Lambda spin polarization in QGP

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Primary References: arXiv:2011.14907, Phys.Rev.C 99, 044910 (2019)

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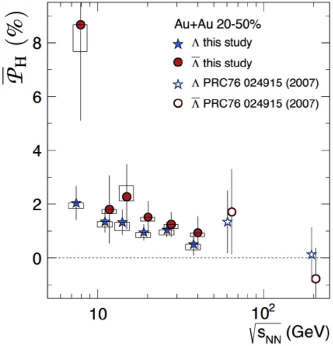
XI Conference of Young Scientists "Problems in Theoretical Physics"
Virtual

First positive measurements of global spin polarization of Λ hyperons by STAR



thermal approach $\longrightarrow$ $P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T}$ $\quad P_{\bar{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$

Becattini, F., Karpenko, I., Lisa, M., Upsal, I. , Voloshin, S. , PRC 95, 054902 (2017)

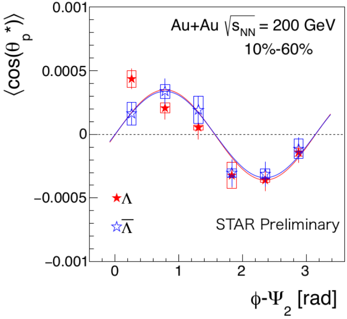
... the hottest, least viscous – and now, most vortical – fluid produced in the laboratory ...

$$\omega = \left(P_\Lambda + P_{\bar{\Lambda}}\right) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22} \mathrm{s}^{-1}$$

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

What?Why?How?: Why:

- Issue with longitudinal polarization.



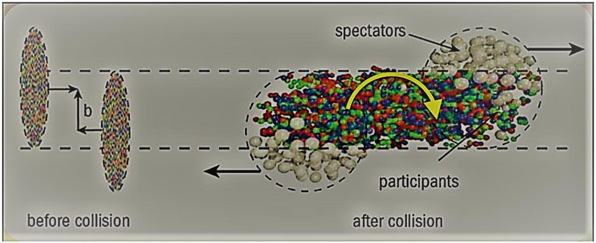
- This study will help to know the formation and characteristics of the QGP.
- Detecting and understanding the QGP allows us to understand better the universe in the moments after the Big Bang.

- Our approach: include spin degrees of freedom into the ideal standard hydrodynamics
- $J^{\mu,\alpha\beta}(x) = x^\alpha T^{\mu\beta}(x) - x^\beta T^{\mu\alpha}(x) + S^{\mu,\alpha\beta}(x)$
- And, conservation of total angular momentum, $\partial_\lambda J^{\lambda,\mu\nu}(x) = 0$ gives $\partial_\lambda S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$
- For symmetric energy-momentum tensor, $T^{\nu\mu}(x) = T^{\mu\nu}(x)$, we have $\partial_\lambda S^{\lambda,\mu\nu}(x) = 0$
- Hence conservation of the angular momentum implies the conservation of its spin part in de Groot, van Leeuwen, and van Weert (GLW) formulation.

Motivation:

- Non-central relativistic heavy-ion collisions creates global rotation of matter which may induce spin polarization.

- Emerging particles are expected to be globally polarized with their spins on average pointing along the systems angular momentum.



Source: CERN Courier

Our hydrodynamic framework:

- We use relativistic hydrodynamic equations for polarized spin 1/2 particles to determine the space-time evolution of the spin polarization based on GLW forms of the energy-momentum and spin tensors.

  S. R. De Groot,Relativistic Kinetic Theory. Principles and Applications (1980).

- Boost-invariant and transversely homogeneous setup is assumed.

- We show how the formalism of hydrodynamics with spin can be used to determine physical observables related to the spin polarization required for the modelling of the experimental data.

  Wojciech Florkowski *et.al.*(Phys. Rev. C 99, 044910), Wojciech Florkowski *et.al.*(Phys. Rev. C 97, 041901), Wojciech Florkowski *et.al.*(Phys. Rev. D 97, 116017).

- We have restricted ourselves to the leading order terms in the spin polarization tensor.

