

Tumour growth model: Lie symmetries and exact solutions

Wednesday, 5 December 2018 16:00 (20 minutes)

We examine the tumour growth model proposed in [1]. In the 2D case, the governing equations after some simplifications take the form

$$\begin{aligned} \alpha_t + (\alpha u^1)_x + (\alpha u^2)_y &= S(\alpha), \quad u_x^1 + u_y^2 = \nabla \cdot (D(\alpha) \nabla p), \\ \left[(2 + \lambda) \alpha u_x^1 + \lambda \alpha u_y^2 \right]_x + \left[\alpha u_y^1 + \alpha u_x^2 \right]_y &= p_x + (\alpha \Sigma(\alpha))_x, \\ \left[\alpha u_y^1 + \alpha u_x^2 \right]_x + \left[(2 + \lambda) \alpha u_y^2 + \lambda \alpha u_x^1 \right]_y &= p_y + (\alpha \Sigma(\alpha))_y, \end{aligned} \quad (1)$$

where D , S and Σ are some functions and their typical forms are listed in [1]. Assuming that the tumour boundary is prescribed by a curve $\Gamma(t, x, y) = 0$, where Γ is an unknown function, the boundary conditions have the form

$$\begin{aligned} u^1 \Gamma_x + u^2 \Gamma_y &= -\Gamma_t, \quad p = 0, \\ \left[(2 + \lambda) u_x^1 + \lambda u_y^2 \right] \Gamma_x + \left[u_y^1 + u_x^2 \right] \Gamma_y &= 0, \\ \left[u_y^1 + u_x^2 \right] \Gamma_x + \left[(2 + \lambda) u_y^2 + \lambda u_x^1 \right] \Gamma_y &= 0. \end{aligned} \quad (2)$$

So, we have the nonlinear boundary value problem (1)-(2) with the unknown moving boundary $\Gamma(t, x, y) = 0$. Using the definition proposed in [2] and assuming Γ to be a closed curve for any $t \geq 0$, we examined the Lie symmetry and constructed the exact solutions of the boundary value problem (1)-(2). For instance, the following statement takes place:

The system of nonlinear PDEs (1) with arbitrary functions D , S and Σ is invariant with respect to the infinite-dimensional Lie algebra generated by the Lie symmetry operators

$$\begin{aligned} \partial_t, \quad F(t) \partial_p, \quad G_g = g(t) \partial_x + \dot{g} \partial_{u^1}, \quad G_h = h(t) \partial_y + \dot{h} \partial_{u^2} \\ J_f = f(t) \left[y \partial_x - x \partial_y + (u^2 + \frac{\dot{f}}{f} y) \partial_{u^1} - (u^1 + \frac{\dot{f}}{f} x) \partial_{u^2} \right]. \end{aligned}$$

Here F , f , g , and h are arbitrary smooth functions and the upper dot means differentiation with respect to time.

[1] H. Byrne, J.R. King, D.L.S. McElwain, L. Preziosi. A two-phase model of solid tumour growth. *Appl. Math. Letters*. **16** (2003) 567-573.

[2] R. Cherniha, S. Kovalenko. Lie symmetries and reductions of multi-dimensional boundary value problems of the Stefan type. *J. Phys. A: Math. Theor.* **44** (2011) 485202 (25 pp.)

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Session Classification: Mathematical Physics

Track Classification: Mathematical Physics