Hadronic and partonic composition of QCD matter across the crossover

Artemiy Lysenko,[1,2] Mark I. Gorenstein,[2] Marek Gazdzicki,[3] Roman Poberezhniuk,[4,2] and Volodymyr Vovchenko[4]

[1] *Physics Department, Taras Shevchenko National University of Kyiv, 03022 Kyiv, Ukraine*
[2] *Bogolyubov Institute for Theoretical Physics, 03680 Kyiv, Ukraine*
[3] *Jan Kochanowski University, 25-369 Kielce, Poland*
[4] *Physics Department, University of Houston, Box 351550, Houston, Texas 77204, USA*

We construct a simple equation of state of strongly interacting matter at zero chemical potentials that provides a unified description of lattice QCD thermodynamics in terms of hadronic and partonic degrees of freedom. The hadronic phase is described by the quantum van der Waals hadron resonance gas, extended by excluded-volume repulsion between mesons, while the quark–gluon plasma is modeled as an ideal gas of quarks and gluons supplemented with a phenomenological interaction term proportional to T^3. The two regimes are connected by a smooth crossover switching function. The three model parameters—the meson hard-core radius, the strength of the partonic interaction term, and the switching temperature—are determined from a fit to lattice QCD results for the trace anomaly. The resulting equation of state reproduces the lattice data on the pressure, entropy density, energy density, and speed of sound in the temperature range $T = 100$–500 MeV. The fit yields a meson hard-core radius $r_M \simeq 0.2$ fm, a partonic interaction scale $A \simeq 600$ MeV, and a switching temperature $T_0 \simeq 216$ MeV, substantially exceeding both the pseudocritical temperature of the QCD chiral crossover and the chemical freeze-out temperature. This finding suggests that the transition from hadronic to partonic degrees of freedom is considerably more gradual than indicated by the chiral pseudocritical temperature alone, with hadronic states remaining an important component of strongly interacting matter up to temperatures of about 250 MeV, well above the QCD chiral crossover.

I. INTRODUCTION

Exploring the phase diagram of quantum chromodynamics (QCD), i.e., the structure of phases of strongly interacting matter (SIM) in the plane of temperature T and baryon chemical potential μ_B, is a central theoretical goal of high-energy nuclear physics. Effective models of SIM typically predict a first-order phase transition between hadronic matter and the quark–gluon plasma (QGP) at large baryon chemical potentials. The corresponding transition line in the μ_B–T plane ends at the QCD critical point [1, 2]. Available estimates of its location typically give $\mu_B \gtrsim 500$ MeV [3–5]. At small chemical potentials, lattice QCD simulations have established that the transition between hadronic matter and the QGP is a smooth crossover [6–8]. The crossover region was traversed by the early Universe during its QCD epoch [9]. The same region is probed experimentally in heavy-ion collisions at the highest available energies, $\sqrt{s_{\rm NN}} \gtrsim 200$ GeV, at RHIC and the LHC [10–12].

Lattice QCD provides first-principles results for the thermodynamic functions at zero chemical potentials [13, 14]. These data alone, however, do not reveal the underlying microscopic degrees of freedom. Thus, connecting the thermodynamic functions to hadronic and partonic constituents requires modeling. Several approaches have recently addressed this question from different perspectives. One class of approaches constructs phenomenological equations of state that interpolate between hadronic and partonic matter while remaining consistent with lattice QCD thermodynamics [2, 15].

Another line of research emphasizes that the microscopic composition above the pseudocritical temperature may differ substantially from that of a weakly interacting quark–gluon plasma. In particular, chiral spin symmetry has been interpreted as evidence for an intermediate strongly correlated "stringy fluid" regime in which confining chromoelectric interactions remain important [16–18]. Complementary studies of quarkyonic matter suggest that confinement may coexist with quark degrees of freedom over a broad range of thermodynamic conditions [19] and have recently explored possible connections of this picture to the finite-temperature QCD phase diagram and lattice simulations [20, 21]. The present work follows this line of research from a different perspective. Rather than proposing a new microscopic phase of strongly interacting matter, we construct a simple equation of state constrained by lattice QCD thermodynamics and use it to quantify the temperature dependence of the hadronic and partonic contributions across the QCD crossover.