Our hydrodynamic framework:

- Solve the standard perfect-fluid hydrodynamic equations.
- Determine the spin evolution in the perfect-hydrodynamic background.
- Determine the Pauli-Lubański (PL) vector on the freeze-out hypersurface.
- Calculate the spin polarization of particles in their rest frame which can be directly compared with the experiment.

Perfect fluid background:

- To get the perfect fluid background we need to solve conservation laws for charge and energy-linear momentum
- They provide closed system of five equations for five unknown functions with EoS (Equation of State)
- Baryon chemical potential (μ), Temperature (T), and three independent components of hydrodynamic flow vector (U^μ)

Conservation laws:

- Charge: $\partial_\alpha N^\alpha(x) = 0$

  where, $N^\alpha = nU^\alpha, \quad n = 4\sinh(\frac{\mu}{T})\; n_{(0)}(T).$

- Energy and linear momentum: $\partial_\alpha T^{\alpha\beta}_{GLW}(x) = 0$

  where, $T^{\alpha\beta}_{GLW}(x) = (\varepsilon + P)U^\alpha U^\beta - Pg^{\alpha\beta}$

- Spin: $\partial_\alpha S^{\alpha,\beta\gamma}_{GLW}(x) = 0$

  where, $S^{\alpha,\beta\gamma}_{GLW} = \cosh(\xi)\left(n_{(0)}(T)U^\alpha \omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta GLW}\right)$

Spin polarization tensor:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be defined by the four-vectors κ^{μ} and ω^{μ},

$$\omega_{\mu\nu} = \kappa_{\mu} U_{\nu} - \kappa_{\nu} U_{\mu} + \epsilon_{\mu\nu\alpha\beta} U^{\alpha} \omega^{\beta},$$

where,

$$\kappa_{\mu} = C_{\kappa X}\ X_{\mu} + C_{\kappa Y}\ Y_{\mu} + C_{\kappa Z}\ Z_{\mu}\ ,$$

$$\omega_{\mu} = C_{\omega X}\ X_{\mu} + C_{\omega Y}\ Y_{\mu} + C_{\omega Z}\ Z_{\mu}$$

These 6 spin components are to be studied for the evolution of spin polarization for spin $1/2$ fermions in our relativistic spin hydrodynamic framework.

Conservation of Charge:

$$\partial_\alpha N^\alpha(x) = 0,$$
$$\text{where,} \quad N^\alpha = nU^\alpha, \quad n = 4\sinh(\xi)\; n_{(0)}(T).$$

The quantity $n_{(0)}(T)$ defines the number density of spinless and neutral massive Boltzmann particles,

$$n_{(0)}(T) = \langle p \cdot U \rangle_0 = \frac{1}{2\pi^2}\, T^3\, \hat{m}^2 K_2\left(\hat{m}\right)$$

where, $\langle \cdots \rangle_0 \equiv \int dP\, (\cdots)\, e^{-\beta\cdot p}$ denotes the thermal average, $\hat{m} \equiv m/T$ denotes the ratio of the particle mass (m) and the temperature (T), and $K_2\left(\hat{m}\right)$ denotes the modified Bessel function.

The factor, $4\,\sinh(\xi) = 2\left(e^{\xi} - e^{-\xi}\right)$ accounts for spin degeneracy and presence of both particles and antiparticles in the system and the variable ξ denotes the ratio of the baryon chemical potential μ and the temperature T, $\xi = \mu/T$.

Conservation of energy and linear momentum:

$$\partial_\alpha T^{\alpha\beta}_{GLW}(x) = 0$$

where the energy-momentum tensor $T^{\alpha\beta}_{GLW}$ has the perfect-fluid form:

$$T^{\alpha\beta}_{GLW}(x) = (\varepsilon + P)U^\alpha U^\beta - Pg^{\alpha\beta}$$

with energy density $\varepsilon = 4\cosh(\xi)\varepsilon_{(0)}(T)$ and pressure
$P = 4\cosh(\xi)P_{(0)}(T)$
The auxiliary quantities are:
$\varepsilon_{(0)}(T) = \langle (p\cdot U)^2\rangle_0$ and $P_{(0)}(T) = -(1/3)\langle p\cdot p - (p\cdot U)^2\rangle_0$
are the energy density and pressure of the spin-less ideal gas respectively.
In case of ideal relativistic gas of classical massive particles,

$$\varepsilon_{(0)}(T) = \frac{1}{2\pi^2}\, T^4\, \hat{m}^2 \Big[3K_2\left(\hat{m}\right) + \hat{m}K_1\left(\hat{m}\right)\Big], \quad P_{(0)}(T) = Tn_{(0)}(T)$$

where, K_1 and K_2 are the modified Bessel functions of 1st and 2nd kind respectively.

