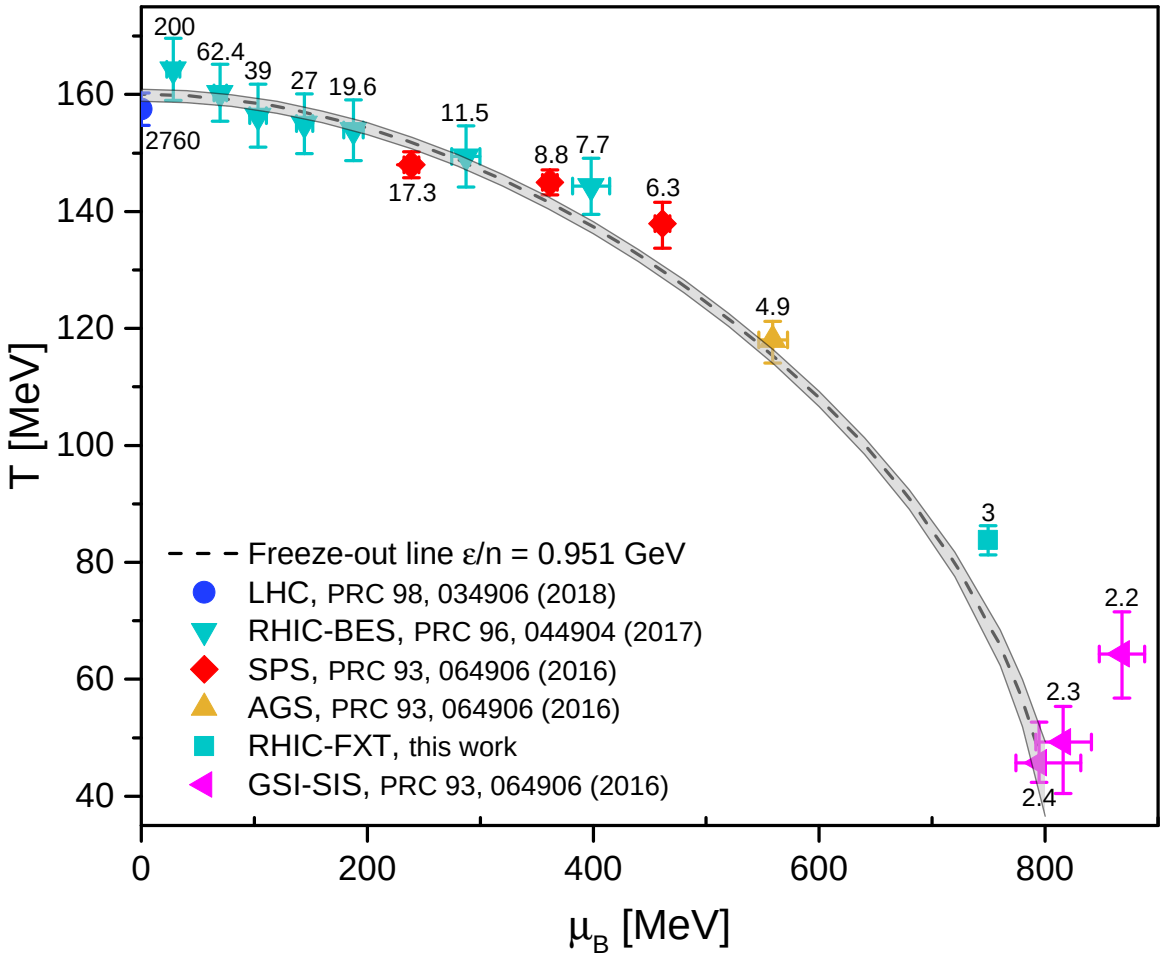
$$\frac{\kappa_3[B]_Q}{\kappa_2[B]_Q} = \frac{4z}{\pi(9 + 2z^2)}\ , \tag{14}$$

$$\frac{\kappa_4[B]_Q}{\kappa_2[B]_Q} = \frac{2}{\pi^2(9 + 2z^2)}\ , \tag{15}$$

where $z \equiv \mu_B/(\pi T)$.

