National Academy of Sciences of Ukraine
Bogolyubov Institute for Theoretical Physics

International Conference

XI Bolyai-Gauss-Lobachevsky (BGL-2019) Conference:
Non-Euclidean, Non-Commutative Geometry and Quantum Physics

19 May - 24 May, 2019

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Abstracts

Zero-order processes in QED with inverse square electric fields

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In this talk, I review general aspects about zero-order processes in Quantum Electrodynamics with t-electric steps as external backgrounds and discuss, in particular, the most important quantities characterizing vacuum instability, such as the differential mean numbers of pairs created from the vacuum, total numbers and the probability of a vacuum remains a vacuum. All these quantities are specialized to a new exactly solvable case, referred to inverse square electric field, that corresponds an electric field inversely proportional to time squared.

Scattering d-waves on distorted black holes

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We consider a generalization of the Regge-Wheeler equation for the case of distorted black holes in Minkowski and AdS spaces, the metric potential of which obeys the Liouville equation [1]. The absorption cross-section is computed for spin-2 particles (the axial perturbations over the background metric) in the small black hole and long-wave approximations. The subsequent analysis of the problem results in finding the natural restriction for the maximum angular momentum of the scattering/absorbed waves and in establishing the spectrum of the absorbed frequencies in AdS_4 space. Finally, we find a good agreement of the established results with that of early obtained in [2].

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Supersymmetric action for multiple D0-brane system

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We will describe a complete action for the system of N D0-branes in flat 10D type IIA superspace. It is invariant under the rigid spacetime supersymmetry and local worldline supersymmetry (κ–symmetry). This latter can be considered as supersymmetry of maximal 1d SU(N) SYM model which is made local by coupling to supergravity induced by embedding of the center of energy worldline into the target superspace.

Statement. *The talk is based on [1].*

This work was supported in part by the Spanish MINECO/FEDER (ERDF EU) grant FPA 2015-66793-P, by the Basque Government Grant IT-979-16, and the Basque Country University program UFI 11/55.

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Categories: between cubes and globes

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For a finite partially ordered set I we define an abstract polytope $\mathcal{P}_I$ which is a cube or a globe in the cases of discrete or linear poset respectively. Then for a poset P we build a small category $\Diamond_P$ with finite lower subsets in P as objects. This category $\Diamond_P = \Diamond_P^+ \Diamond_P^-$ is factorized into a product of two wide subcategories $\Diamond_P^+$ of *faces* and $\Diamond_P^-$ of *degenerations*. One can imagine a degeneration from I to $J \subset I$ as a projection of abstract polytope $\mathcal{P}_I$ to the subspace spanned by J. Morphisms in $\Diamond_P^+$ with fixed target I are identified with faces of $\mathcal{P}_I$. Composition in $\Diamond_P$ admits natural geometric interpretation.

On the category $\widehat{\Diamond_I}$ of presheaves on $\Diamond_I$ we construct a monad of free category in two steps: for a terminal presheaf the free category is obtained via a generalized nerve construction; for a general case cells of the nerve are colored by elements of the initial presheaf. Strict P-fold categories are defined as algebras over this monad.

All constructions are functorial in P. The usual theory of globular and cubical higher categories in natural way can be translated into our general context.

Effective Hadronic Supersymmetry

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Division algebras, specifically octonions, Jordan and related algebras have arisen in a concerted manner in unified theories of basic interactions, interwoven with supersymmetry, realized linearly and non-linearly with superstrings and supermembranes. We show that the quark model with potentials derived from QCD, including quark-diquark model for excited hadrons gives mass formulae in very good agreement with experiment and goes a long way in explaining the approximate symmetries and supersymmetries of the hadronic spectrum, including the symmetry breaking mechanism.

Helstrom bound with non-standard coherent states

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In quantum information processing, when one tries to distinguish between two non-orthogonal states through some receiver device, there exists a quantum error probability. The latter is bounded below by a quantum limit named Helstrom bound. In this talk we analyze and compare quantum limits for states which generalize the Glauber-Sudarshan coherent states, like non-linear and (modified) Susskind-Glogower coherent states. We show that for the latter ones the Helstrom bound can be significantly lowered.

Radiation reaction and action-at-a-distance of point-like particles in de Sitter space

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Developments of astrophysics of the expanding Universe makes it actual classical problems of electrodynamics in de Sitter space.

A charged particle radiates when it is accelerated. The radiation carries energy, momentum, and angular momentum. The lost of this conserved quantities modifies particles trajectory [1]. We use the covariant electromagnetic Green function in de Sitter space [2] in order to derive equation of motion of a point charge in an external electromagnetic field where the radiation reaction is taken into account. We give the details of the calculations for the specific case when the particle is accelerated by a static electromagnetic field.

Similarly to the case of flat space-time, the electromagnetic interaction of particle systems in curved space-time can be treated as an action-at-a-distance [3].

Classical mechanics of interacting particles in the de Sitter space-time is constructed in frame of the formalism of Fokker action integrals. It is generalized to a wide class of interactions. The well known examples of scalar and electromagnetic interactions are particular cases. It is known that for the time-asymmetric retarded-advanced electromagnetic and the other interactions the Fokker-type action in Minkowski space $\mathbb{M}_4$ reduces to the Lagrangian one [4]. Here this is shown for de Sitter space. Using the representation of de Sitter space-time as a hyperboloid in the 5-dimensional Minkowski space $\mathbb{M}_5$ we construct the covariant action principle for time-asymmetric particle dynamics with constraint and Hamiltonian description. It is invariant with respect to de Sitter group and integrable. A formal solution for this dynamics is built.

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On classification of symmetry reductions and invariant solutions for the Euler-Lagrange-Born-Infeld equation

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We study a connection between the structural properties of the low-dimensional ($dimL \leq 3$) nonconjugate subalgebras of the Lie argebra of the generalized Poincaré group $P(1,4)$ and the results of symmetry reductions for the Euler-Lagrange-Born-Infeld equation.

To classify symmetry reductions and invariant solutions of the above mentioned equation, we need a classification of the non-singular manifolds in the space $M(1,3) \times R(u)$, which are invariant with respect to the low-dimensional ($dimL \leq 3$) nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$. To perform this classification, we suggested the use of the structural properties of those subalgebras. Here, and in what follows, $M(1,3)$ is the four-dimensional Minkowsky space, $R(u)$ is the real number axis of the dependent variable u.

To date, we have performed the classification of non-singular manifolds in the space $M(1,3) \times R(u)$ invariant with respect to three-dimensional nonconjugate subalgebras of the Lie algebra of the group $P(1,4)$ and we have used the results obtained for the classification of symmetry reductions and invariant solutions for the Euler-Lagrange-Born-Infeld equation.

In my report, I plan to present some of the results obtained.