Conservation of spin angular momentum:

$$\partial_\alpha S^{\alpha,\beta\gamma}_{GLW}(x) = 0$$

GLW spin tensor in the leading order of $\omega_{\mu\nu}$ is:

$$S^{\alpha,\beta\gamma}_{GLW} = \cosh(\xi)\left(n_{(0)}(T)U^\alpha\omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta GLW}\right)$$

Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor, whereas the auxiliary tensor $S^{\alpha,\beta\gamma}_{\Delta GLW}$ is:

$$S^{\alpha,\beta\gamma}_{\Delta GLW} = \mathcal{A}_{(0)}\, U^\alpha U^\delta U^{[\beta}\omega^{\gamma]}{}_{\delta}$$
$$+\mathcal{B}_{(0)}\left(U^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_{\delta} + U^\alpha\Delta^{\delta[\beta}\omega^{\gamma]}{}_{\delta} + U^\delta\Delta^{\alpha[\beta}\omega^{\gamma]}{}_{\delta}\right),$$

with,

$$\mathcal{B}_{(0)} = -\tfrac{2}{\hat{m}^2}s_{(0)}(T)$$

$$\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2n_{(0)}(T)$$

Boost-Invariant form of fluid dynamics with spin:

- Conservation law of charge can be written as:

$$U^\alpha \partial_\alpha n + n \partial_\alpha U^\alpha = 0$$

Therefore, for Bjorken type of flow we can write,

$$\partial_\tau n + \frac{n}{\tau} = 0$$

- Conservation law of energy-momentum can be written as:

$$U^\alpha \partial_\alpha \varepsilon + (\varepsilon + P) \partial_\alpha U^\alpha = 0$$

Hence for the Bjorken flow,

$$\partial_\tau \varepsilon + \frac{(\varepsilon + P)}{\tau} = 0$$

Boost-Invariant form of fluid dynamics with spin:

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3,$

$\mathcal{P}(\tau) = \mathcal{A}_1,$

$\mathcal{Q}_1(\tau) = -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{Q}_2(\tau) = -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right),$

$\mathcal{R}_1(\tau) = -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{R}_2(\tau) = -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right).$

$$\mathcal{A}_1 = \cosh(\xi)\left(n_{(0)} - \mathcal{B}_{(0)}\right),$$
$$\mathcal{A}_2 = \cosh(\xi)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right),$$
$$\mathcal{A}_3 = \cosh(\xi)\,\mathcal{B}_{(0)}$$

Six spin components evolve independently in this set-up, which will not be the case in full 3+1D geometry.

Background evolution:

Initial baryon chemical potential $\mu_0 = 800$ MeV
Initial temperature $T_0 = 155$ MeV
Particle (Lambda hyperon) mass $m = 1116$ MeV

Initial and final proper time is $\tau_0 = 1$ fm and $\tau_f = 10$ fm, respectively.

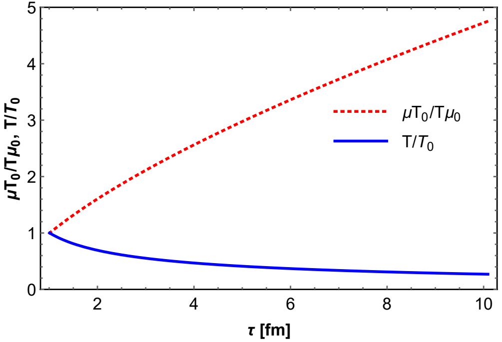


Figure: Proper-time dependence of T divided by its initial value T_0 (solid line) and the ratio of baryon chemical potential μ and temperature T re-scaled by the initial ratio μ_0/T_0 (dotted line) for a boost-invariant one-dimensional expansion.

