

Calculation of thermodynamic potential for Bose system near condensate point

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Bose system in the presence of the condensate is investigated in the Bogolyubov model of the separated condensate (see about in [1]). In this state occupation number n_0 of one-particle state with momentum $p = 0$ is macroscopic one. In this model the system is described by the statistical operator

$$w(n_0) = e^{\beta[\Omega(n_0) - \hat{H}(n_0) + \mu \hat{N}(n_0)]}, \quad \text{Sp} w(n_0) = 1$$

where operators $\hat{H}(n_0)$, $\hat{N}(n_0)$ are given by Hamiltonian of the system \hat{H} and operator of particle number \hat{N} after substitution $n_0^{1/2}$ instead of the Bose operators a_0, a_0^+ (β, μ are the inverse temperature and the chemical potential). According to Bogolyubov equilibrium value n_0^0 of the occupation number n_0 can be found from the minimum condition of the thermodynamic potential $\Omega(n_0)$ i.e. it is the non-equilibrium one of the system. Near transition point from normal state to the state with the condensate occupation number n_0 is small in comparison with the total number of particles and the potential $\Omega(n_0)$ can be calculated in a perturbation theory in powers of $n_0^{1/2}$.

The purpose of this paper is calculating of the potential $\Omega(n_0)$ in a modified thermodynamic perturbation theory with small parameter n_0 . The obtained expression for $\Omega(n_0)$ can be used as the potential Landau in his theory of the phase transitions of the second kind for the system under consideration. The statistical operator $w(n_0)$ can be written in the form $w(n_0) = \exp \beta [F - (\hat{\mathbf{H}}_0 + \hat{U}_1 + \hat{U}_2)]$ where the operators $\hat{U}_1 \sim n_0^{1/2}, \hat{U}_2 \sim n_0$. The calculation is substantially simplified by the fact that the independent on n_0 operator $\hat{\mathbf{H}}_0$ commutes with the operator of the number of particles. It is shown that only integer powers of this number are present in expansion of the potential. Relatively compact expressions for the coefficients of this series are obtained because operators describing the interaction in the system commute under T-product. For a Bose gas they are calculated in an additional perturbation theory in interaction between particles. An analyze of the results with connection to the Landau theory of phase transitions is given.

[1] Akhiezer, A.I. Methods of Statistical Physics [Text] / A.I. Akhiezer, A.I. and S.V. Peletminskii. – Oxford: Pergamon Press, 1981. – 450 p.

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