At low temperatures, we describe SIM by the hadron resonance gas (HRG) model with the van der Waals interactions between baryons and between antibaryons [22], extended here by an excluded-volume repulsion between mesons with a hard-core radius r_M. At high temperatures, the QGP pressure is modeled by an ideal gas of quarks and gluons supplemented with a phenomenological term $-AT^3$, with $A = \text{const} > 0$. The two regimes are connected using the switching function method of Refs. [15, 23], which involves a temperature parameter T_0. The three model parameters, r_M, A, and T_0, are fixed by fitting the lattice data on the trace anomaly, $(\varepsilon - 3p)/T^4$. The resulting equation of state (EoS) is then confronted with the lattice data on the pressure, entropy density, energy density, and speed of sound at temperatures $T = 100$–500 MeV. The fit yields the switching temperature $T_0 \simeq 216$ MeV, well above the pseudocritical temperature of the chiral transition, $T_{\rm pc} \approx 155$ MeV [14, 24], and the chemical freeze-out temperature $T_{\rm ch} = 150$–160 MeV [10, 25]. This implies that hadronic degrees of freedom continue to make a sizable contribution to SIM thermodynamics up to $T \approx 250$ MeV.

The paper is organized as follows. The HRG model with the meson excluded volume is formulated in Sec. II. The QGP model is presented in Sec. III. The full EoS based on the switching function is constructed in Sec. IV, where the model parameters are fixed by fitting the lattice data. Section V summarizes the results and discusses their microscopic interpretation.

II. HADRON RESONANCE GAS

To describe the hadronic phase of SIM, we employ the quantum van der Waals hadron resonance gas (QvdW-HRG) model proposed in Ref. [22]. Unlike the ideal HRG, this model incorporates non-resonant attractive and repulsive interactions between baryons in a thermodynamically consistent manner while preserving quantum statistics. It reproduces the liquid–gas phase transition and ground-state properties of nuclear matter, providing a useful phenomenological description of hadronic matter.

Following Ref. [22], van der Waals interactions are included only among baryons and, separately, among antibaryons. The baryon–antibaryon, meson–meson, and meson–(anti)baryon van der Waals interactions are neglected in the original formulation. The pressure in the grand canonical ensemble reads

$$p_H(T,\mu) = P_M + P_B + P_{\bar{B}} \,, \tag{1}$$

where the meson and (anti)baryon partial pressures are

$$P_M = \sum_{i\in M} p_i^{\rm id}(T,\mu_i) \,, \qquad P_X = \sum_{i\in X} p_i^{\rm id}(T,\mu_i^{X*}) - a\, n_X^2 \,, \quad X \in \{B,\bar{B}\} \,. \tag{2}$$

Here M denotes the set of all mesons, while B and $\bar{B}$ denote the sets of baryons and antibaryons, respectively. The chemical potentials $\mu = (\mu_B, \mu_Q, \mu_S)$ are conjugate to the three conserved charges of strong interactions: baryon number, electric charge, and strangeness. The chemical potential of the i-th hadron species is $\mu_i = b_i\mu_B + q_i\mu_Q + s_i\mu_S$, where b_i, q_i, and s_i denote the corresponding conserved charges.

The ideal-gas pressure of the i-th hadron species is

$$p_i^{\rm id}(T,\mu_i) = \frac{d_i}{6\pi^2} \int dm\, f_i(m) \int_0^\infty \frac{dk\, k^4}{\sqrt{k^2+m^2}} \left[\exp\left(\frac{\sqrt{k^2+m^2} - \mu_i}{T} \right) \pm 1 \right]^{-1} \,, \tag{3}$$

where the plus sign corresponds to fermions and the minus sign to bosons. Here d_i is the spin degeneracy factor, while the mass integration over the distribution $f_i(m)$ accounts for the finite widths of resonances, with $f_i(m) \to \delta(m - m_i)$ for stable hadrons. The resonance widths are treated using the energy-independent Breit–Wigner prescription [26]. We use the PDG2020 hadron list implemented in the `Thermal-FIST` package [27], which contains all established hadrons and resonances listed in the 2020 edition of the Particle Data Group. We do not include light nuclei in the list.

The shifted chemical potentials μ_i^{X*} of (anti)baryons are determined from the transcendental equation

$$\mu_i^{X*} = \mu_i - b\, P_X - ab\, n_X^2 + 2a n_X \,, \tag{4}$$

which accounts simultaneously for the repulsive excluded-volume correction and the attractive mean-field contribution. The (anti)baryon number densities satisfy

$$n_X = (1 - b n_X) \sum_{i\in X} n_i^{\rm id}(T,\mu_i^{X*}) \,, \tag{5}$$

where $n_i^{\rm id}(T,\mu_i) = (\partial p_i^{\rm id}/\partial\mu_i)_T$ is the ideal-gas number density.

The parameters $a > 0$ and $b > 0$ characterize, respectively, the mean-field attractive interaction and the excluded-volume repulsion between (anti)baryons. The parameters are fixed by reproducing the empirical saturation properties of symmetric nuclear matter at zero temperature [28]. The resulting values, $a = 329$ MeV fm^3 and $b = 3.42$ fm^3, are assumed to be universal for all baryon and antibaryon species [22].

