

Low-order momentum correlation functions of the symmetric Tamm-Dancoff q -bosons, derived via under-trace operations

Tuesday, 22 September 2026 17:45 (5 minutes)

Using the set of independent Symmetric Tamm-Dancoff (STD) type q -deformed quantum oscillators [1] with the standard linear-in- N Hamiltonian, we construct respective gas of q -bosons. Such a deformed q -Bose gas model admits [2] exact analytic expressions for the r -particle momentum correlation function, and corresponding intercepts, of any r th order. This is the third example of deformed oscillators' based Bose gas model, along with the qp -Bose gas model [3] and the μ -deformed one [4] (based on Jannussis's oscillator), where correlation average $\langle (a_k^\dagger)^r (a_k)^r \rangle$ was calculated explicitly. In view of the formal resemblance between the qp -bracket and the STD deformation structure function, with N -multiplier ignored, we apply the idea of the derivation from [3] for qp -bosons, to the current case of STD q -bosons.

Indeed, using just under-trace algebraic operations, for the STD q -bosons in a fixed $\mathbf{k}$ -mode we find

$$\langle a^\dagger a \rangle_{\text{STD}} = \frac{1}{(e^{\beta\omega} - q)^2} \left\{ \left\langle \frac{[a, [a, a^\dagger]_q]_q a^\dagger}{\varphi_{\text{STD}}(N+1)} \right\rangle + \frac{[1]_q^{\text{STD}}! e^{\beta\omega}}{\text{Tr } e^{-\beta\omega N}} \right\}$$

versus $\langle a^\dagger a \rangle_{qp} = \frac{[1]_{qp}!}{e^{\beta\omega} - q} \langle [a, a^\dagger]_q \rangle$ for qp -bosons, see [3]. What concerns the quadratic correlations, using the same approach of under-trace transformations we derive

$$\begin{aligned} \langle a^{\dagger 2} a^2 \rangle_{\text{STD}} &= \frac{1}{(e^{\beta\omega} - q^2)^3 (e^{\beta\omega} - 1)^3} \left\{ \left\langle \sum_{i=0}^4 A_i(q) g_q(N+1+i) g_q(N+i) \cdot q^{-2N} \right\rangle \right. \\ &\quad \left. - (e^{\beta\omega} - 1) g_q(0) P_2(q; e^{\beta\omega}) \right\} + \varphi_{\text{STD}}(2)! \frac{e^{\beta\omega} - 1}{(e^{\beta\omega} - q^2)^3} \end{aligned}$$

where underlying role belongs to the double commutator as well, through $g_q(N)$ -function, $[a, [a, a^\dagger]_q]_q = -g_q(N+1) q^{1-N} \cdot a$, and $P_2(q; e^{\beta\omega})$ is some quadratic polynomial in $e^{\beta\omega}$, with coefficients depending on q .

1. W.S. Chung, A.M. Gavriliuk, I.I. Kachurik, A.P. Rebesh: J. Phys. A: Math. Theor. **47**, 305304 (2014).
2. A.M. Gavriliuk, Yu.A. Mishchenko, arXiv preprint, arXiv:1410.0538 (2014).
3. L.V. Adamska, A.M. Gavriliuk, J. Phys. A: Math. Gen. **37**, 4787-4796 (2004).
4. A.M. Gavriliuk, Yu.A. Mishchenko, Phys. Lett. A **376**, 2484-2489 (2012).

Primary authors: MISHCHENKO, Yuriy (Bogolyubov Institute for Theoretical Physics of NAS of Ukraine); GAVRI-LIK, Alexandre (Bogolyubov Institute for Theoretical Physics of NAS of Ukraine)

Presenter: MISHCHENKO, Yuriy (Bogolyubov Institute for Theoretical Physics of NAS of Ukraine)

Session Classification: Poster Session

Track Classification: MATHEMATICAL PHYSICS