$T = T(\sqrt{s_{NN}}), \quad \mu_B = \mu_B(\sqrt{s_{NN}})$

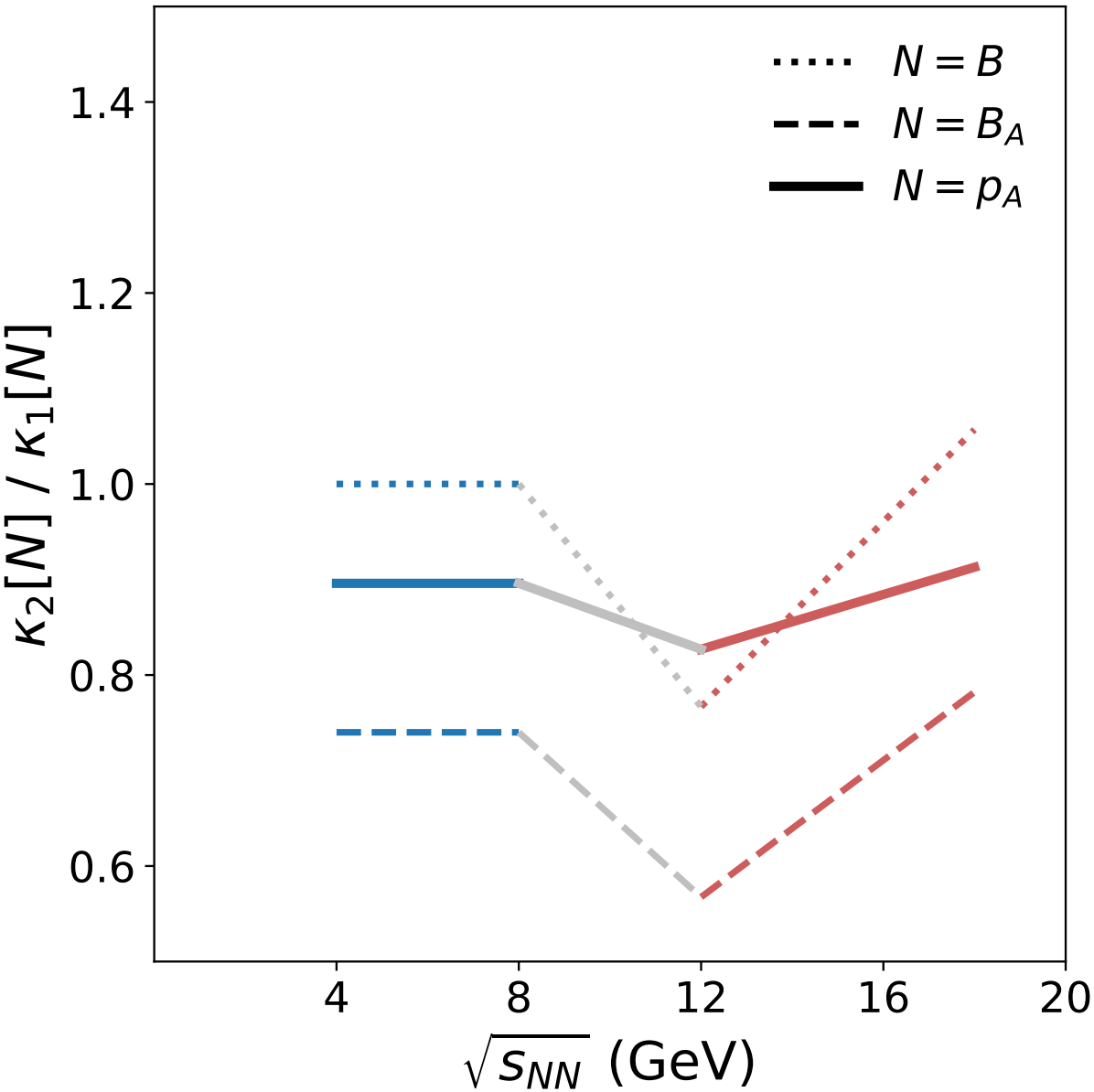
Chemical Freeze-out Line



