

Entanglement in parameterized quantum circuits and its quantification on quantum computers

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We study entanglement in multi-qubit variational quantum circuits with different structures of entangling blocks. For two- and multi-qubit variational states generated by circuits with RY gates and entangling CZ operators, we derive analytical expressions and recurrence relations for quantum correlators and the entanglement distance [1]. For a closed one-dimensional chain of n qubits, an explicit analytical expression for the entanglement of a qubit with the rest of the system for a two-layer variational protocol is obtained. It is shown that the entanglement of a given qubit is determined not only by its local gate parameters but also by the parameters of the gates acting on its nearest and next-nearest neighbors [1]. We also consider multi-qubit variational states represented by vertex- and edge-weighted graphs and constructed using single-layer circuits with RX rotations and RZZ gates. For quantum graph states of arbitrary structure, we derive the geometric measure of entanglement and quantum correlators and establish their relation to the edge- and vertex-weight structure in the neighborhoods of the corresponding graph vertices [2]. We show that the entanglement of a qubit in quantum graph states is determined by the weights of the incoming and outgoing edges associated with the corresponding vertex, as well as by its indegree and outdegree [3].

The analytical results are validated using quantum computing on the IBM Marrakesh and IBM Kingston quantum processors and the Qiskit AerSimulator, including noisy simulations [1-3]. The corresponding quantum circuits are implemented to investigate the dependence of the entanglement measures on circuit parameters and graph structure. The results of quantum computing are in a good agreement with the analytical results.. It is worth noting that the obtained dependencies of entanglement in variational quantum states on the parameters of variational quantum circuits and the structure of their entangling blocks are of fundamental importance. At the same time, they are relevant to potential applications in quantum computing. Variational quantum states and quantum graph states underly in a wide range of quantum algorithms, including quantum machine learning, variational quantum protocols, and quantum error correction. The analytical characterization of their entanglement, together with quantum protocols for its quantification using quantum computing, can therefore contribute to the design and improvement of quantum algorithms and to the identification of circuit architectures capable of generating and controlling a desired degree of entanglement. Also, the obtained relations between entanglement of quantum graph states and structural properties of graphs, vertex and edge weights, open up the possibility of studying them using quantum computing.

[1] Kh. P. Gnatenko, A. Kaczmarek Properties of multi-qubit variational quantum states representing weighted graphs and their computing with quantum programming, arXiv:2604.00958 (2026).

[2] Kh. P. Gnatenko, R. O. Hredil, V. Y. Pinchuk, M. Z. Seniak, Y. T. Shevchuk Entanglement distance of two- and multi-qubit variational states and its quantification with quantum computing, arXiv:2605.00259 (2026).

[3] Kh. P. Gnatenko Entanglement of multi-qubit states representing directed networks and its detection with quantum computing Phys. Lett. A 521, 129815 [5 p.] (2024).

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