Spin polarization evolution:

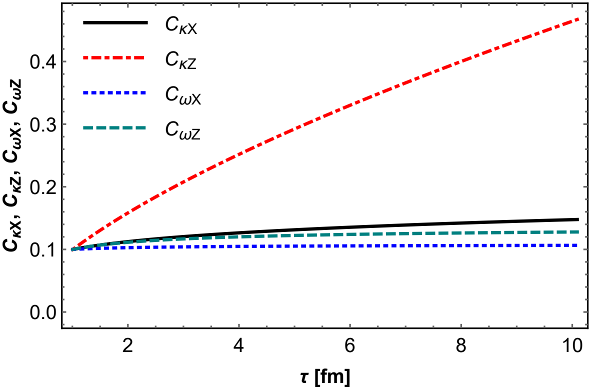


Figure: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$. The coefficients $C_{\kappa Y}$ and $C_{\omega Y}$ satisfy the same differential equations as the coefficients $C_{\kappa X}$ and $C_{\omega X}$.

Momentum dependence of polarization:

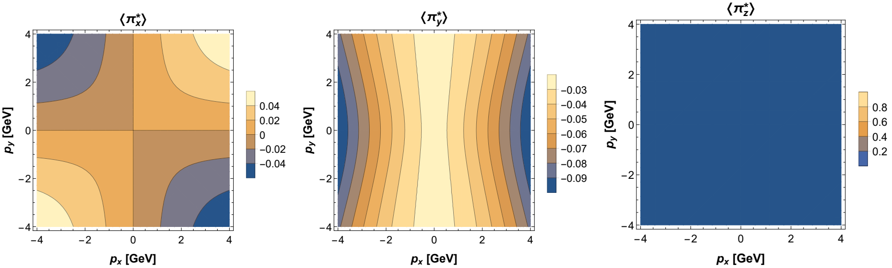
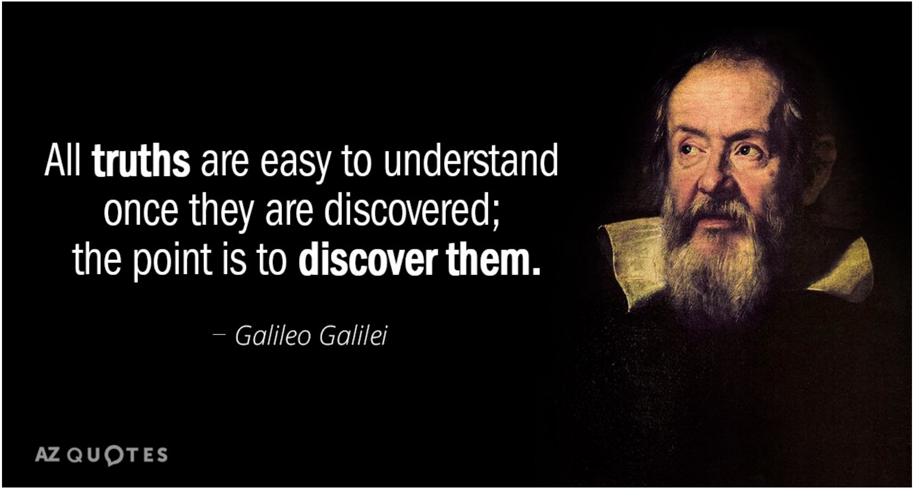


Figure: Components of the PRF mean polarization three-vector of Λ's. The results obtained with the initial conditions $\mu_0 = 800$ MeV, $T_0 = 155$ MeV, $\boldsymbol{C}_{\kappa,0} = (0,0,0)$, and $\boldsymbol{C}_{\omega,0} = (0,0.1,0)$ for $y_p = 0$.

Summary:

- We have discussed relativistic hydrodynamics with spin based on the GLW formulation of energy-momentum and spin tensors.
- For boost invariant and transversely homogeneous set-up we show how our hydrodynamic framework with spin can be used to determine the spin polarization observables measured in heavy ion collisions.
- Currently, we are working on the extension of our hydrodynamic approach for 1+3 dimensions.
- We have also studied spin polarization for the Gubser expanding background (see arXiv:2011.14907) and got some interesting results for spin dynamics. This could be used as another check beside Bjorken for our 3+1D code.