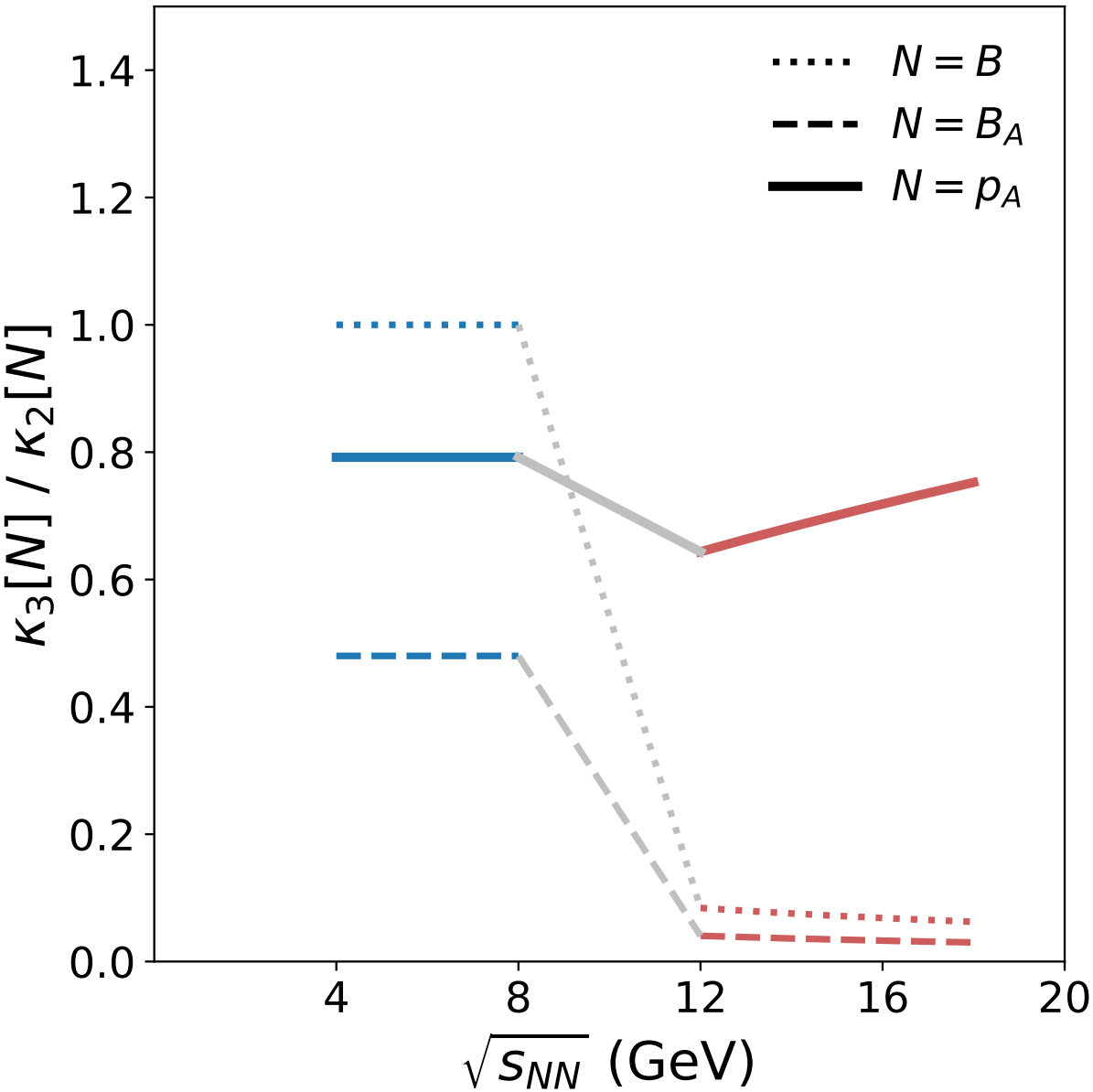
J. Cleymans and K. Redlich, Phys. Rev. Lett. (1998): $E/N = 1$ GeV