The QvdW-HRG pressure at vanishing chemical potentials, $\mu = 0$, corresponds to the $r_M = 0$ curve in Fig. 1(a). All numerical calculations are performed using the `Thermal-FIST` package [27]. The model reproduces lattice QCD thermodynamics[1] [13] up to temperatures of about $T \lesssim 200$ MeV, but predicts a much steeper increase of the pressure at higher temperatures. Such behavior prevents a thermodynamically consistent matching to the deconfined phase because, if the HRG and QGP represent different phases of SIM, the Gibbs criterion requires the QGP pressure to exceed the HRG pressure at sufficiently high temperatures [30].

Including excluded-volume repulsion only for baryons is insufficient because the thermodynamics becomes dominated at high temperatures by point-like mesons, whose number of effective degrees of freedom exceeds that of the QGP. This indicates that the assumption of point-like mesons becomes inadequate at high particle densities. We therefore extend the QvdW-HRG model by introducing short-range excluded-volume repulsion also among mesons. This modification is not intended as a microscopic description of meson interactions. Rather, it provides a simple phenomenological parametrization of the reduction of the effective hadronic degrees of freedom at high temperatures while preserving the successful description of low-temperature thermodynamics. Such a description has been applied in several studies to suppress hadrons at large temperatures and densities, in particular in the context of building a hadron-quark description of QCD thermodynamics [31–33]. The meson chemical potentials in Eq. (2) are replaced according to

$$\mu_i \to \mu_i^{M*} = \mu_i - b_M \sum_{j\in M} p_j^{\rm id}(T,\mu_j^{M*}) \,, \tag{6}$$

where, unlike in the baryonic sector, only the excluded-volume correction is included and no attractive mean-field contribution is introduced. The meson number density satisfies

$$n_M = (1 - b_M n_M) \sum_{i\in M} n_i^{\rm id}(T,\mu_i^{M*}) \,. \tag{7}$$

The excluded-volume parameter is related to the meson hard-core radius by $b_M = \frac{16}{3}\pi r_M^3$.

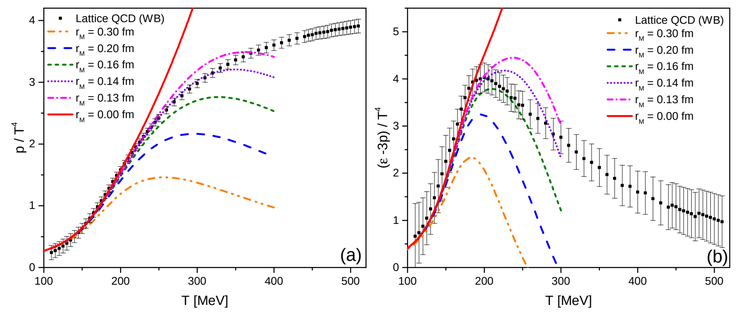


Figure 1. Temperature dependence of (a) the scaled pressure p/T^4 and (b) the trace anomaly $(\varepsilon - 3p)/T^4$ in the QvdW-HRG model at $\mu = 0$ for several values of the meson hard-core radius r_M. The case $r_M = 0$ corresponds to the original model of Ref. [22] with point-like mesons. The lattice QCD data of the Wuppertal–Budapest Collaboration are taken from Ref. [13].

In contrast to the nucleon–nucleon interaction, which is well constrained by nuclear physics, little empirical information is available on short-range repulsion between mesons, although interacting meson matter has been described within the van der Waals equation of state [34, 35]. Furthermore, the attractive meson–meson and meson–(anti)baryon

[1] We use the Wuppertal–Budapest (WB) data [13] as they span a broader temperature range (100–500 MeV) compared to the HotQCD data [14]. More accurate lattice data for (2+1+1)-flavor matter have been presented recently [29]; they are consistent within errors with the data of Ref. [13] that we use.

interactions are expected to be largely encoded in the resonance spectrum included in the HRG [36]. Because the short-range mesonic repulsion remains poorly constrained experimentally, we treat the meson hard-core radius r_M as a phenomenological parameter. Figure 1(a) illustrates the temperature dependence of the HRG pressure for several values of r_M. Even a modest meson excluded volume suppresses the rapid increase of the HRG pressure at high temperatures, ensuring that the QGP pressure eventually becomes dominant.