Thank you for your attention!

Back-Up Slides

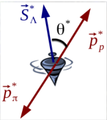
Measuring polarization in experiment:

Parity-violating decay of hyperons

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\mathrm{H}} \mathbf{P}_{\mathrm{H}} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

P_H: Λ polarization
p_p^*: proton momentum in the Λ rest frame
α_H: Λ decay parameter
($\alpha_{\Lambda} = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$)



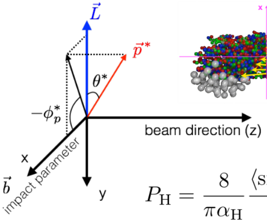
$$\Lambda \to p + \pi^-$$

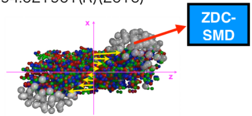
(BR: 63.9%, $c\tau$ ~7.9 cm)

C. Patrignani et al. (PDG), Chin. Phys. C 40, 100001 (2016)

Projection onto the transverse plane

Angular momentum direction can be determined by spectator deflection (spectators deflect outwards)
- S. Voloshin and TN, PRC94.021901(R)(2016)





$$P_{\mathrm{H}} = \frac{8}{\pi \alpha_{\mathrm{H}}} \frac{\langle \sin(\Psi_1 - \phi_p^*) \rangle}{\mathrm{Res}(\Psi_1)}$$

Ψ_1: azimuthal angle of b
ϕ_p^*: ϕ of daughter proton in Λ rest frame
STAR, PRC76, 024915 (2007)

Source: T. Niida, WWND 2019

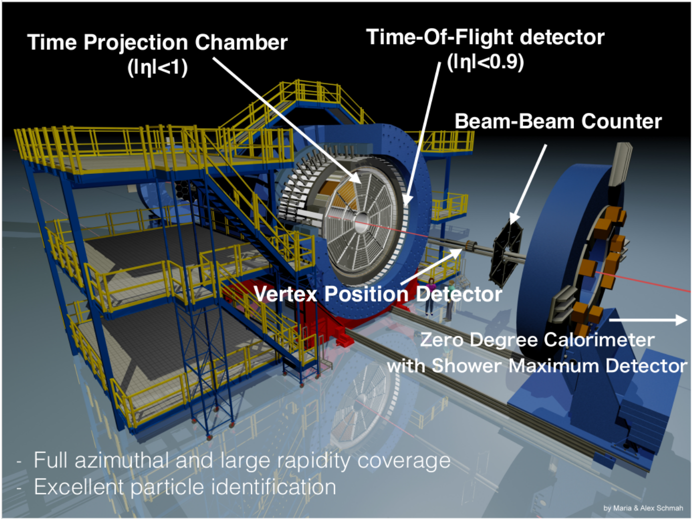


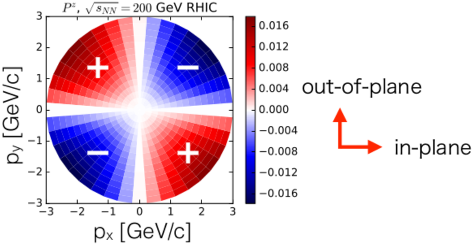
Figure: Schematic view of STAR Detector

Other works:

- Other theoretical models used for the heavy-ions data interpretation dealt mainly with the spin polarization of particles at freeze-out, where the basic hydrodynamic quantity giving rise to spin polarization is the 'thermal vorticity' expressed as $\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu)$.