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Footprint of spatial noncommutativity in resonant detectors of gravitational wave

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The present day gravitational wave (GW) detectors strive to detect the length variation $\delta L = hL$, which, owing to the smallness of the metric perturbation $\sim h$, is an extremely small length $\sim 10^{-18} - 10^{-21}$ meter. The recently proposed noncommutative structure of space has a characteristic length-scale $\sqrt{\theta}$ which has an estimated upper bound in similar length-scale range. We therefore propose that GW data can be used as an effective probe of noncommutative structure of space and demonstrate how spatial noncommutativity modifies the responding frequency of the resonant mass detectors of GW and also the corresponding probabilities of GW induced transitions that the phonon modes of the resonant mass detectors undergo. In this paper we present the complete perturbative calculation involving both time-independent and time-dependent perturbation terms in the Hamiltonian.

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Hamiltonians built from pseudo-hermitian position/momentum operators, and nonlinear Bogolyubov transformations

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In the recent works [1], [2], [3], the two- and three-parameter (p, q)- and (p, q, μ)-deformed extensions of the Heisenberg algebra (HA) were studied. The two parameters p and q appeared due to (p, q)-deformation $[pXP - qPX]$ of the commutator in the l.h.s. of basic relation $[X, P] = i\hbar$, while the parameter μ enters through the extra term $i\hbar\mu H$, involving the Hamiltonian H, in its r.h.s. For the both deformed algebras, we explored and proved their realizability in terms of the corresponding nonstandard (i.e. other than well known ones) deformed quantum oscillator algebras, each being defined by its specified deformation structure function $\phi(N)$. There appears unusual aspect that concerns the 3-parameter case: namely, it turns out [1] that the multiplier μ in the deformed HA depends explicitly on the excitation number operator N. Such N-dependence of μ is special for the two-sided (p, q, μ)-deformation of HA just due to its *connectedness with*, or realizability by, respective deformed oscillator (DO) algebras.

The most important property stemming from just mentioned connectedness between those algebras is that such link dictates unusual $\eta_a(N)$-pseudo-Hermitian conjugation rule for the creation and annihilation operators, with $\eta_a(N)$ depending on the excitation number operator N (note that usually η-pseudohermitian quantum mechanics exploits the η being a function of P, or X, or the both, see e.g. [4]). In turn, that leads to the related $\eta(N)$-pseudo-Hermiticity of the position/momentum operators. Different possible cases are studied, and interesting features implied by the use of such $\eta(N)$-based conjugation rule are inferred. It turns out that it is possible to construct (i) the Hermitian Hamiltonian $\mathcal{H}$ and (ii) the Hamiltonian which is $\eta_H(N)$-pseudo-Hermitian (the two Hamiltonians look rather similar). The both Hamiltonians are nontrivial generalizations of the familiar operator $\frac{1}{2}(P^2 + X^2)$ of quantum harmonic oscillator.

As another important task, we develop our main tools needed for further study — the generalized nonlinear (with "coefficients" nonlinearly depending on the excitation number operator N) one-mode Bogolyubov transformations. Its application enables to obtain the spectrum [5] of our "almost free" (but essentially nonlinear) Hermitian $\mathcal{H}$. The slightly more involved case of the $\eta_H(N)$-pseudo-Hermitian Hamiltonian built from the same, as in the (i) case, $\eta(N)$-pseudo-Hermitian position and momentum operators, is considered as well.

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Time reversal symmetry in noncommutative space and parameters of noncommutativity

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Time reversal symmetry is examined in noncommutative phase space of canonical type. As an apparent example of time reversal symmetry breaking in the space we study the circular motion. We conclude that because of noncommutativity of coordinates and noncommutativity of momenta the period of the motion depends on its direction.

To recover the time-reversal and rotational symmetries in noncommutative phase space we propose to construct tensors of noncommutativity involving additional coordinates and additional momenta. As a result noncommutative algebra which is rotationally-invariant and invariant under the time reversal is constructed. Besides the algebra is equivalent to noncommutative algebra of canonical type.

In addition we show that the weak equivalence principle is recovered in noncommutative phase space if we consider the tensors of noncommutativity corresponding to a particle to be proportional inversely to its mass [2]. It is worth noting that the idea to consider parameters of deformed algebra to be dependent on mass gives a possibility to obtain the list of important results in deformed space with minimal length [3,4], in Lie-algebraic noncommutative space [5], in twist-deformed spacetime [6]. Among the results are solution of problem of description of macroscopic body motion, recovering of the weak equivalence principle.

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Polarization of the vacuum of quantized spinor field by a topological defect in two-dimensional space

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Two-dimensional space with a topological defect is a transverse section of three-dimensional space with the Abrikosov-Nielsen-Olesen vortex, i.e. a gauge-flux-carrying tube which is impenetrable for quantum matter. Charged spinor matter field is quantized in this section with the most general mathematically admissible boundary condition at the edge of the defect. We show that a current and a magnetic field are induced in the vacuum. These vacuum polarization effects depend periodically on the value of the gauge flux, providing a quantum-field-theoretical manifestation of the Aharonov-Bohm effect. We find that the requirement of finiteness of the total induced vacuum magnetic flux restricts the set of boundary conditions at the edge of the topological defect.

THE GAUSS'S, THEOREMA EGREGIUM

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To understand Gauss's theory of the non-Euclidean geometry we have to reestablish some definitions of the coordinate system, and also introduce the so called Gaussian coordinates. We show here that the two points distance as a postulate can establish a metric geometry. If we able to show the validity of this postulate on any surface than its have his own geometry, not necessarily Euclidean. Gauss showed in The Theorema Egregium that a surface have such attributes. The different geometries of the regular surfaces written here: Euclidean, spherical, and hyperbolic. This theorem has been presented at 1827. (Based on the lectures of K. Lanczos: Department of Physical Sciences and Applied Mathematics, North Carolina State University, Raleigh, 1968.) The importance of this lecture is to make clear and understandable how and why the physicians use non-Euclidean geometry.

Dalitz decays of unflavored mesons in Poincare covariant quark model

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In the course of the work, the authors, using the previously obtained parameters of the proposed model [1,2], investigate the behavior of the decay $V \to P\ell^+\ell^-$ constant for various dilepton momentum. As a result, a comparative analysis of the decay constant was performed taking into account the anomalous magnetic moments of quarks with both experimental values and the VMD-model.

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On Solutions of the Fuji-Suzuki-Tsuda System

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We derive Fredholm determinant and series representation of the tau function of the Fuji–Suzuki–Tsuda system and its multivariate extension, thereby generalizing to higher rank the results obtained for Painlevé VI and the Garnier system. As a byproduct we obtain a direct isomonodromic proof of the AGT-W relation for W_N-algebras with central charge $c = N - 1$.

The talk is based on the results of papers [1,2].

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Integral quasi-potential equations in the relativistic configuration representation

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In spite of the integral quasi-potential equations in the momentum representation (MR) are widely used, integral quasi-potential equations in the relativistic configuration representation (RCR) have their advantages. Three-dimensional equations can be reduced to partial equations for a specific value of the orbital quantum number under consideration of local quasipotentials in the Lobachevsky momentum space. In the MR, the partial expansion problem reduces to a search for partial potentials. In the RCR, it is necessary to have the explicit form of the partial waves and the partial Greens functions.