A. Lysenko et al., Phys. Rev C (2025): $E/N \cong 0.95$ GeV

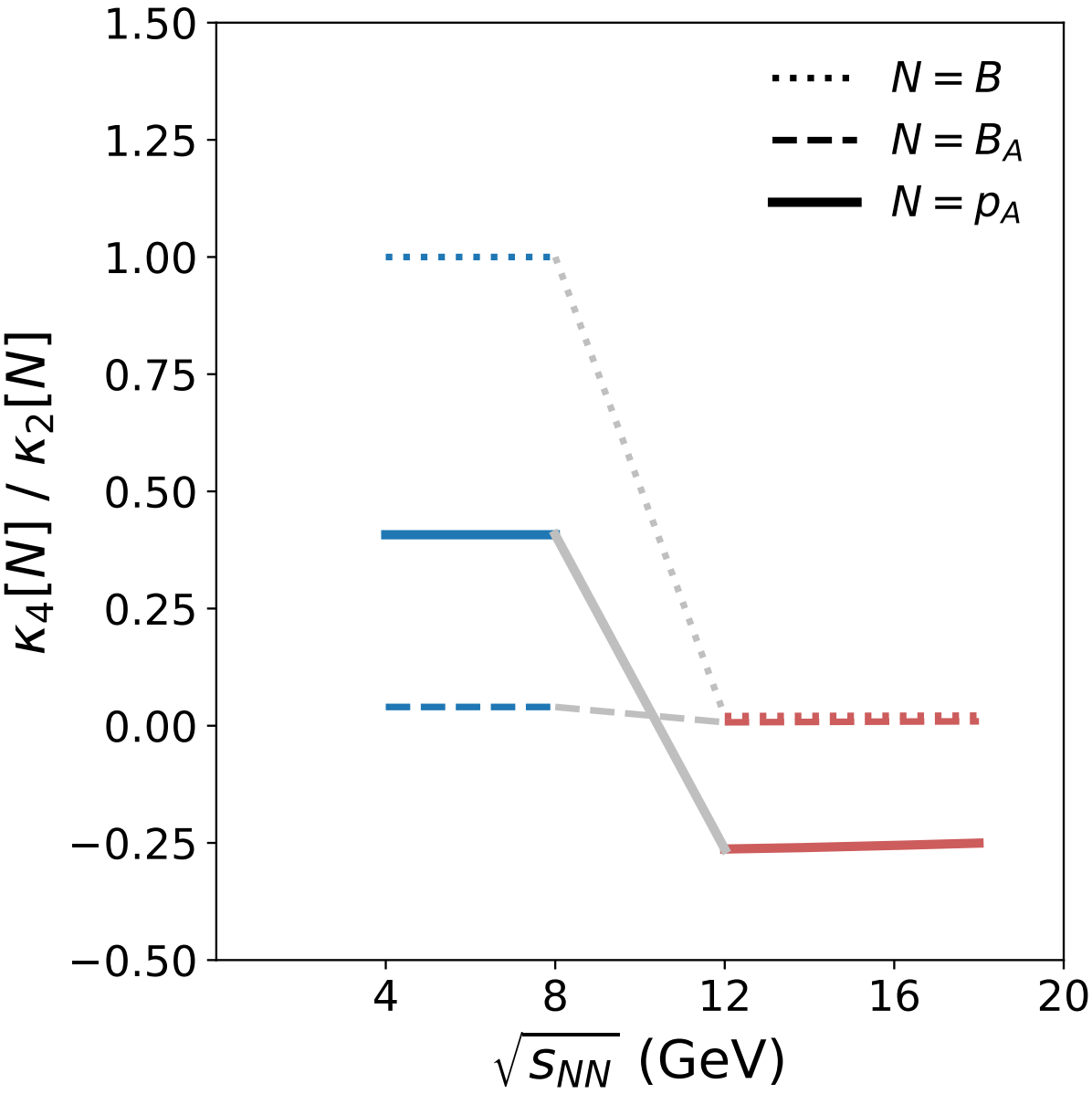
Scaled Variance κ_2/κ_1



Skewness κ_3/κ_2



Kurtosis κ_4/κ_2



Sub-Ensemble Acceptance Method, $\alpha = 0.26$

V. Vovchenko, O. Savchuk, R. V. Poberezhnyuk, M. I. Gorenstein, V. Koch, Connecting fluctuation measurements in heavy-ion collisions with the grand-canonical susceptibilities, Phys. Lett. B **811** (2020) 135868.