At vanishing chemical potentials, the entropy density and energy density follow from the thermodynamic identities

$$s = \frac{dp}{dT} \,, \qquad \varepsilon = T\frac{dp}{dT} - p \,. \tag{8}$$

Figure 1(b) shows the corresponding trace anomaly, $(\varepsilon - 3p)/T^4$, for the same values of r_M. Unlike the original QvdW-HRG model with point-like mesons, finite values of r_M generate a pronounced maximum in the trace anomaly. For $r_M \simeq 0.14$ fm, the HRG model alone reproduces the position and magnitude of the lattice QCD peak around $T \simeq 200$ MeV. This underscores that a pronounced maximum of the trace anomaly is not by itself a unique signature of deconfined matter but may also arise from strong hadronic interactions [37–39]. In Sec. IV, the parameter r_M is determined together with the QGP parameters by fitting the complete equation of state to lattice QCD thermodynamics.

III. QUARK-GLUON PLASMA

To describe the QGP phase of strongly interacting matter we start from an ideal gas of quarks and gluons and introduce a simple phenomenological correction that accounts for nonperturbative interactions remaining above the QCD transition. The resulting parametrization is intended to provide a minimal thermodynamically consistent description of the lattice QCD equation of state while keeping the number of free parameters as small as possible.

The pressure of an ideal gas of quarks and gluons at $\mu = 0$ is

$$p_Q^{\rm id}(T) = \frac{d_g\pi^2}{90}T^4 + \sum_{q=u,d,s} \frac{d_q}{6\pi^2} \int_0^\infty \frac{dk\, k^4}{\sqrt{k^2+m_q^2}} \left[\exp\left(\frac{\sqrt{k^2+m_q^2}}{T} \right) + 1 \right]^{-1} \,, \tag{9}$$

where $d_g = 16$ and $d_q = 12$ are the numbers of internal degrees of freedom of gluons and of each quark flavor (including antiquarks), respectively. We include the three lightest quark flavors, u, d, and s, consistent with the $N_f = 2 + 1$ lattice QCD calculations [13] analyzed below.

In the considered temperature range the bare masses of the u and d quarks can be neglected, yielding

$$p_{u/d}^{\rm id}(T) = \frac{7}{8}\frac{d_q\pi^2}{90}T^4 \,. \tag{10}$$

The bare strange-quark mass is taken from Particle Data Tables, $m_s \simeq 93.5$ MeV [40].

The quantity $p_Q^{\rm id}/T^4$ corresponds to the $A = 0$ curve in Fig. 2(a). It approaches the Stefan–Boltzmann limit of a weakly interacting quark–gluon plasma,

$$\frac{p}{T^4} = \frac{\pi^2}{90}\left(d_g + \frac{21}{8}d_q \right) \simeq 5.2,$$

the small deviation originating from the finite strange-quark mass. In contrast, the lattice QCD pressure remains significantly below this limit even at the highest temperature considered, $T = 500$ MeV, indicating that sizable interaction effects persist well above the crossover region.

To account for these nonperturbative effects while preserving the simplicity of the model, we supplement the ideal-gas pressure by a phenomenological interaction term,

$$p_Q(T) = p_Q^{\rm id}(T) - AT^3, \tag{11}$$

where $A > 0$ is a temperature-independent parameter with the dimension of energy. The correction decreases as $1/T$ relative to the dominant ideal-gas contribution,

$$\frac{AT^3}{T^4} = \frac{A}{T},$$

and therefore vanishes asymptotically, ensuring that the Stefan–Boltzmann limit is recovered at sufficiently high temperatures. The dependence T^3 provides a simple one-parameter correction that possesses this property while providing a good description of lattice QCD thermodynamics.

Figure 2(a) shows p_Q/T^4 for several values of A. For $A = 700$ MeV the model provides a reasonable description of the lattice pressure above $T \simeq 200$ MeV. Later on, by fitting the combined hadron-quark model to the lattice data, we obtain a somewhat smaller value, $A \simeq 600$ MeV, which reflects the residual contribution of the hadronic phase in the crossover region.

The entropy density and the energy density are calculated from the thermodynamic identities (8). Figure 2(b) shows the resulting trace anomaly. Neglecting the small strange-quark mass correction, Eq. (11) yields

$$\frac{\varepsilon_Q - 3p_Q}{T^4} \simeq \frac{A}{T},$$

which decreases monotonically with temperature. Consequently, the QGP model alone cannot reproduce the pronounced peak of the trace anomaly observed in lattice QCD. As demonstrated in the next section, this peak emerges naturally from the matching of the QGP and HRG equations of state. The parameter A is determined simultaneously with the meson hard-core radius r_M by fitting the complete equation of state to the lattice QCD results.