New determination of relativistic partial waves and explicit form of the partial free Green's functions of the quasi-potential Logunov-Tavkhelidze, Gross, and Kadyshevsky equations in the RCR for two-particle systems with an arbitrary orbital angular momentum were recently established [1]. It opens new opportunities for describing relativistic two-particle systems. The latest results of our progress will be presented.

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Two particle bound state problem in case of the Lobachevsky-local Coulomb potential

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Numerical solutions of the quasipotential type equations describing coupled spherically-symmetric states of two scalar particles in case of the Lobachevsky-local Coulomb potential are obtained in the relativistic configuration representation. The elastic scattering form-factors and the decay constants of two-particle systems into two photons are calculated on the basis of the solutions found. It is determined that the number of zeros of elastic form-factors is equal to the "number of the system quantum state minus one". The obtained spectra of energy and decay constants are compared with experimentally measured values for positronium.

Galaxy rotation curves in the μ-deformation based approach

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Recently, a model of dark matter (DM) viewed as μ-Bose condensate, an analogue of Bose-Einstein condensate (BEC) occurring within certain μ-deformed Bose gas model, was proposed [1] such that it possesses some advantages with respect to the well-known BEC DM model [2]. First, its prediction of the total mass of galactic DM halo is in a better agreement with observations with respect to the BEC DM model. Second, the critical temperature of condensation $T_c(\mu)$ is higher in the deformed case, that makes the condensate state (phase) more stable against possible heating sources.

In the present work, we consider galaxy stellar kinematics in the framework of the μ-deformed model. Our approach bases on the deformation of the Lane-Emden (LE) equation. As it is characteristic for any deformed models, different versions of particular μ-deformation of the initial model can be proposed. Herein we consider two simplest types of deformation. Namely, the first one implies minimal μ-deformation of the LE equation resulting from replacement of radial derivative with its μ-deformed generalization. The second μ-analogue of LE equation is derived by means of the requirement that it admits as its solution the simplest possible modification of the well-known solution of LE equation, that was used for DM density profiles previously.

Then, least square fitting of the observational low surface galaxies rotation curves with the model predictions has been performed. As result, we obtain a better agreement between our theoretical rotation curves and observational data, within the both types of deformed model, in comparison with classical BEC DM. Moreover, this tendency remains valid even in the case if the deformation parameter is fixed at some critical (characteristic) value.

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Free motion of the particles in the Lobachevsky space in the term of the scattering theory

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The problems of the motion of a free particles in the three dimensional Lobachevsky space are interpreted as scattering by space. The classical and quantum-mechanical cases are considered. A mechanical interpretation of parallel straight lines of Lobachevsky space is given as the trajectories of non-interacting material points emitted from a point at infinity. Due to properties of parallel lines in the Lobachevsky space, they can be considered as trajectories of particles scattered at an infinitely distant point. The concept of differential scattering cross sections in the horosphere element for the classical and quantum-mechanical problems is introduced. An analytical expression for the differential cross section in the quantum-mechanical problem is obtained. To derive this expression, we used solutions of the Schrödinger equation in horospherical coordinates.

The quantum-mechanical case is considered also on the basis of the integral equation derived from the Schrödinger equation. After the separation of variables in quasi-Cartesian coordinate system, the integral equation is derived for movement along the axis of symmetry horosphere, which coincides with the z axis. The relationship between the scattering amplitude and analytical functions is established. The iteration method and finite differences for the solution of the integral equation is proposed.

It is noted that part of the horosphere is a secant beam of parallel trajectories, can be considered as a model of a two-dimensional flat universe in three-dimensional space with curvature – Lobachevsky space.

Geometry of quantum state of spin-s system with long-range Ising-type interaction

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The evolution of a spin-s system described by the long-range Ising-type interaction is studied. We explore the topology of the quantum state manifold defined by this evolution and show that it corresponds to a sphere [1]. The speed of evolution of the system is calculated. The conditions for achieving the minimal and maximal values of the speed of evolution are obtained. We examine the properties of such system in the thermodynamic limit. Also in the case of spin-1/2 system we study the entanglement of one spin with the remaining system [2]. The scalar curvature of the quantum state manifold is represented by speed of evolution [1] and entanglement [2] of the system. This is important for an experimental measurement of the curvature. We apply the results to the physical system of methane molecule. Finally, the influence of an external magnetic field on the metric of the state manifold and on the speed of evolution is studied. So, the conditions on the magnetic field which should be applied to the system for achieving the minimal and maximal possible speeds of evolution are obtained.

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Generalized uncertainty principle in quantum cosmology

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The Heisenberg uncertainty principle plays a fundamental role in quantum mechanics. Possible modifications of the Heisenberg uncertainty relations which take into account effects of gravity have been debated since the middle 1980's (see references in [1]). Here the new uncertainty relation is derived in the context of the canonical quantum theory with gravity for the case of the maximally symmetric space, where the geometrical properties of the system are determined by a single variable, namely the cosmic scale factor a. From the commutation relation $[a, \pi] = i\hbar$ between a and its conjugate momentum $\pi = -i\hbar\,\partial_a$, it follows the uncertainty relation [1]

$$\Delta a \Delta \pi \geq \frac{\hbar}{2} \tag{1}$$

in ordinary Heisenberg form, but it has a different physical meaning. The uncertainty relation (1) establishes a connection between fluctuations of the quantities which determine the intrinsic and extrinsic curvatures of the spacelike hypersurface in spacetime. By associating quantum operators to the scalar curvature ${}^{(3)}R$ and the extrinsic curvature tensor $K_{ij} = -\frac{1}{2}\partial^{(3)}g_{ij}/\partial\tau$, where ${}^{(3)}g_{ij}$ is the 3-metric and τ is the proper time, Eq. (1) can be written explicitly in terms of curvature fluctuations,

$$\Delta^{(3)}R\ \Delta K \gtrsim 4\pi\hbar\ \frac{|{}^{(3)}R|}{{}^{(3)}V}, \tag{2}$$

where $K = K_i^i$ and ${}^{(3)}V \sim \frac{4}{3}\pi a^3$ is the 3-volume of the measurement.

By supposing that the Einstein equations (with quantum correction terms) are valid in the quantum regime, one can reduce the uncertainty relation (1) to the Unruh's uncertainty relation between the metric and the curvature. Such a connection between Eq. (1) and the Unruh's uncertainty relation may be interpreted as clarifying the physical meaning of the latter.