For the considered cumulants' ratios, the relations read:

$$\frac{\kappa_2[B_A]}{\kappa_1[B_A]} = (1-\alpha)\,\frac{\kappa_2[B]}{\kappa_1[B]}\,, \tag{16}$$

$$\frac{\kappa_3[B_A]}{\kappa_2[B_A]} = (1-2\alpha)\,\frac{\kappa_3[B]}{\kappa_2[B]}\,, \tag{17}$$

$$\frac{\kappa_4[B_A]}{\kappa_2[B_A]} = [1-3\alpha(1-\alpha)]\,\frac{\kappa_4[B]}{\kappa_2[B]} - 3\alpha(1-\alpha)\left(\frac{\kappa_3[B]}{\kappa_2[B]}\right)^2\,, \tag{18}$$

where B_A is the net-baryon number in the acceptance and α is a fraction of net-baryons within the momentum acceptance.

$$\omega[B_A] = (1-\alpha)\,\omega[B]\,, \qquad \alpha = 0.26$$

For the HRG: $\omega[B] = 1 \quad \rightarrow \quad \omega[B_A] = 0.74$

For the QGP: $\omega[B] = 0.8 \quad \rightarrow \quad \omega[B_A] = 0.6$

Binomial distribution, $\beta = 0.4$

$$p(p_A, \beta) \;=\; \sum_{B_A=p_A}^{\infty} \frac{B_A!}{p_A!(B_A-p_A)!}\beta^{p_A}(1-\beta)^{N_A-p_A}\; P(B_A) \qquad (19)$$

$$\omega_\beta[p_A] \;=\; 1 \;-\; \beta \;+\; \beta\,\omega[B_A] \qquad (20)$$

1). $1 < \omega_\beta[p_A] < \omega[B_A]$, 2). $\omega[B_A] < \omega_\beta[p_A] < 1$

$$\mathrm{HRG}: \quad \omega[B_A] = 0.75 \quad \rightarrow \quad \omega_\beta[p_A] = 0.9\ , \qquad (21)$$

$$\mathrm{QGP}: \quad \omega[B_A] = 0.6 \quad \rightarrow \quad \omega_\beta[p_A] = 1 \;-\; 0.84`\beta \qquad (22)$$

The binomial thinning, $\beta = 0.4$

See, e.g., O. Savchuk, R. Poberezhnyuk, V. Vovchenko, M. Gorenstein, Binomial acceptance corrections for particle number distributions in high-energy reactions, Phys. Rev. C **101** (2020) 024917.

$$\frac{\kappa_2[p_A]}{\kappa_1[p_A]} = 1 - \beta + \beta \, \frac{\kappa_2[B_A]}{\kappa_1[B_A]} \, , \tag{23}$$

$$\frac{\kappa_3[p_A]}{\kappa_2[p_A]} = \frac{\kappa_2[B_A]/\kappa_1[B_A]}{\kappa_2[p_A]/\kappa_1[p_A]} \left[\beta^2 \frac{\kappa_3[B_A]}{\kappa_2[B_A]} + 3\beta(1-\beta)\right] + \frac{1-\beta}{\kappa_2[p_A]/\kappa_1[p_A]}(1-2\beta) \, , \tag{24}$$

$$\begin{aligned}\frac{\kappa_4[p_A]}{\kappa_2[p_A]} = \frac{\kappa_3[B_A]/\kappa_2[B_A]}{\kappa_3[p_A]/\kappa_2[p_A]} \Bigg[& \beta^3 \frac{\kappa_4[B_A]}{\kappa_2[B_A]} \\ & + 6\beta^2(1-\beta)\frac{\kappa_3[B_A]}{\kappa_2[B_A]} + \beta(1-\beta)(7-11\beta)\Bigg] \\ & + \frac{1-\beta}{\kappa_2[p_A]/\kappa_1[p_A]}\left[1 - 6\beta(1-\beta)\right] \, .\end{aligned} \tag{25}$$

Factorial Cumulants

The relation between factorial cumulants C_n and ordinary cumulants κ_n is mediated by the signed Stirling numbers of the first kind $s(n, j)$

