

Resonant tunneling through one-point singular potentials

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In one-dimensional quantum mechanics, shrinking a system to isolated points (set of Lebesgue's measure zero) leads to exactly solvable models. These models are referred to as point (contact or zero-range) interactions (PIs). In these cases, resolvents and spectra of Schroedinger operators, scattering coefficients and other characteristics can analytically be computed. Currently, because of the rapid progress in fabricating nanoscale quantum devices, of particular importance is the point modeling of different structures like quantum waveguides, spectral filters, or infinitesimally thin sheets.

We have initiated a new project in the domain of PIs regarding a resonant tunneling through zero-range potentials being more singular than a traditional Dirac's delta potential. A typical example is the distributional potential in the form of a spatial derivative of the delta function.

Using here a very simple two-scale (two-parameter) regularization scheme, we observe that different pathways $\Delta'_\varepsilon(x) \rightarrow \delta'(x)$ as $\varepsilon \rightarrow 0$ realize different PIs with the following three types of boundary conditions on the wave function $\psi(x)$:

- i) $\psi(-0) = \psi(+0) = 0$ (separated with full reflection);
- ii) $\psi(+0) = \theta_n \psi(-0)$, $\psi'(+0) = \theta_n^{-1} \psi'(-0)$, $\theta_n \in R$, $n \in Z$ (non-separated with resonant tunneling);
- iii) $\psi(+0) = \theta_n \psi(-0)$, $\psi'(+0) = \alpha_n \psi(-0) + \theta_n^{-1} \psi'(-0)$, $\alpha_n \in R$ (non-separated with resonant tunneling having a single bound state at each n).

For each of these types, the regions on the plane of two squeezing parameters are indicated explicitly.

Recent literature:

1. A.V. Zolotaryuk and Y. Zolotaryuk, "Scattering data and bound states of a squeezed double-layer structure", *J. Phys. A: Math. Theor.* **54** (2021) 035201; doi.org/10.1088/1751-8121/abd156
2. Y. Zolotaryuk and A.V. Zolotaryuk, "Influence of a squeezed prewell on tunneling properties and bound states in heterostructures", *Annals of Physics* **477** (2025) 169999; doi.org/10.1016/j.aop.2025.169999
3. Y. Zolotaryuk and A.V. Zolotaryuk, "Conditions for perfect transmission of quantum particles across layered heterostructures and resonance effects of an auxiliary well potential", *Front. Appl. Math. Stat.* **11** (2015) 1636414; doi.org/10.3389/fams.2025.1636414

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