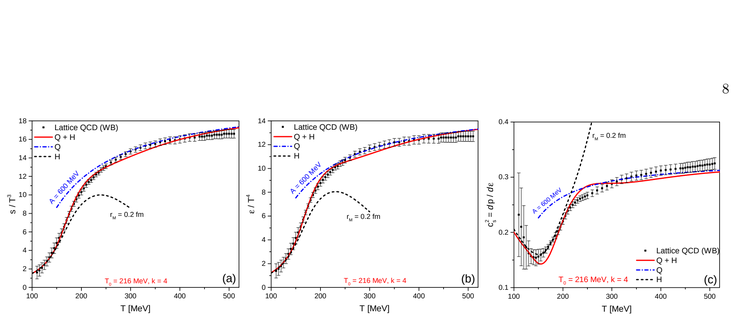


Figure 2. Temperature dependence of (a) the scaled pressure p/T^4 and (b) the trace anomaly $(\varepsilon - 3p)/T^4$ in the QGP model (11) for several values of the interaction parameter A. The case $A = 0$ corresponds to the ideal quark–gluon gas (9). The lattice QCD data of the Wuppertal–Budapest Collaboration are taken from Ref. [13].

IV. UNIFIED EQUATION OF STATE

To describe the smooth crossover between the hadronic and quark–gluon plasma phases, we employ the switching-function method proposed in Ref. [15]. Unlike the Maxwell construction appropriate for a first-order phase transition, the switching-function approach provides a thermodynamically smooth interpolation between the HRG and QGP equations of state without introducing discontinuities in thermodynamic observables. At vanishing chemical potentials the full equation of state of strongly interacting matter is written as

$$p(T) = \mathscr{K}(T)\, p_Q(T) + [1 - \mathscr{K}(T)]\, p_H(T), \tag{12}$$

where p_H (1) and p_Q (11) denote the HRG and QGP pressures, respectively. The switching function

$$\mathscr{K}(T) = \exp\left[-\left(\frac{T_0}{T} \right)^k \right] \tag{13}$$

increases smoothly from nearly zero at low temperatures ($T \sim 100$ MeV) to unity at high temperatures ($T \sim 500$ MeV). It may be interpreted as the temperature-dependent interpolation weight of the QGP contribution to the pressure, while $1 - \mathscr{K}(T)$ gives the corresponding weight of the hadronic component. Following Ref. [15], we fix the exponent to $k = 4$, whereas the switching temperature T_0 is treated as a free parameter.

Interpolating the pressure is particularly convenient because all remaining thermodynamic observables follow uniquely from standard thermodynamic identities, ensuring internal thermodynamic consistency. The entropy density is obtained from Eq. (8),

$$s = \frac{dp}{dT} = \mathscr{K}(T)\, s_Q + [1 - \mathscr{K}(T)]\, s_H + \frac{d\mathscr{K}(T)}{dT}\,(p_Q - p_H) \,, \tag{14}$$

where

$$\frac{d\mathscr{K}(T)}{dT} = \frac{k}{T}\left(\frac{T_0}{T} \right)^k \mathscr{K}(T). \tag{15}$$

Although the pressure is simply a weighted average of the HRG and QGP pressures, derivatives of the switching function generate additional crossing contributions to the entropy density and, consequently, to the energy density. These terms originate entirely from the temperature dependence of $\mathscr{K}(T)$.

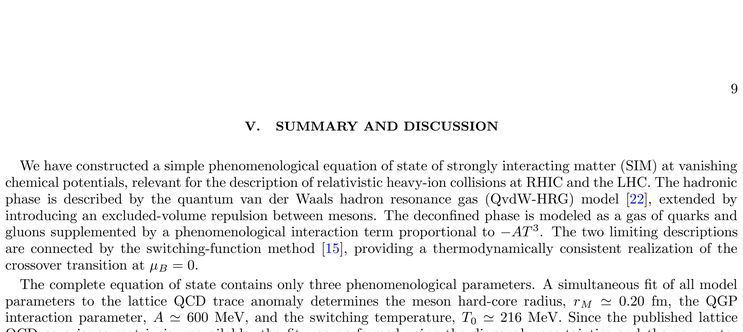


Figure 3. Temperature dependence of (a) the switching function $\mathscr{K}(T)$ defined by Eq. (13) and (b) its derivative $d\mathscr{K}/dT$ for the fitted switching temperature $T_0 = 216$ MeV. The derivative reaches its maximum at $T \simeq 204$ MeV.

Figure 3 shows the switching function and its temperature derivative. At $\mu_B = 0$, the only derivative contribution is the crossing term in Eq. (14). Numerically, it remains below approximately 10% of the total entropy density over the entire temperature range considered, indicating that the thermodynamic observables are dominated by the HRG and QGP components rather than by the interpolation itself. We note that a choice of the exact analytical form of the switching function remains an open question and may deserve further studies [41, 42]. For instance, recently, a statistical-mixture formulation [43] was proposed in which the mixing weight becomes an internal thermodynamic variable determined by minimizing the grand potential. In particular, it allows one to introduce a first-order phase transition with a critical endpoint. However, for the present purposes, restricted to zero chemical potentials, the simple parametrization (13) provides an accurate description while keeping the crossing contributions small.