The influence of gravity on quantum fluctuations of position Δx and momentum Δp of a test particle is considered. Under the assumption that the deviations Δa and $\Delta\pi$ can be represented in the form of the normal frequency functions of deviations of position $x - \langle x\rangle$ and momentum $p - \langle p\rangle$, the uncertainty relation (1) transforms into the generalized Heisenberg-type uncertainty relation with corrections to gravity,

$$\Delta x \geq \frac{1}{2}\left[\left(1 + \frac{\Lambda}{6}(\Delta x)^2\right)\frac{\hbar}{\Delta p} + \frac{G}{2c^3}\left(1 + \frac{\Lambda}{6}(\Delta x)^2\right)\Delta p\right], \tag{3}$$

where G is Newton's gravitational constant, Λ is a cosmological constant. In the case $\Lambda = 0$, the relation (3) reduces to the form arising from string theory.

The new generalized uncertainty relation of energy and time is formulated and its connection with the geometrical variables of the theory is shown.

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Quantum corrections to the dynamics of the gravitational system

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The study of matter-energy density distribution in a quantum gravitational system (QGS) is of interest in connection with the problem of the mechanism of nucleation of the expanding universe from the initial cosmological singularity point. We consider a model with a finite number of degrees of freedom, which, notwithstanding, may provide a reasonable framework for addressing the problems of quantum gravity. In the case of the maximally symmetric geometry with the Robertson-Walker metric, the geometric properties of the system are determined by a single variable, namely the cosmic scale factor a. The matter sector of the homogeneous isotropic QGS is taken in the form of a uniform scalar field ϕ. This field can be interpreted as a surrogate of all possible real physical fields of matter averaged with respect to spin, space and other degrees of freedom. In addition, it is accepted that QGS is filled with a perfect (reference) fluid in the form of a relativistic matter (radiation) which defines a material reference frame enabling us to introduce the time variable (recognize the instants of time).

The basic equations of the homogeneous QGS can be reduced to the following simple set of two differential equations in partial derivatives for the state vector Ψ [1,2]

$$\left(-i\partial_T - \frac{2}{3}E\right)\Psi = 0, \tag{1}$$

$$\left(-\partial_a^2 + \mathrm{k}a^2 - 2aH_\phi - E\right)\Psi = 0, \tag{2}$$

where T is a conformal time, $E = a^4\rho_\gamma$ is a real constant which is determined by the energy density of radiation ρ_γ, $\mathrm{k} = +1, 0, -1$ is the curvature constant, H_ϕ is the Hamiltonian of the field ϕ. It is shown that the state vector Ψ can be normalized to unity.

The state vector Ψ averaged over the states of matter ϕ has the form $\langle\phi|\Psi\rangle \sim \exp(iS(a))$, where the function S satisfies the generalized Hamilton-Jacobi equation [2]

$$(\partial_a S)^2 + \mathrm{k}a^2 - E = \frac{3}{4}\left(\frac{\partial_a^2 S}{\partial_a S}\right)^2 - \frac{1}{2}\frac{\partial_a^3 S}{\partial_a S}. \tag{3}$$

The right-hand side of the equation (3) is proportional to $\hbar^2$ (in ordinary physical units) and responsible for quantum corrections to the dynamics of the system.

The generalization of the wave equations to the domain of negative values of the cosmic scale factor is made. For the arrow of time from past to future, the state vector describes the QGS contracting for the negative values of the scale factor and expanding for its positive values. The intensity distributions of matter are calculated for two exactly solvable models of spatially closed and flat QGSs formed by dust and radiation. The analogies with the motion in time of minimum wave packet for spatially closed QGS and with the phenomenon of diffraction in optics for flat QGS are drawn.

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Generalized uncertainty from geometric superpositions

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Phenomenological approaches to quantum gravity implement a minimum resolvable length-scale but do not link it to an underlying formalism describing geometric superpositions. Here, we introduce an intuitive approach in which points in the classical spatial background are delocalized, or "smeared", giving rise to an entangled superposition of geometries. The model uses additional degrees of freedom to parameterize the superposed classical backgrounds. Our formalism contains both minimum length and minimum momentum resolutions and we naturally identify the former with the Planck length. In addition, we argue that the minimum momentum is determined by the de Sitter scale, and may be identified with the effects of dark energy in the form of a cosmological constant. Within the new formalism, we obtain both the Generalized Uncertainty Principle (GUP) and Extended Uncertainty Principle (EUP), which may be combined to give an uncertainty relation that is symmetric in position and momentum. Crucially, our approach does not imply a significant modification of the position-momentum commutator, which remains proportional to the identity matrix, in contradistinction to existing models in the literature. Implications for the black hole uncertainty principle correspondence and cosmology are briefly discussed, and prospects for future work on the smeared-space model are outlined.

Probing non-commutative quantum mechanics by nanoscale physics

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Abstract

The fundamental difficulties in theoretical physics, such as the singularity in particle physics, nonlocality and dark energy in quantum gravity, imply the existence of the finite length and time (Planck scale), by which physicist can understand consistently all phenomena in physics world[1] The finite Planck length and time unit hint the noncommutative spacetime, namely noncommutative algebra and geometry. [1] The noncommutative algebra and geometry can be generalized further to noncommutative phase space as a generalization of the Heisenberg algebra in quantum mechanics. The noncommutative concepts and techniques can be expected to improve the singularity problem in the particle physics and nonlocality in quantum gravity and quantum cosmology.[1] In fact, the noncommutative phase-space phenomena also arise in condensed matter physics, [1] such as the analogy between the two-dimensional electron gas in the presence of a magnetic field and the free-particle system in noncommutative phase space. [1] It implies that the noncommutative-space phenomenon can occur beyond the Planck scale even though physics in noncommutative space has not been understood completely. However, there has not been any direct experimental evidence to demonstrate the existence of noncommutative space. The difficulty to observe directly phenomena in noncommutative space is because the effects of noncommutative space or phase space are too weak in the Planck scale.

In this presentation we propose an experimental scheme based on the Aharonov-Bohm effect in the nano-scale system to probe the noncommutative phase space phenomena in the Planck's scale world. [2] We consider a nano-scale ring with an external magnetic field along the axis of the ring in noncommutative phase space. The Seiberg-Witten map makes this system equivalent to the ring with an extra effective magnetic flux in the quantum mechanical (Heisenberg algebra) phase space. We introduce two variables related to the persistent current in the ring to probe the noncommutative phase space effect. We give a value-independent criterion to detect the existence of the noncommutative phase space. [2] Namely the answer for existence or nonexistence of the noncommutative phase space depends only on the trend of the curves, rather than the values of the observation data. Since the persistent current and magnetic flux in mesoscopic rings have been implemented by nanotechnology,[3] it can be expected to detect the noncommutative phase-space effect by this scheme.

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Advances on Snyder spacetime

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We review recent results on the relativistic Snyder model. In particular we discuss the construction of a quantum field theory on Snyder background and the generalization to a curved spacetime.