$$C_n = \sum_{j=1}^{n} s(n, j)\, \kappa_j, \tag{26}$$

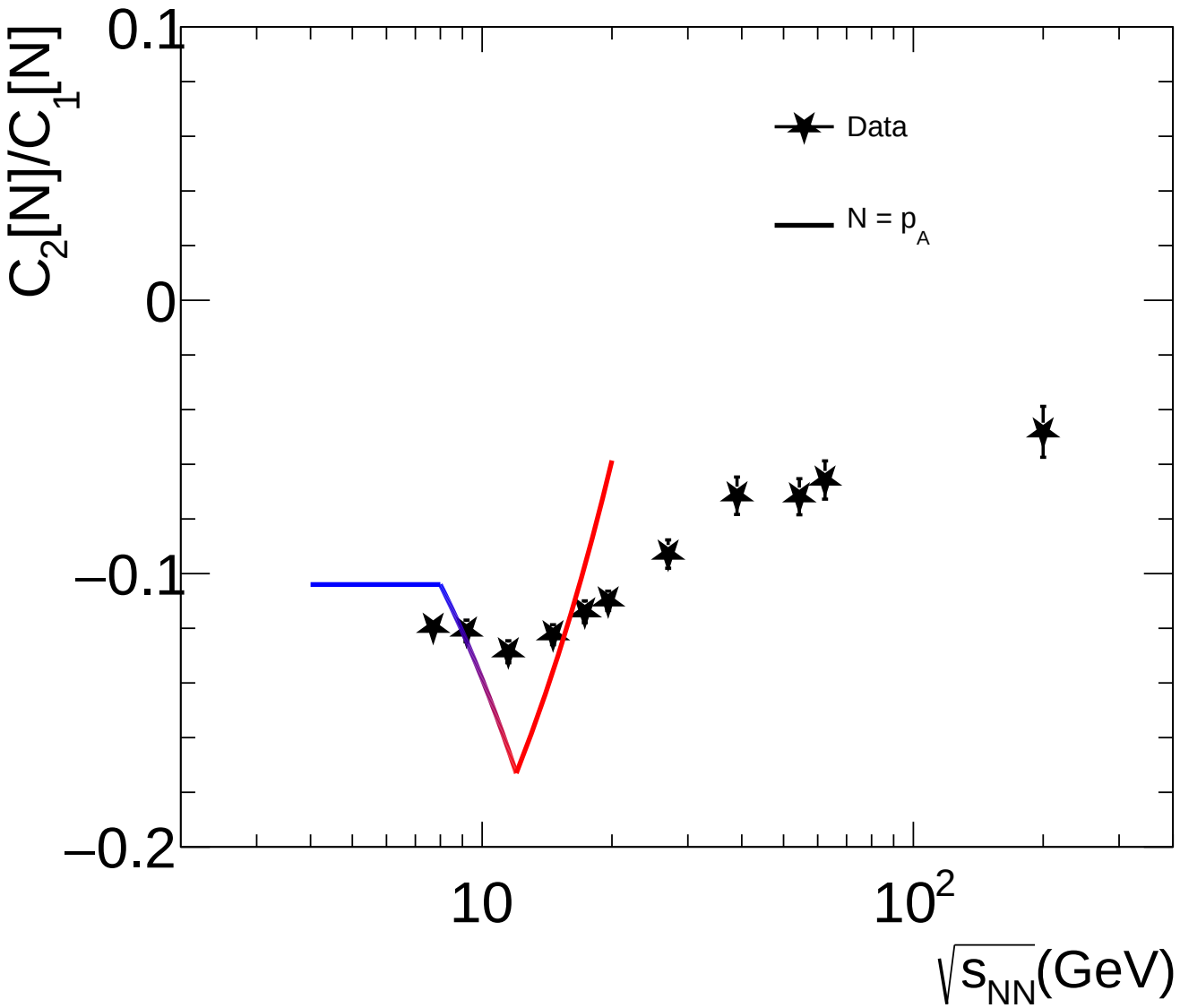
For the first four cumulants Eq. 26 yields

$$\begin{aligned} C_1 &= \kappa_1 && (27)\\ C_2 &= -\kappa_1 + \kappa_2, && (28)\\ C_3 &= 2\kappa_1 - 3\kappa_2 + \kappa_3, \\ C_4 &= -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4. \end{aligned}$$