Our present analysis is restricted to zero chemical potentials. Extending it to finite chemical potentials is a possible future extension, which will require modifications. These modifications include both the formulation of the QGP phase EoS at finite μ_B, and the possible explicit dependence of the switching function $\mathscr{K}$ on μ_B (or any other chemical potentials). In turn, the derivatives of $\mathscr{K}$ with respect to μ_B will contribute to the baryon density and higher-order baryon-number susceptibilities. This is the reason why we omit the consideration of conserved-charge susceptibilities, such as χ_2^B, in this study. The susceptibilities receive contributions from the chemical potential dependence of both the switching function and the QGP interaction term, neither of which is constrained by the thermodynamics at vanishing chemical potentials. However, susceptibilities at $\mu_B = 0$ are available from lattice QCD and could be used in such a study. We leave such an extension to finite chemical potentials for future work.

Our complete equation of state (12) of QCD matter at vanishing chemical potentials contains only three phenomenological parameters: the meson hard-core radius r_M, the QGP interaction parameter A, and the switching temperature T_0. They are determined to be $r_M \simeq 0.2$ fm, $A \simeq 600$ MeV and $T_0 \simeq 216$ MeV from a χ^2 fit to the lattice QCD data for the trace anomaly $(\varepsilon - 3p)/T^4$ [13]. All remaining thermodynamic observables are then obtained without further adjustment. The value of χ^2/NDF at the minimum is 0.445. The quoted χ^2/NDF is calculated using the published uncertainties shown in Ref. [13], which correspond to the diagonal elements of the covariance matrix. The lattice QCD results at different temperatures are known to be strongly correlated, but the full covariance matrix is not publicly available. Therefore, the quoted χ^2 should be regarded primarily as a measure of the overall quality of the description rather than as a statistically rigorous goodness-of-fit estimator. The extracted parameter values are driven mainly by the global temperature dependence of the trace anomaly and are expected to be considerably less sensitive to the neglected correlations than the absolute value of χ^2. Rigorous estimates of the parameter uncertainties cannot presently be obtained without the full covariance matrix, and are therefore not quoted.

The resulting pressure and trace anomaly are shown in Fig. 4, together with the separate HRG and QGP contributions. They are shown to describe the lattice data within uncertainties.

Figure 5 compares the full equation of state with the lattice QCD results for the entropy density, s/T^3, the energy density, ε/T^4, and the speed of sound squared, $c_s^2 = dp/d\varepsilon$. The model reproduces the lattice data within uncertainties over nearly the entire temperature range, including the characteristic minimum of the speed of sound around $T \simeq 150$ MeV. Only small overestimates of the entropy and energy densities, at the level of a few percent, appear in particular for $T \gtrsim 450$ MeV.

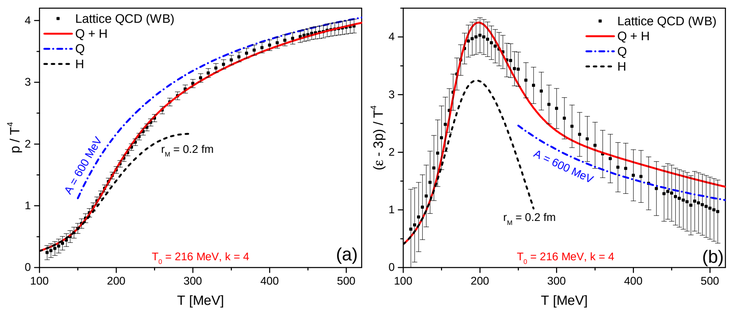


Figure 4. Temperature dependence of (a) the scaled pressure p/T^4 and (b) the trace anomaly $(\varepsilon - 3p)/T^4$ for the full equation of state (12). The dash-dotted and dashed curves show separately the QGP (11) and HRG (1) contributions, respectively. The lattice QCD data of the Wuppertal–Budapest Collaboration are taken from Ref. [13].

Let us now discuss the physical interpretation of the three model parameters, r_M, A, and T_0.

The hard-core meson radius r_M. The fitted meson hard-core radius, $r_M \simeq 0.2$ fm, is considerably smaller than the (anti)baryon hard-core radius, $r_b \simeq 0.59$ fm (corresponding to $b_b = 3.42$ fm^3). Consequently, meson excluded-volume effects are expected to have only a negligible influence at low temperatures. Indeed, Fig. 1(a) shows that the HRG pressures obtained with $r_M = 0$ and $r_M = 0.2$ fm are almost indistinguishable below $T \lesssim 160$ MeV. Therefore, meson excluded-volume effects are expected to have little influence on thermal fits of hadron multiplicities at the chemical freeze-out temperature, $T_{\rm ch} = 150$–160 MeV [10, 25, 44] and other standard applications of the HRG model. On the other hand, above $T \gtrsim 200$ MeV the mesonic excluded volume effects substantially suppress the HRG pressure. The values of meson hard-core radius we obtain are comparable to those studied in the literature in various contexts [32, 43, 45, 46].

