

Quanta Measurements are Indirect only

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BITP

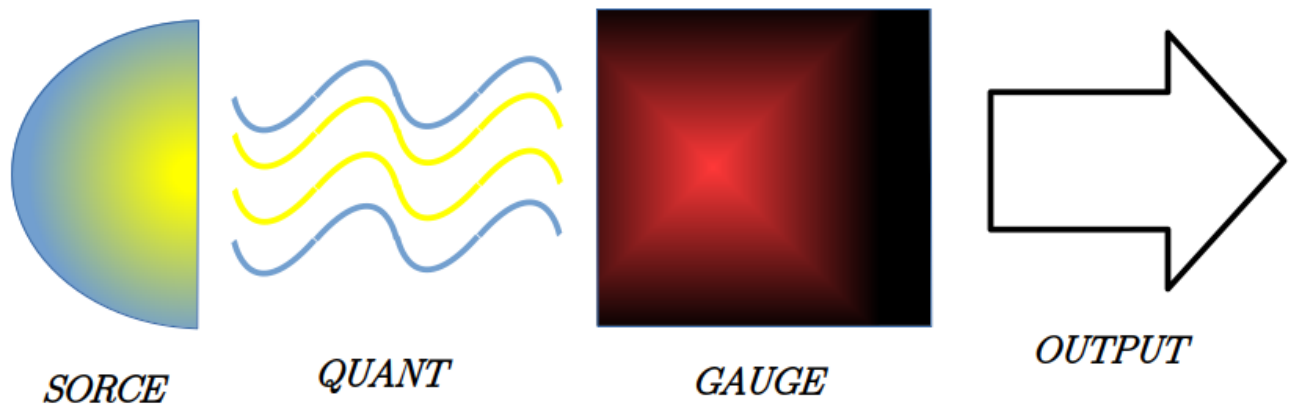
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Common Measurement

Indirect Measurement

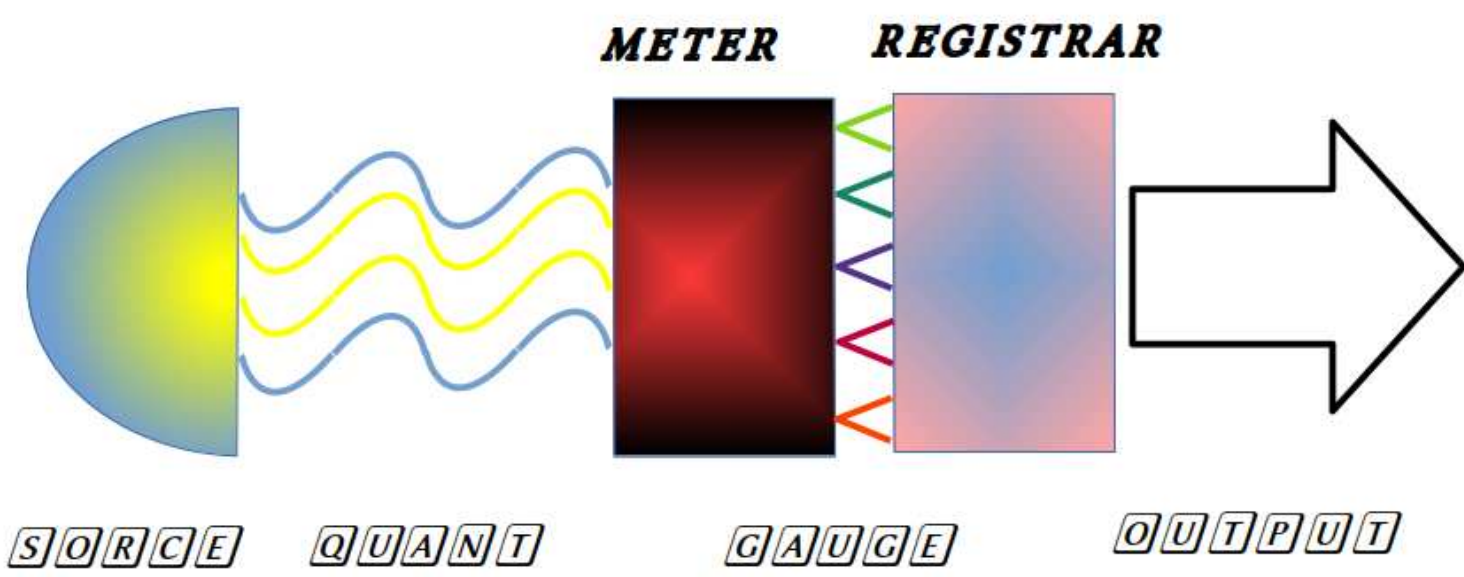
Summary

Common Measurement Process



Navigation icons for the Common Measurement Process diagram.