Composite Quantum Particles as Deformed Oscillators: Wavefunctions, Entanglement and Implications

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Composite structure of particles is important, – since it somewhat modifies their statistics, compared to pure Bose- or Fermi-particles. Moreover, the spin-statistics theorem is not valid anymore. Say, π-mesons, excitons, Cooper pairs are not ideal bosons and, likewise, baryons are not pure fermions. Quasibosons composed of two bosons or two fermions were studied enough in detail, see e.g. [1-4]. In this work we develop analogous study [5] of composite Fermi-type particles (i.e. quasifermions), and explore them in two major cases: (i) "boson + fermion" composite quasifermions; (ii) "deformed-boson + fermion" composite quasifermions. As we show, in the both cases, quasifermions admit only the realization by ordinary fermions. While the case (i) was solved explicitly, and admissible wavefunctions found along with entanglement measures, the case (ii) was treated upto the first orders of approximation in deformation parameter of the constituent deformed boson. Entanglement entropy and purity of composite quasifermions are explored, their dependence on the relevant parameters established, and for some particular two- or three-mode cases depicted graphically. The entanglement entropy in the examined cases varies between zero and $\ln 2$ (or $\ln 3$). Formal comparison is performed with composite bosons, realized by deformed oscillators, in particular what concerns suitable wavefunctions, cf. [1,2], and entanglement entropy, see [3]. Possible application to such systems as trions or baryons is briefly discussed.

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Energy observable for a quantum system with a dynamical state space and a geometric extension of quantum mechanics

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We address the problem of defining the energy observable for a quantum system whose state space depends on time. The solution leads to a moderate geometric extension of QM where the role of the Hilbert space and the Hamiltonian operator is played by a complex Hermitian vector bundle E endowed with a metric-compatible connection and a global section of a real vector bundle determined by E. The axioms of QM are not replaced by others but elevated to the level of the relevant bundles. The standard description of quantum systems in terms of a Hilbert space and a Hamiltonian operator, which respectively determine the kinematical and dynamical properties of the system, is recovered locally, i.e., in local patches of E.

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Statistics Effects in Extremal Black Holes Ensemble

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We consider a grand canonical ensemble of the static and extremal black holes (EBHs), when "extremality"consists in equivalence of electric charge and mass of individual black hole [1-3]. Assuming a homogeneous distribution of black holes in space, we find the effective mass of test particle and the mean time dilation at the admissible points of space, taking into account the gravitational action of surrounding black holes. Using these characteristics, we are mainly studying an effect of quantum statistics, governing the extremal black holes. The type of quantum statistics was first discussed in [4] in the context of EBHs quantum properties. We concentrate here on the Bose–Einstein, the classical and the infinite statistics, when each of them specifies the statistical weight for a configuration of certain number of black holes. Using a mean field approximation, main termodynamical characteristics are calculated and displayed, what permits us to evaluate the visible effect of each statistics.

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Supersymmetry and superintegrabilitry of PDM Schroedinger equations

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The contemporary results concerning symmetries and supersymmetries in generalized Schrödinger equations are presented. Namely, position dependent mass Schrödinger equations admitting the standard Lie and generalized symmetries are classified, as well as the equations with matrix potentials. A number of superintegrable systems is presented together with their exact solutions.

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Dark sector in cosmology: Dark Matter, Dark Energy, Dark Ages

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In the brief review talk I will discuss the theoretical and observational aspects of the dark sector in cosmology: dark matter and dark energy, which are dominant energy density components of our Universe, as well as their possible manifestations in the Dark Ages and Cosmic Dawn. The first sources of light at the redshift $z \sim 10 - 50$ can be important fount of information about properties of dark matter and dark energy which give the possibility to shed a light on their physical nature. The intensity of Dark Ages halos in the redshifted 21 cm hyperfine structure line of atomic hydrogen and lines of the first molecules are sensitive to the free-streaming scale of dark matter particles and their mass and half-life, if they are decaying or self annihilating, as well as to the parameters of models of early dark energy. The key experiments, ongoing and upcoming projects will be mentioned.

Geometry meets Quantum: from phase transitions to black holes

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We review two remarkable examples of quantum description, implementation of which exploit the hidden/manifest geometry of the physical problem. The first one is the so-called quantum metric and quantum curvature, widely used in classification of quantum phase transitions. The second one is the loop corrections to the standard General Relativity action, account of which becomes important in resolving some of the long-standing problems of Black Holes Physics.

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Wave optics in Kerr space-time

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For obtaining polarization effects in the Kerr space-time we have used algebraically special approach for Maxwell equations [1]. As a consequence, in Kinnersley tetrad Maxwell field is described only by one extremal component φ_2, and the Maxwell equations have closed-form solution [2]

$$\varphi_2 = C\frac{e^{i\omega(t-\tilde{r})+im\phi}}{\sin\theta(r-ia\cos\theta)}e^{-a\omega\cos\theta}\left(\frac{1-\cos\theta}{\sin\theta}\right)^m, \qquad (1)$$

$t > 0$, $r_+ < r < \infty$, $0 < \theta < \pi$, $0 \le \phi < 2\pi$, $\tilde{r} = r + M\ln\Delta + \frac{M^2}{\sqrt{M^2-a^2}}\ln\left(\frac{r-r_+}{r-r_-}\right) + \frac{am}{2\omega\sqrt{M^2-a^2}}\ln\left(\frac{r-r_+}{r-r_-}\right)$, $\omega \in \mathbb{R}$ is a frequency of the wave, $m \in \mathbb{Z}$ is an azimuthal number, M is a mass of gravitating body, a is an angular momentum per unit mass ($a < M$), $\Delta = r^2-2Mr+a^2$, $r_+ = M + \sqrt{M^2-a^2}$, $r_- = M - \sqrt{M^2-a^2}$, C is a complex constant.

From the solution (1) for outgoing waves we have obtained formulas for Stokes parameters, ellipticity angle and polarization angle, and gravitational analog of Faraday effect. There are distinguished two polarization effects in Kerr field. The first one is the rotation of the plane of polarization (RPP), and the second is the influence of angular momentum of rotating body on an amplitude of right or left circularly polarized waves, discovered for low and high frequencies in [3,4]. Obtained in our approach results confirm the formula of Gnedin and Dymnikova [5] for the RPP, dispersion of the RPP is absent. The influence of angular momentum on amplitude is established by a closed-form expression in the full range of frequencies.

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On reaction of a spinning particle on the spacetime curvature

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For description of the physical meaning of the Riemann tensor in general relativity it is appropriate to take into account the definition of the so called gravitoelectric $E^{(i)}_{(k)}$ and gravitomagnetic $B^{(i)}_{(k)}$ components of the gravitational field [1]

$$E^{(i)}_{(k)} = R^{(i)(4)}{}_{(k)(4)}, \tag{1}$$

$$B^{(i)}_{(k)} = -\frac{1}{2} R^{(i)(4)}{}_{(m)(n)} \varepsilon^{(m)(n)}{}_{(k)}, \tag{2}$$

where the indices in the parentheses correspond to the local tetrad coordinates, i and k run 1, 2, 3. Components (1) determine the tidal forces, whereas components (2) cause the spin-gravity coupling: this result follows directly from the Mathisson-Papapetrou equations in the representation of the comoving tetrads. It is important that the values of $E^{(i)}_{(k)}$ and $B^{(i)}_{(k)}$ significantly depend on the velocity of the observer relative to the source of the gravitational field. For example, in the case of a Schwarzschild mass the components of $E^{(i)}_{(k)}$ and $B^{(i)}_{(k)}$ are proportional to γ or γ^2, where γ is the Lorentz factor as calculated by an observer who is at rest relative to the source of the gravitational field.