The phenomenological interaction term $-AT^3$. The behavior of the thermodynamic functions in lattice QCD indicates the presence of interaction effects in the QGP even at the highest temperature considered, $T = 500$ MeV. Rather different explanations of these effects can be found in the literature. In Refs. [2, 15] the quark-gluon interactions were described by perturbative correction terms up to order α_s^3, with two free parameters adjusted to fit the lattice results. In our modeling of the QGP EoS we instead describe all interaction effects by the phenomenological term $-AT^3$, which contains a single free parameter. The fitted value, $A \simeq 600$ MeV, corresponds to substantial interaction effects: at $T = 200$ MeV, the QGP pressure (11) is reduced by a factor of about 2.4 and the energy density by about 40% relative to the ideal quark–gluon gas. This is not a weakly interacting quark–gluon gas but a strongly interacting quark–gluon liquid. The applicability of a perturbative description of the QGP in the temperature range 150–300 MeV therefore remains questionable. The parameter A is of the order of the QCD scale,

$$A \simeq 1.8\, \Lambda_{\overline{\rm MS}}^{N_f=3} \,,$$

using the value reported in Ref. [47].

Power-law corrections to the QGP pressure have been discussed before. A linear contribution to the QGP pressure, $-aT$, suggested in Ref. [48], suppresses the pressure but does not affect the energy density. The arguments based on the fuzzy-bag picture and dimension-two condensates suggest a correction quadratic in the temperature, $-BT^2$ [49–51], whereas perturbation theory modifies the coefficient of T^4 only logarithmically through the running coupling [52]. The $-AT^3$ term should thus be viewed as a phenomenological parametrization preferred by the lattice data in the considered temperature window; its scale of order $\Lambda_{\rm QCD}$ is, however, qualitatively consistent with the picture in which nonperturbative confining correlations survive in the QGP well above the crossover [16, 19].

We note that for $T \lesssim 340$ MeV the interaction term AT^3 exceeds the ideal-gas pressure of gluons, $d_g\pi^2T^4/90$. The QGP pressure (11) in this temperature range therefore stays below that of an ideal gas of quarks alone. This is consistent with the expectation that gluonic degrees of freedom are strongly suppressed just above the crossover, whether because thermal gluon masses exceed the quark ones [53, 54] or because gluons remain confined into heavy glueballs up to temperatures of about 285 MeV [20].

It is instructive to compare this parametrization with the MIT bag model [55],

$$p_Q = p_Q^{\rm id} - B, \qquad \varepsilon_Q = \varepsilon_Q^{\rm id} + B, \tag{16}$$

where $B > 0$ is the bag constant. While the bag constant lowers the pressure, it simultaneously increases the energy density. The lattice QCD results require both quantities to be suppressed relative to the ideal-gas limit. The conventional bag-model equation of state is therefore inconsistent with the lattice QCD thermodynamics in the considered temperature range. Note that the bag model can be improved, for instance, by considering effective or constituent quark masses, which are much heavier than bare masses [53, 54, 56–58].

The crossover transition temperature T_0. The pseudocritical temperature of the chiral crossover at $\mu_B = 0$, $T_{\rm pc} \approx 155$ MeV [6, 14, 24], is close to the chemical freeze-out temperature, $T_{\rm ch} = 150$–160 MeV. The fitted switching temperature, $T_0 \simeq 216$ MeV, is considerably larger, in agreement with the findings of Ref. [15]. A similar picture emerges in the chiral mean-field model [33, 59], where the hadronic and quark contributions to the total pressure become equal at a comparable temperature [59]. Consequently, the hadronic contribution remains substantial well above the pseudocritical temperature: $1 - \mathscr{K} \simeq 0.98$ at $T = 155$ MeV and still $1 - \mathscr{K} \simeq 0.43$ at $T = 250$ MeV. The fitted equation of state therefore predicts a significant admixture of hadronic degrees of freedom throughout the temperature interval 150–250 MeV. It will be interesting to investigate alternative forms of the switching function [2, 42] and the statistical-mixture approach proposed in Ref. [43], as well as to incorporate conserved-charge susceptibilities into the analysis.

