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Representations of solutions for a first order quasilinear equation in a form of series in powers of the copolynomial δ-function

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By a *copolynomial* we mean here a linear functional on the space of polynomials $\mathbb{R}[x]$. The space of copolynomials is denoted by $\mathbb{R}[x]'$. If $T \in \mathbb{R}[x]'$ and $p \in \mathbb{R}[x]$, then the result of application of T to p is written as (T, p). The derivative T' of a copolynomial $T \in \mathbb{R}[x]'$ is defined in the same way as in the classical theory of generalized functions: $(T', p) = -(T, p')$, $p \in \mathbb{R}[x]$. An important example of a copolynomial is the δ-function, which is defined by $(\delta, p) = p(0)$, $p \in \mathbb{R}[x]$.

The *Cauchy-Stieltjes transform* of a copolynomial $T \in \mathbb{R}[x]'$ is defined as the following formal Laurent series in the ring $s^{-1}\mathbb{R}[[s^{-1}]]$: $C(T)(s) = \sum_{k=0}^{\infty}(T, x^k)/s^{k+1}$. The mapping $C : \mathbb{R}[x]' \to s^{-1}\mathbb{R}[[s^{-1}]]$ is an isomorphism of these vector spaces. The multiplication of copolynomials is defined through the multiplication of their Cauchy-Stieltjes transforms. The theory of linear and nonlinear PDEs in the ring $\mathbb{R}[x]'[[t]]$ was studied in [1,2], respectively.

Let $f(z) = \sum_{j=1}^{\ell} f_j z^j \in \mathbb{R}[z]$. We prove that the following Cauchy problem for the first order quasilinear equation

$ $\frac{\partial u}{\partial t} + f(u)\frac{\partial u}{\partial x} = 0, \qquad u(0, x) = \delta(x),$

in \mathbb{R}$[x]'[[t]]$ has a unique solution

u(t,x)=$\sum_{k=0}^{\infty}\sum_{|\alpha|=k} f_1^{\alpha_1}\ldots f_\ell^{\alpha_\ell}\frac{(\sum_{j=1}^{\ell} j\alpha_j+k)!}{\alpha_1!\ldots\alpha_\ell!(\sum_{j=1}^{\ell} j\alpha_j+1)!}\delta^{\sum_{j=1}^{\ell} j\alpha_j+k+1}(x)t^k$, where \alpha=(\alpha_1,\ldots,\alpha_{\ell})\in\ma index, |\alpha|=\sum\{j=1}^{\ell}\alpha_j$.

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[1] Gefter S. L., Piven' A. L.: Partial differential equations in module of copolynomials over a commutative ring, *J. Math. Phys. Anal. Geom.* **21** (2025), No. 1, 56–83.

[2] Gefter S. L., Piven' A. L.: Nonlinear Partial Differential Equations in Module of Copolynomials over a Commutative Ring. *J. Math. Phys. Anal. Geom.* **21** (2025), No. 3, 319–345.

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