In the linear spin approximation the 3-acceleration $a_{(i)}$ of the deviation of the spinning particle from free geodesic fall, measured by the comoving observer, is determined by the expression

$$a_{(i)} = \frac{s_{(k)}}{m} B^{(k)}_{(i)}, \tag{3}$$

where $s_{(k)}$ is the components of the spin of the particle and m is its mass. The absolute value of the 3-acceleration is proportional to γ^2, i.e. this value becomes much greater for very high velocities.

Using the Mathisson-Papapetrou equations we investigated different cases of significant influence of the spin-gravity coupling on trajectories of the highly relativistic spinning particle in the Schwarzschild and Schwarzschild-de Sitter spacetimes [2-5].

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Fradkin Equation for a Spin 3/2 Particle in Presence of External Electromagnetic and Gravitational Fields

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Within the general theory of the relativistic wave equations, in addition to simplest and commonly used equations for particles of spins 0, 1/2, 1, 3/2, and 2 many other and more complicated equations may be proposed. These generalized models are based on the use of extended sets of irreducible representations of the Lorentz group. There are known equations with several mass parameters (S=0, 1/2, 1), with several spin parameters, with additional intrinsic electromagnetic characteristics: anomalous magnetic moment (S=1/2, 1, 2), quadrupole moment (S=1), polarisability (S=0,1), Darwin-Cox structure (S=0), and other. Such additional intrinsic characteristics physically manifest themselves in presence of external fields, for instance electromagnetic and gravitational. Many year ago, a special generalized model for a spin 3/2 particle different from Pauli–Fierz and Rarita–Schwinger model, was proposed by Fradkin. To the present time it is not clear which additional structure underlies this extended wave equation. We investigate this model systematically, applying the general Gel'fand–Yaglom formalism. Applying standard set of requirements: relativistic invariance, single nonzero mass and spin S=3/2, P-symmetry, existence of Lagrangian for the model, we derive a set of spinor equations, first in absence of external fields. The 20-component wave function consists of bispinor and vector-bispinor. It is shown that in free case the Fradkin extended model reduces to minimal Pauli–Fierz and Rarita–Schwinger model. Then we take into account the presence of external electromagnetic fields, it turn out that in minimal form the Fradkin equation contains a special and additional interaction term governed by electromagnetic tensor $F_{\alpha\beta}$. Finally we take into account the external curved space-time background, in generally covariant case the Fradkin equation contains additional gravitational interaction term, governed by Ricci tensor $R_{\alpha\beta}$. If the electric charge of the particle is zero, The Fradkin model remains correct and describes a neutral spin 3/2 particle of Majorana type interacting additionally with geometrica background through Ricci tensor.

Fermi-gas with nonadditive statistics

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We analyze two approaches to the generalization of the Fermi-distribution using the nonadditive Tsallis q-exponential

$$e_q^x = \begin{cases} e^x & \text{for } q = 1, \\ [1+(1-q)x]^{1/(1-q)} & \text{for } q \neq 1 \text{ and } 1+(1-q)x > 0, \\ 0^{1/(1-q)} & \text{for } q \neq 1 \text{ and } 1+(1-q)x \leq 0. \end{cases}$$

Various approaches are known in the literature to make such a generalization [1]. We consider the Gibbs factor in the expressions for occupation numbers written in the following two ways,

$$n_1(\varepsilon,\mu,T) = \frac{1}{e_q^{(\varepsilon-\mu)/T}+1} \qquad \text{and} \qquad n_2(\varepsilon,z,T) = \frac{1}{z^{-1}e_q^{\varepsilon/T}+1}.$$

Since the factorization rule breaks for the q-exponentials, $e_q^{x+y} \neq e_q^x e_q^y$, one cannot establish a simple connection $z = e^{\mu/T}$ between the chemical potential and fugacity.

Having obtained μ or z from

$$N = \int_0^\infty g(\varepsilon) n_{1,2}(\varepsilon,\cdot,T)\, d\varepsilon,$$

where the density of states $g(\varepsilon) = NA\varepsilon^{s-1}$, we can calculate energy

$$E = \int_0^\infty \varepsilon\, g(\varepsilon) n_{1,2}(\varepsilon,\cdot,T)\, d\varepsilon \qquad (1)$$

and other thermodynamic functions. In the figure to the right, the isochoric specific heat is shown.

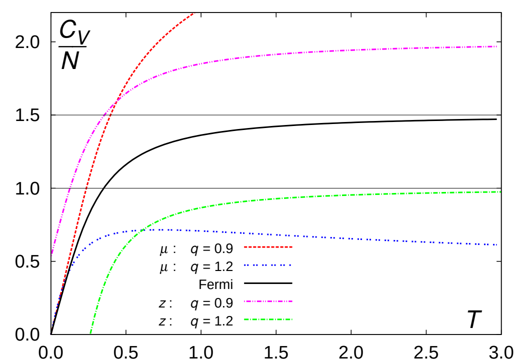

Figure: Isochoric specific heat for different values of q compared to the ideal Fermi gas.

Some peculiarities of the two models will be discussed. In particular, $C_V \sim T^\gamma$ (not constant, see [2]) at high temperatures for the n_1 case and finite (non-zero) minimum temperatures for n_2 at $q > 1$, cf. [3]. The results from the second case, though exotic from the first glance, might be applicable in effective modeling of physical phenomena in various domains [4,5].

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Quasiposition Representation in Deformed Space with Minimal Length

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In quantum mechanics the introduction of a minimal length can be accomplished through a generalization of Heisenberg's uncertainty principle. Generalized uncertainty principle, in turn, can be obtained by modification of the usual canonical commutation relations. An important feature of quantum theory with minimal length is that eigenstates of the position operator are no longer physical states. As a result, we cannot work with the position representation anymore. One of the possible ways to proceed is to use the quasiposition representation.

In this work, we consider the properties of the quasiposition representation and discuss the problem of discription of the singular potentials in quasiposition space. We also obtain a solution of the Schroedinger equation with Dirac -function potential in quasiposition representation, which resolves the discussion presented in [1-3].