Indirect Measurement Process



Navigation icons for the Indirect Measurement Process diagram.

States and Observables

Definition
Observable and its Resolution of Identity

$$\mathbb{A} = \sum_{n=1}^{N^{(Q)}} A_n \Pi_n^{\{\mathbb{A}\}} \quad \sum_{n=1}^{N^{(Q)}} \Pi_n^{\{\mathbb{A}\}} = \hat{I} \quad (1)$$

Nondegenerate Observable

$$\Pi_n^{\{\mathbb{A}, n.d.\}} = |\psi_n; \mathbb{A}\rangle \langle \psi_n; \mathbb{A}|, \quad \langle \psi_n; \mathbb{A} | \psi_n; \mathbb{A} \rangle = 1. \quad (2)$$

$$\text{Density Matrix} \quad \rho = \sum_{n=1}^{N^{(p)}} p_n \Pi_n^{\{p\}}; \quad \sum_{n=1}^{N^{(p)}} \Pi_n^{\{p\}} = \hat{I}, \quad (3)$$

$$\text{Pure state} \quad p_{n'} = 1 \mapsto \rho = \Pi_{n'}^{\{p\}} = |\psi_{n'}; \mathbb{A}\rangle \langle \psi_{n'}; \mathbb{A}|$$

Navigation icons for the States and Observables section.

Interaction Quantum-Meter

Hamiltonian

$$H_I = \sum_{n=1}^{N^{(Q)}} R_n \times \Pi_n^{\{\mathbb{A}\}} \quad \mapsto \quad H^{\{QM\}} = \sum_{n=1}^{N^{(Q)}} \left(H_{in}^{\{M\}} + R_n \right) \times \Pi_n^{\{\mathbb{A}\}} \\ H_n^{\{M\}} = H_{in}^{\{M\}} + R_n, \quad \left[H_n^{\{M\}} H_{in}^{\{M\}} \right] \neq 0, \quad \left[H_n^{\{M\}} H_{n'}^{\{M\}} \right] \neq 0 \quad (4)$$

Responses R_n are the observables of the Meter.
 $H_n(\mathbb{A})$ is the Meter Hamiltonian, provided the Quantum is in the state $\Pi_n(\mathbb{A})$.

Subspaces of stationary states $V_{in}^{\{M\}}, V_{n'}^{\{M\}}$

Let $f : R^{N_M} \mapsto T_{N_M}, T_{N_M}$ is simplex $\sum p_k = 1, p_k \geq 0$.

$$\forall f : \quad \rho_{free}^{\{M\}} = f \left(H_{in}^{\{M\}} \right) \in V_{in}^{\{M\}}, \quad \rho_n^{\{M\}} = f \left(H_n^{\{M\}} \right) \in V_n^{\{M\}} \quad (5)$$

$$\text{Before interaction} \quad \left[\rho_{free}^{\{M\}}, H_{in}^{\{M\}} \right] = 0 \quad \text{Output states} \quad \left[\rho_n^{\{M\}}, H_n^{\{M\}} \right] = 0$$

Navigation icons for the Interaction Quantum-Meter section.

Probability

Rigorous Definition

Random variable is the probability triple $(\Omega, \mathbb{P}, \sigma(\Omega))$ (Kolmogorov triple)

- Sample space Ω : $\omega_n \equiv (\rho^{\{out\}} = \rho_n[\mathbb{A}])$
- Probability distribution

$$\text{Born rule} \quad P[\psi; res] = |\langle res | \psi \rangle|^2$$

$$P_n = \langle \psi | \Pi_n | \psi \rangle, \quad \text{OR} \quad P_n = \text{Tr}(\Pi_n \rho), \\ \mathbb{P}[\mathbb{A}] = \left\{ P_n, n \in 1, N^{(Q)} \right\}. \quad (6)$$

Von Neumann-Lüders Rule

$$|n; \mathbb{A}\rangle = P_n^{-1/2} \Pi_n | \psi \rangle \quad \text{OR} \quad \rho_n[\mathbb{A}] = P_n^{-1} \Pi_n \rho \Pi_n.$$

- $\sigma(\Omega)$ is sigma-algebra over Ω (here — the set of all subsets)

Navigation icons for the Probability section.

Classicality of the Quantum State Measurement

Navigation icons for the Classicality of the Quantum State Measurement section.