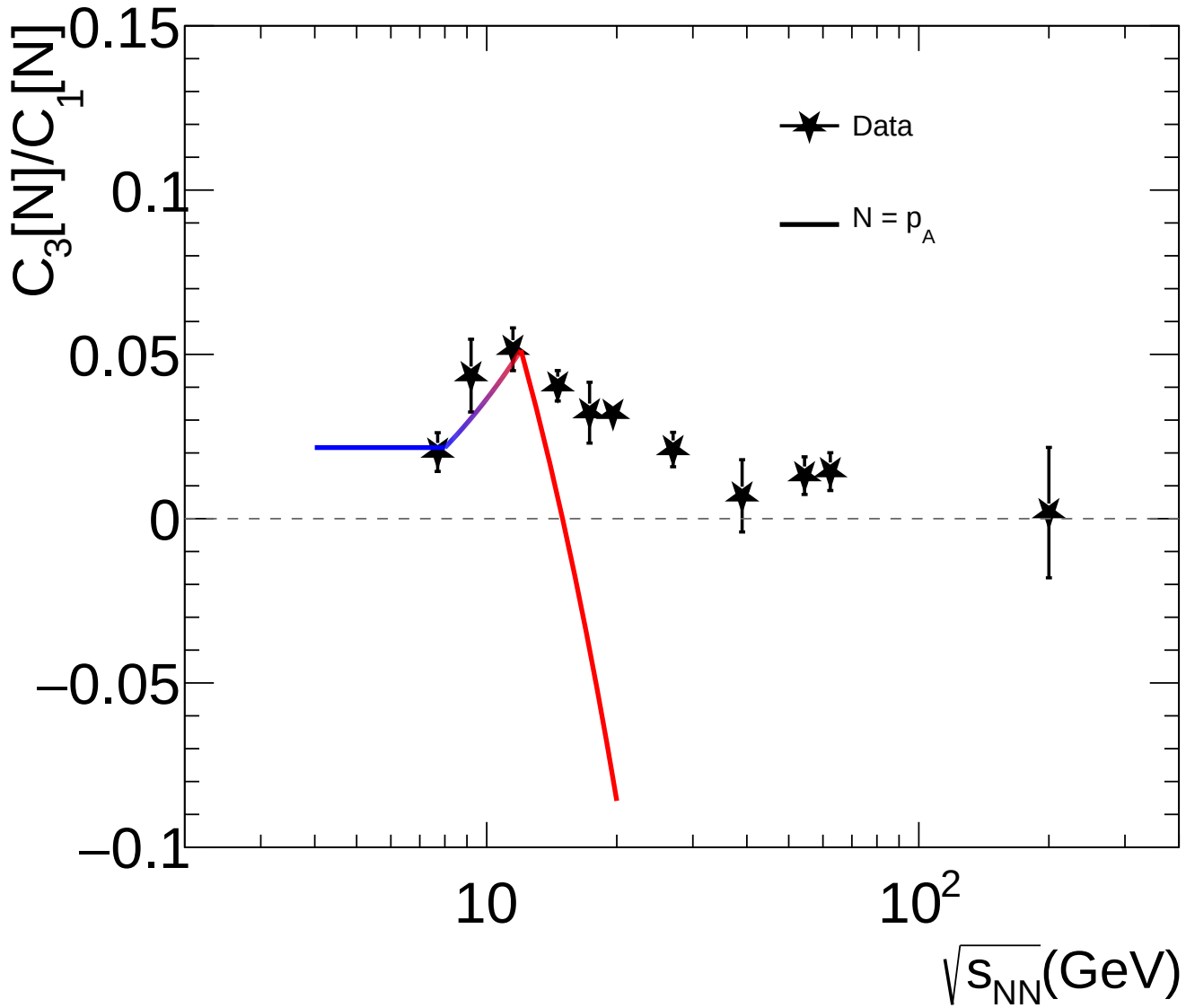
The considered cumulant ratios are

$$\begin{aligned} C_2/C_1 &= -1 + \kappa_2/\kappa_1, \\ C_3/C_1 &= 2 - 3\kappa_2/\kappa_1 + \kappa_3/\kappa_1, && (29)\\ C_4/C_1 &= -6 + 11\kappa_2/\kappa_1 - 6\kappa_3/\kappa_1 + \kappa_4/\kappa_1. \end{aligned}$$