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Early versus late inflation: similarities and differences

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Current accelerated expansion of the Universe is sometimes called the "late inflation" by analogy with the inflationary stage in the early Universe. We constrain jointly the parameters of dynamical dark energy and the contribution of tensor-mode perturbations for the slow-roll inflation scenario. The dark energy is assumed to be a minimally coupled classical scalar field with barotropic equation of state. The used datasets include Planck data on CMB anisotropy and lensing, BICEP2/Keck Array data on B-mode polarization, BAO from SDSS and 6dFGS, RSDs from BOSS-DR12, weak lensing from DES and SN Ia data from the Pantheon compilation. For each obtained set of cosmological parameters we reconstruct the potential of a dark energy scalar field. We discuss the similar and different properties of the inflaton and dark energy scalar fields.

Inflationary Magnetogenesis with Helical Coupling

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We consider helical coupling to electromagnetism with the Lagrangian

$$\mathcal{L}_{\rm em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} f F_{\mu\nu} \tilde{F}^{\mu\nu} \,, \tag{1}$$

where $\tilde{F}^{\mu\nu}$ is the Hodge dual of $F^{\mu\nu}$, and present a simple scenario of evolution of the coupling function f leading to a viable inflationary magnetogenesis without the problem of back-reaction. In our scenario, the coupling $f(\eta)$ evolves linearly with the conformal time η, interpolating between two constant values in the past and in the future. Since the absolute value of f does not have any significance (the second term in (1) with constant f is topological), the strong-coupling problem does not arise in this model. The duration of the transition $\Delta\eta$ and the corresponding change Δf are the two parameters of the model that can be adjusted to produce magnetic field of any strength in a narrow spectral band centered at any reasonable comoving wavenumber $k_{\rm m} = |f'|/2 = |\Delta f|/2\Delta\eta$. Constraint on the magnitude of the magnetic field comes from the considerations of back-reaction on the inflationary dynamics and, in the simple inflation based on a massive scalar field, allows for production of magnetic fields with extrapolated current values up to $B_0 \sim 10^{-7}\,$G. The dependence of $|\Delta f|$ on B_0 is logarithmic: for B_0 in the range 10^{-30}–$10^{-7}\,$G with spectrum peaked on the comoving scale $k_{\rm m} \simeq \mathrm{Mpc}^{-1}$, one requires $|\Delta f| \simeq 131$–238.

In our scenario, the generated electromagnetic field is close to maximally helical, with the magnetic and electric fields having the same magnitude and spectral densities. The electromagnetic spectrum is exponentially peaked around the comoving wavenumber $k_{\rm m} = |f'|/2$ with narrow width $\Delta k = k_{\rm m}/|\Delta f| \ll k_{\rm m}$. It is reasonable to think that evolution of the helical coupling in which f' slowly varies in time will produce electromagnetic field with spectrum in broader range of wavenumbers, spanned by the values of $|f'|/2$.

Primordial helical hypermagnetic fields may be responsible for generating baryon asymmetry of the universe. This imposes a post-inflationary constraint

$$B_0 \lesssim 10^{-21} \left(\frac{\mathrm{Mpc}}{\lambda_0} \right)^{1/2} \mathrm{G} \tag{2}$$

on the admissible values of B_0 and $k_{\rm m}$ in our simple scenario of monotonic evolution of f. This constraint can probably be circumvented by assuming more complicated (non-monotonic) evolution of the coupling f producing magnetic fields of sufficient strength but with conveniently limited helicity. Other constraints on models of this type may arise from the considerations of the created baryon number inhomogeneities that can affect the cosmic microwave background and primordial nucleosynthesis and from the Schwinger effect.

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Symmetries of relativistic hydrogen atom

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The spin (1,0) Bose symmetry of the Dirac equation for the free spinor field, proved recently in [1, 2], is extended for the Dirac equation interacting with external Coulomb field. Relativistic hydrogen atom is modeling here by the equation

$$\left(i\partial_0 - \widehat{H}\right) = 0; \quad \widehat{H} \equiv \gamma^0 \vec{\gamma} \cdot \vec{p} + \gamma^0 m - \frac{e^2}{|\vec{x}|}. \tag{1}$$

Fermionic spin 1/2 SO(4) symmetry of equation (1) was found in [3, 4] and repeated in [5]. We are able to add both the fermionic and bosonic symmetries known from [1, 2] for the case of non-interacting spinor field. New symmetry operators were found on the basis of new gamma matrix representations of the Clifford and SO(8) algebras, which were presented recently in [6].

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Relativistic equations for arbitrary spin, especially for the spin s=2

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Further approbation of the equation for the particles of arbitrary spin introduced recently in [1–3] is under consideration. The comparison with the known equations suggested by Bhabha, Bargmann–Wigner, Rarita–Schwinger (for spin s = 3/2) and other authors is discussed. The advantages of the new equations are considered briefly. The advantages follow from the fact that our equation does not contain the redundant components. The important partial example of spin s=2 case is considered in details. The 10-component Dirac-like wave equation for the spin s=(2,2) particle-antiparticle doublet is suggested. The link with 10-component Maxwell-like equation for this doublet is considered.

The Poincaré invariance is proved. The way of introduction of interaction with external field is demonstrated in [1, 2].

The three level consideration (relativistic canonical quantum mechanics

$$\left(i\partial_0 - \sqrt{-\Delta + m^2}\right) f(x) = 0, \quad f \equiv \mathrm{column}(f^1, f^2, ..., f^{10}), \tag{1}$$

$$f(x) = \left| \begin{array}{c} f_{\mathrm{part}} \\ f_{\mathrm{antipart}} \end{array} \right| = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k e^{-ikx} b^{\mathrm{A}}(\overrightarrow{k}) \mathrm{d}_{\mathrm{A}}^{\mathrm{Cartesian}}, \quad \mathrm{A} = \overline{1, 10}, \tag{2}$$

canonical Foldy–Wouthuysen type field theory

$$\left(i\partial_0 - \Gamma_{10}^0 \sqrt{-\Delta + m^2}\right) \phi(x) = 0, \quad \Gamma_{10}^0 = \left| \begin{array}{cc} \mathrm{I}_5 & 0 \\ 0 & -\mathrm{I}_5 \end{array} \right|, \tag{3}$$

$$\phi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \left[e^{-ikx} g^{\mathrm{A}}(\overrightarrow{k}) \mathrm{d}_{\mathrm{A}}^{\mathrm{Cartesian}} + e^{ikx} g^{*\mathrm{B}}(\overrightarrow{k}) \mathrm{d}_{\mathrm{B}}^{\mathrm{Cartesian}} \right], \quad \mathrm{A} = \overline{1, 5}, \, \mathrm{B} = \overline{6, 10}, \tag{4}$$

and locally covariant field theory) is presented.

The operator link between the relativistic canonical quantum mechanics and locally covariant field theory of arbitrary spin is found. Such link is given by the extended Foldy–Wouthuysen transformation between the 2(2s+1)-component local field theory and the corresponding relativistic canonical quantum mechanics. The hypothesis on the spin s=5/2 particle is discussed.