Peculiarities of measurement of state

- Measurement of quantum state requires realization of a series of measurements for incompatible observables.
- Measurement for an observable requires a series of acts of measurement.

Information processing

- The act of measurement is a physical process, whereas the result of a series of measurement acts is obtained through processing information from the results for a set of independent physical processes.
- The measurement of an observable or a quantum state is not only a physical process, but also an informational one, with an Observer involved. Such a measurement is not an act of Quanta measuring.
- The Observer can be a biological one, or of any other kind.

Navigation icons for the Information processing section.

Meter as a Very Large Quantum

Measurement of the Meter state requires numerous operations

The practical task is to detect the instant of the Meter state approach to one of the known stationary states.
This is a problem of comparing hypotheses regarding the properties of a set of random variables.

Example

For each subspace ω_k a value $D^{\{\omega_k\}}$ is assigned as a quasi-measure of distance between the state $\rho^{\{M\}}(t)$ and the subspace.

Measurement algorithm

1. Sequential measurements of values $D^{\{\omega_k\}}$ for all $k \in 1, \dots, N^{(A)}$ are repeated a sufficient number of times to reduce the uncertainty of each value up to an acceptable level.
2. If the average for one of the values $D^{\{\omega_{k'}\}}$ is substantially smaller than that for any other:
 - Then its index k' is accepted as the result, and the measurement is completed.
 - Else the measurement is repeated.

Navigation icons for the Meter as a Very Large Quantum section.

Random Element

Definition by Maurice Fréchet

Random element is the probability triple $(\Omega, \mathbb{P}, \sigma(\Omega))$ supplemented by a map from the sample space Ω to the set of results $\mathbb{E}$.

Application for Indirect Measurement

- The measurement of the Meter comes to the end at state approaching one of the stationary states,

$$\Omega = \left\{ \forall_n \omega_n = \left(\rho_{out}^{\{M\}} \approx \rho_n^{\{M\}} \right) \right\}. \quad (7)$$

- As the result of the measurement corresponding state of Quantum,

$$\mathbb{E} = \left\{ \forall_n e_n \equiv (\rho^{\{out\}} = \rho_n[\mathbb{A}]) \right\}, \quad (8)$$

is taken.

Technical requirements for the Observer

Observer must have a definition of distance between the state of the Meter and each set of stationary states (5).

Navigation icons for the Random Element section.

Degenerate Observable

$$\mathbb{A} = \sum_{n=1}^{N^{(A)}} A_n \Pi_n^{\{\mathbb{A}\}}, \quad \sum_{n=1}^{N^{(A)}} \Pi_n^{\{\mathbb{A}\}} = \hat{I}, \quad N^{(A)} \ll \dim H^{(M)} \quad (9)$$

Eigenspaces of Hilbert space of Meter

$$H_n^{\{M\}} = \Pi_n^{\{\mathbb{A}\}} H^{(M)}, \quad N_n^{\{A\}} = \dim H_n^{\{M\}}, \quad \bigcup_{n=1}^{N^{(A)}} H_n^{\{M\}} = H^{(M)}$$

Bases of eigenspaces generate a basis for the Hilbert space.

$$\forall_n \sum_{k=1}^{N_n^{\{A\}}} \psi_n |k; n, \mathbb{A}\rangle \in H_n^{\{M\}} \quad \sum_{n=1}^{N^{(A)}} \sum_{k=1}^{N_n^{\{A\}}} \psi_{n,k} |k; n, \mathbb{A}\rangle \in H^{(M)} \quad (10)$$

Navigation icons for the Degenerate Observable section.

Assessment of deviation

Diagonality criterion

Let the transition operator $J_{n,m}$ is $J_{n,m} = |n\rangle \langle m| = X_{n,m} + iY_{n,m}$.

$$\text{If } \forall_{n,m} (\langle X_{n,m} \rangle = 0) \& (\langle Y_{n,m} \rangle = 0) \quad \text{then} \quad \rho = \sum p_n |n\rangle \langle n|$$

Approximate deviation distance

Let p_n are ordered by decrease, and

$$p_{resd} = 1 - \sum_{k=1}^{2K} p_k, \quad J[K] = \sum_{n=1}^{2K} \sqrt{n(2K-n)} |n+1\rangle \langle n| = X[K] + iY[K]$$

The quantity

$$D^{\{K\}}[\rho] = \sqrt{\langle X[K] \rangle^2 + \langle Y[K] \rangle^2} \quad (11)$$

is the approximation for the distance with accuracy not worse than p_{resd} .
Determination of distance requires approximately $4K/p_{resd}^2$ measurement events.

Navigation icons for the Assessment of deviation section.