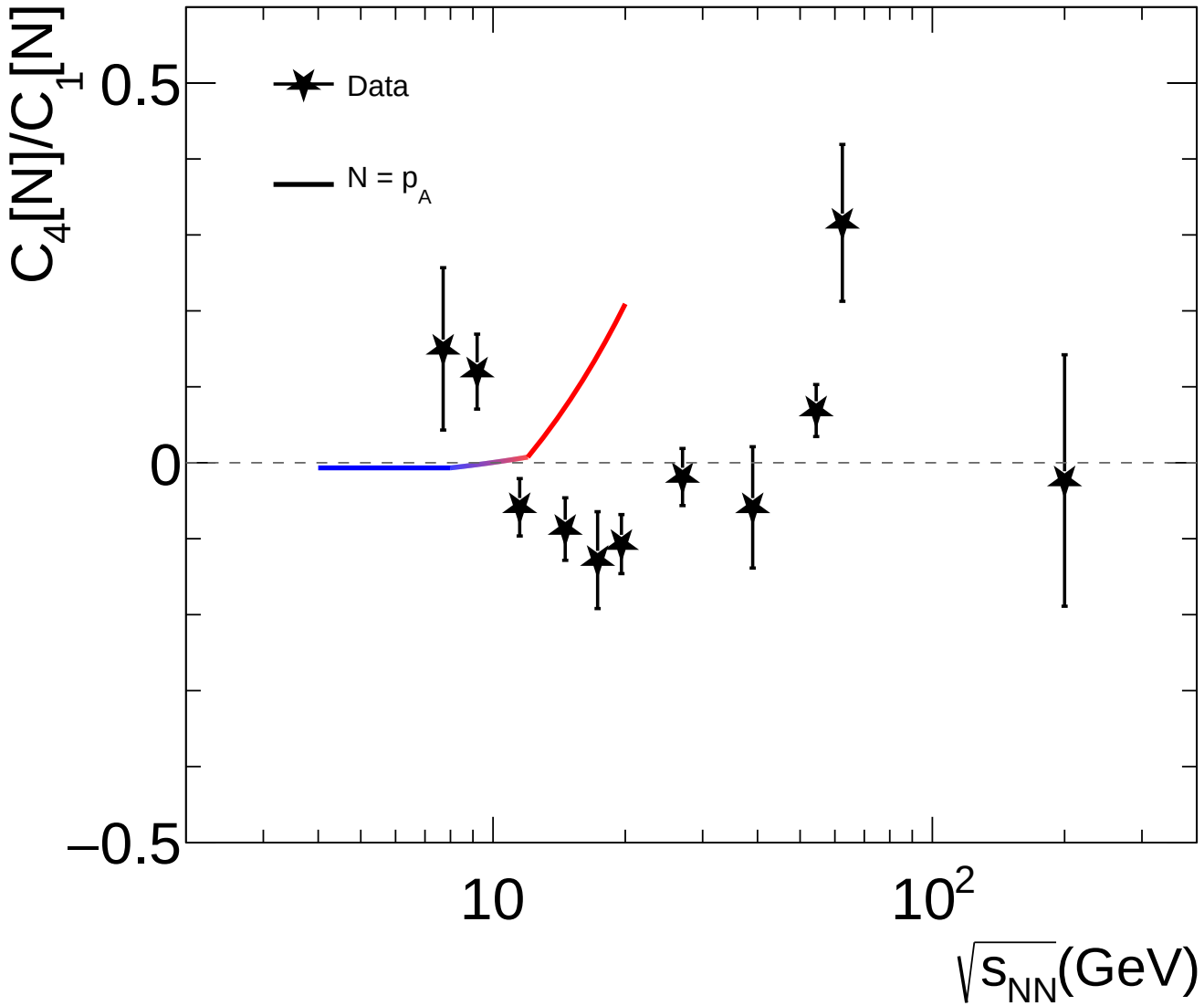
Comparison with the STAR Data, C_2/C_1:



C_3/C_1:



C_4/C_1



Summary

A quantitative comparison of the onset of deconfinement predictions with the experimental results requires, in particular,

(i) using experimentally estimated values of the parameter α, which significantly varies between data points,

(ii) using experimentally estimated values of the parameter β, which varies between data points,

(iii) improved modelling of fluctuations of the fraction of protons for a given number of accepted baryons,

(iv) taking into account the production of nuclear clusters,

(v) taking into account the nuclear critical point, which can significantly enhance the baryon number fluctuations at low collision energies,

(vi) studying the sensitivity of the predictions to the QGP equation of state,

(vii) investigating the sensitivity of the results to the assumption that net-baryon fluctuations freeze out in the vicinity of the chemical freeze-out line.