Figure 5. Temperature dependence of (a) the entropy density s/T^3, (b) the energy density ε/T^4, and (c) the speed of sound squared, $c_s^2 = dp/d\varepsilon$, for the full equation of state (12). The dash-dotted and dashed curves show separately the QGP and HRG contributions, respectively. The lattice QCD data of the Wuppertal–Budapest Collaboration are taken from Ref. [13].

V. SUMMARY AND DISCUSSION

We have constructed a simple phenomenological equation of state of strongly interacting matter (SIM) at vanishing chemical potentials, relevant for the description of relativistic heavy-ion collisions at RHIC and the LHC. The hadronic phase is described by the quantum van der Waals hadron resonance gas (QvdW-HRG) model [22], extended by introducing an excluded-volume repulsion between mesons. The deconfined phase is modeled as a gas of quarks and gluons supplemented by a phenomenological interaction term proportional to $-AT^3$. The two limiting descriptions are connected by the switching-function method [15], providing a thermodynamically consistent realization of the crossover transition at $\mu_B = 0$.

The complete equation of state contains only three phenomenological parameters. A simultaneous fit of all model parameters to the lattice QCD trace anomaly determines the meson hard-core radius, $r_M \simeq 0.20$ fm, the QGP interaction parameter, $A \simeq 600$ MeV, and the switching temperature, $T_0 \simeq 216$ MeV. Since the published lattice QCD covariance matrix is unavailable, the fit was performed using the diagonal uncertainties and the parameter uncertainties are not quoted; for details, see the discussion in Sec. IV. Without further adjustment, the resulting equation of state reproduces the lattice QCD pressure, trace anomaly, entropy density, energy density, and speed of sound over the temperature range $T = 100$–500 MeV.

The analysis leads to three notable physical conclusions. First, the peak of the trace anomaly can be reproduced within the hadronic sector alone once short-range repulsive interactions between mesons are included, demonstrating that this feature is not by itself a unique signature of deconfined matter. In this respect, our approach differs from previous switching-function constructions, such as those of Refs. [2, 15, 23]. Second, the fitted interaction parameter indicates that substantial nonperturbative effects persist in the QGP well above the crossover region. Third, the switching temperature, $T_0 \simeq 216$ MeV, is considerably higher than both the pseudocritical chiral-crossover temperature, $T_{\rm pc} \approx 155$ MeV, and the chemical freeze-out temperature, $T_{\rm ch} = 150$–160 MeV. Consequently, the equation of state predicts a substantial admixture of hadronic degrees of freedom throughout the temperature interval 150–250 MeV.

Besides providing an accurate parametrization of lattice QCD thermodynamics, the proposed equation of state supplies additional information that is not directly accessible from lattice calculations alone. Examples include the partial contributions of hadrons and quarks to thermodynamic observables, the absolute strangeness content, and the possibility of evaluating quantities relevant for dynamical simulations of heavy-ion collisions, such as thermal photon and dilepton emission.

The present results also raise several open questions. The persistence of a significant hadronic component up to temperatures of about 250 MeV suggests that the properties of hadronic matter in the crossover region may differ qualitatively from those of the hadron resonance gas near chemical freeze-out. This observation is consistent with recent ideas that color-singlet hadronic excitations or hadron-like correlations may survive well above the pseudocritical temperature and coexist with deconfined quark and gluon degrees of freedom in the crossover region [16, 17, 19, 21, 60]. Interpreting the interval 150–250 MeV as a distinct phase of strongly interacting matter would, however, raise new questions about the nature of the two crossover transitions bounding it, near $T \simeq 150$ MeV and $T \simeq 250$ MeV. Our phenomenological equation of state does not introduce a third phase; rather, it treats matter in this temperature range as a mixture of the strongly interacting HRG and QGP components. The successful description of lattice QCD thermodynamics supports this picture.

The present work also has several limitations that indicate natural directions for future studies. First, the phenomenological high-temperature equation of state should be confronted with perturbative QCD descriptions [2, 15] to clarify which features of the lattice thermodynamics genuinely require additional nonperturbative contributions. Second, a global fit including both hadronic and high-temperature parameters, together with alternative switching functions or statistical-mixture formulations would provide an important test of the robustness of the extracted switching temperature. Finally, extending the analysis to finite baryon chemical potential and incorporating conserved-charge susceptibilities will allow one to investigate whether the picture obtained here remains valid away from $\mu_B = 0$ and whether it can provide additional constraints on the microscopic structure of strongly interacting matter.

ACKNOWLEDGMENTS

We are thankful to Musfer Adzhymambetov, Leonid Glozman, Joseph Kapusta, Volodymyr Kuznietsov, Larry McLerran, Robert Pisarski, Lorenzo Salcedo, and Jan Steinheimer, for fruitful comments and discussions. M.I.G. and A.L. are thankful to the National Research Foundation of Ukraine for the support under the grant number 2025.07/0050.