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On Goldstone Fields of Spin Higher than 1/2

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We will discuss properties of 3d non-linear models of vector and vector-spinor Goldstone fields associated with spontaneous breaking of certain higher-spin counterparts of supersymmetry (so-called Hietarinta algebras) whose Lagrangians are of a Volkov-Akulov type. At the quadratic order these Lagrangians contain, respectively, the Chern-Simons and the Rarita-Schwinger term. The vector Goldstone model has a propagating degree of freedom which, in a decoupling limit, is a quartic Galileon scalar field (similar to those appearing in models of modified gravity). On the other hand, the vector-spinor goldstino retains the gauge symmetry of the Rarita-Schwinger action and eventually reduces to the latter by a non-linear field redefinition. We thus find that in three space-time dimensions the free Rarita-Schwinger action is invariant under a hidden rigid symmetry generated by fermionic vector-spinor operators and acting non-linearly on the Rarita-Schwinger goldstino.

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Inflation and pre-inflation in scalar-tensor and $f(R)$ gravity

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Several new results on inflation and pre-inflationary evolution of homogeneous cosmological models in scalar-tensor and $f(R)$ gravity are presented. The $R + R^2$ (Starobinsky) model where R is the Ricci scalar, augmented by small one-loop quantum gravitational corrections, represents the pioneer inflationary model [1] which still remains viable. It contains only one adjustable parameter taken from observations, has a graceful exit from inflation and a natural mechanism for creation and heating of matter after its end, and it produces a very good fit to existing observational data on the power spectrum of primordial scalar (adiabatic density) perturbations. More generally, all viable slow-roll inflationary models in $f(R)$ gravity should be close to this model over some range of R. It also represents a dynamical attractor for slow-rolling scalar fields strongly coupled to gravity, as well as for the mixed R^2-Higgs inflationary model [2]. We consider the inverse problem of reconstruction of inflationary models in $f(R)$ gravity using information on the power spectrum of scalar perturbations only, ambiguity in this procedure and how it can be fixed by some aesthetic assumptions on the absence of new physical scales during and after inflation. The forms of $f(R)$ for which exact constant-roll solutions generalizing slow-roll ones can be realized are found [3]. As follows from observational data on the primordial scalar (matter density) perturbation spectrum, running of the dimensionless coefficient in front of the R^2 term with curvature due to loop quantum-gravitational corrections is small and does not exceed a few percents [4]. The same refers to the $R\Box R$ correction considered perturbatively, without increasing the number of degrees of freedom [5]. Also studied is the problem of inflation formation from preceding generic classical curvature singularity, and which conditions are needed for this [6]. Some exact anisotropic solutions describing it are presented. Since this process is generic, too, for inflation to begin inside a patch including the observable part of the Universe, causal connection inside the whole patch is not necessary. However, it becomes obligatory for a graceful exit from inflation in order to have practically the same number of e-folds during inflation inside this patch.

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Geodesic structure of spherically symmetric space-times around astrophysical relativistic configurations with scalar fields

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We consider static spherically symmetric compact objects in General Relativity with minimally-coupled static scalar fields (SF). The focus is on SF Lagrangians with the canonical kinetic term and massive and massless monomial potentials. The solutions of the joint system of Einstein – SF equations are derived numerically under conditions of asymptotic flatness. Then we study the effective potentials that define properties of geodesics in this space-time. We show that the distribution of the stable circular orbits describing the test particle motions can be qualitatively different of the Schwarzschild case. Possible observational signals of these distributions are discussed.

Soccer-ball problem in quantum space

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Studies in quantum gravity and string theory suggest existence of minimal length or quantum of space. Such a space is called space with minimal length or quantum space and can be described with the help of deformation of the Heisenberg algebra. On the macroscopic level this leads to the deformed Poisson brackets. The problem of description of macroscopic bodies in a quantum space is known as a soccer-ball problem. Let us suppose that the behavior of elementary particles in the quantum space is known. Then we face the question: what is the behavior of the center-of-mass of composite macroscopic bodies which consist of elementary particles in quantum space? This problem is related to list of problems in a space with minimal length. Among them are the problem of violation of the equivalence principle, preserving of properties of the kinetic energy, finding Galilean and Lorenz transformation in quantum space. These questions are discussed and solution of the soccer-ball problem is proposed. The talk is based on the papers [1-3].

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Four-component integrable nonlinear Schrödinger system with background-controlled intersite interactions

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The most featured items characterizing the semi-discrete nonlinear Schrödinger system with background-controlled inter-site resonant interactions are summarized. The system is shown to be integrable in the Lax sense, that make it possible to obtain its soliton solutions in the framework of properly parameterized dressing procedure based on the Darboux transformation and the implicit Bäcklund transformation. The system integrability inspires an infinite hierarchy of local conservation laws, the key ones of which were found explicitly in the framework of generalized recursive approach. The system consists of two basic dynamic subsystems and one concomitant subsystem and it permits the Hamiltonian formulation accompanied by the highly nonstandard Poisson structure. The nonzero background level of concomitant fields mediates the appearance of an additional type of inter-site resonant coupling and as a consequence it establishes the triangular-lattice-ribbon spatial arrangement of location sites for the basic field excitations. Adjusting the governing background parameter, we are able to switch over the system dynamics between two essentially distinct regimes separated by the critical point. The physical implications of system criticality become evident after rather sophisticated canonization procedure of basic field variables. There are two variants of system standardization equal in their rights. Each variant is realizable in the form of two nonequivalent canonical subsystems. The broken symmetry between canonical subsystems gives rise to the crossover effect in the nature of excited states. Thus, in the under-critical region the system supports the bright excitations in both subsystems, while in the over-critical region one of subsystems converts into the subsystem of dark excitations.

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Contact Interactions in Heterostructures: A Squeezed Limit

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Abstract. The heterostructures composed of two and three parallel plane layers are studied in the squeezed limit as both the thickness of the layers and the distance between them simultaneously tend to zero. The presence of a squeezed prewell in the potential profile in the system is shown to be crucial for the appearance of non-zero tunneling transmission through zero-thickness heterostructures. The typical example of such a system is the bilayer for which the squeezed potential profile is the first derivative of Diracs delta function. It is demonstrated that the squeezed potentials are not required to have a limit in the sense of distributions and there exists a whole family of well-defined single-point interactions of one-dimensional Schrödinger equations for which the resonant tunneling transmission occurs at some hypersurfaces in the space formed by the layer strength constants (curves for a bilayer on the 2D plane and surfaces for a trilayer in the 3D space). These hypersurfaces form a countable (discrete) resonance set on which the transmissivity is non-zero, whereas beyond this set the system is fully opaque satisfying the Dirichlet boundary conditions at both the sides of the potential. The conditions for the resonant tunneling and the reflection-transmission coefficients for these interactions are computed explicitly. It is shown that for a given multilayer system, the resonance set crucially depends on the way of one-point shrinking. The notion of point (contact) interactions with bias potentials is introduced. The transmission matrix and reflection-transmission coefficients for biased point systems are interpreted and computed asymptotically. References below exemplify the subject.

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