

The some solution of the Bryan-Pidduck equation

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The Boltzmann equation [1] that describes the evolution of rarefied gases is one of the main equations of the kinetic theory of gases. For a model of rough spheres, the equation has the form:

$$\begin{aligned} &\backslash\mathrm{begin}\{\mathrm{equation}\} \\ &D(f)=Q(f,f), \\ &\backslash\mathrm{end}\{\mathrm{equation}\} \end{aligned}$$

where the left-hand side of the equation is the differential operator:

$D(f) \equiv \frac{\partial f}{\partial t} + (V, \frac{\partial f}{\partial x})$, and the right-hand side of the Boltzmann equation is the collision integral, which for the hard spheres model

$$\frac{d^2}{2} \int_R dV_1 \int_R d\omega_1 \int_{\Sigma} d\alpha B(V - V_1, \alpha) \left[f(t, V_1^*, x, \omega_1^*) f(t, V^*, x, \omega^*) - f(t, V, x, \omega) f(t, V_1, x, \omega_1) \right],$$

where $f(t, V, x, \omega)$ is the distribution function of particles.

The problem of determination of the exact and approximate solutions of the Boltzmann equation in the explicit form is quite urgent. At present, the sole known exact solution of the Boltzmann equation is an expression usually called the Maxwell distribution or simply Maxwellian (after J. C. Maxwell, Scottish physicist). In the case of Maxwellians M , we get

$$D(f) = 0, \quad Q(f, f) = 0. \tag{1}$$

The solution to this equation will be look for in the next form: $f(t, x, V, \omega) = \sum_{i=1}^{\infty} \varphi_i(t, x) M_i(t, x, V, \omega)$.

As a measure of the deviation between the parts of the Boltzmann equation we will consider a uniform-integral error of the form: $\Delta = \sup_{(t,x) \in \mathbb{R}^4} \int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \left| D(f) - Q(f, f) \right|$.

In the paper[2], we were obtained sufficient conditions for the coefficient functions and hydrodynamic parameters appearing in the distribution, which enable one to make the analyzed error as small as desired.

REFERENCES

- [1] S. Chapman and T.G. Cowling. The Mathematical Theory of Non-Uniform Gases. *Cambridge Univ. Press*, Cambridge, 1952.
- [2] V. Gordevskyy and O. Hukalov; The Interaction of a Countable Number of Eddy Flows for the Bryan-Pidduck Model. *J. Math. Phys. Anal. Geom.* 2025, 21, 218-231.

Primary author: HUKALOV, Oleksii (B.Verkin Institute for Low Temperature Physics and Engineering of the NAS of Ukraine)

Presenter: HUKALOV, Oleksii (B.Verkin Institute for Low Temperature Physics and Engineering of the NAS of Ukraine)

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