  F. Becattini *et.al.*(Annals Phys. 338 (2013)), F. Becattini, L. Csernai, D. J. Wang (PRC 88, 034905), F. Becattini *et.al.*(PRC 95, 054902), Iu. Karpenko, F. Becattini (EPJC (2017) 77: 213), F. Becattini, Iu. Karpenko(PRL 120, 012302 (2018))

Hydro calculation of P_z
F. Becattini and I. Karpenko,
PRL.120.012302 (2018)



Conservation of Charge:

$$\partial_\alpha N^\alpha(x) = 0,$$
$$\text{where,} \quad N^\alpha = nU^\alpha, \quad n = 4\sinh(\xi)\; n_{(0)}(T).$$

The quantity $n_{(0)}(T)$ defines the number density of spinless and neutral massive Boltzmann particles,

$$n_{(0)}(T) = \langle p\cdot U\rangle_0 = \tfrac{1}{2\pi^2}\, T^3\, \hat{m}^2 K_2\left(\hat{m}\right)$$

where, $\langle\cdots\rangle_0 \equiv \int dP\,(\cdots)\, e^{-\beta\cdot p}$ denotes the thermal average, $\hat{m} \equiv m/T$ denotes the ratio of the particle mass (m) and the temperature (T), and $K_2\left(\hat{m}\right)$ denotes the modified Bessel function.

The factor, $4\,\sinh(\xi) = 2\left(e^{\xi} - e^{-\xi}\right)$ accounts for spin degeneracy and presence of both particles and antiparticles in the system and the variable ξ denotes the ratio of the baryon chemical potential μ and the temperature T, $\xi = \mu/T$.

Conservation of energy and linear momentum:

$$\partial_\alpha T^{\alpha\beta}_{GLW}(x) = 0$$

where the energy-momentum tensor $T^{\alpha\beta}_{GLW}$ has the perfect-fluid form:

$$T^{\alpha\beta}_{GLW}(x) = (\varepsilon + P)U^\alpha U^\beta - Pg^{\alpha\beta}$$

with energy density $\varepsilon = 4\cosh(\xi)\varepsilon_{(0)}(T)$ and pressure $P = 4\cosh(\xi)P_{(0)}(T)$

The auxiliary quantities are:

$\varepsilon_{(0)}(T) = \langle (p\cdot U)^2 \rangle_0$ and $P_{(0)}(T) = -(1/3)\langle p\cdot p - (p\cdot U)^2 \rangle_0$ are the energy density and pressure of the spin-less ideal gas respectively.

In case of ideal relativistic gas of classical massive particles,

$$\varepsilon_{(0)}(T) = \frac{1}{2\pi^2}\, T^4\, \hat{m}^2 \Big[3K_2\left(\hat{m}\right) + \hat{m}K_1\left(\hat{m}\right) \Big], \quad P_{(0)}(T) = Tn_{(0)}(T)$$

where, K_1 and K_2 are the modified Bessel functions of 1st and 2nd kind respectively.

Conservation of spin angular momentum:

$$\partial_\alpha S^{\alpha,\beta\gamma}_{GLW}(x) = 0$$

GLW spin tensor in the leading order of $\omega_{\mu\nu}$ is:

$$S^{\alpha,\beta\gamma}_{GLW} = \cosh(\xi)\left(n_{(0)}(T)U^\alpha\omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta GLW}\right)$$

Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor, whereas the auxiliary tensor $S^{\alpha,\beta\gamma}_{\Delta GLW}$ is:

$$\begin{gathered} S^{\alpha,\beta\gamma}_{\Delta GLW} = \mathcal{A}_{(0)}\, U^\alpha U^\delta U^{[\beta}\omega^{\gamma]}{}_{\delta} \\ +\mathcal{B}_{(0)}\left(U^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_{\delta} + U^\alpha\Delta^{\delta[\beta}\omega^{\gamma]}{}_{\delta} + U^\delta\Delta^{\alpha[\beta}\omega^{\gamma]}{}_{\delta}\right), \end{gathered}$$

with,

$$\mathcal{B}_{(0)} = -\tfrac{2}{\hat{m}^2}s_{(0)}(T)$$

$$\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2n_{(0)}(T)$$

Basis for boost invariant and transversely homogeneous systems:

For our calculations, it is useful to introduce a local basis consisting of following 4-vectors,

$$\begin{aligned}
U^\alpha &= \frac{1}{\tau}(t,0,0,z) = (\cosh(\eta),0,0,\sinh(\eta))\,,\\
X^\alpha &= (0,1,0,0)\,,\\
Y^\alpha &= (0,0,1,0)\,,\\
Z^\alpha &= \frac{1}{\tau}(z,0,0,t) = (\sinh(\eta),0,0,\cosh(\eta))\,.
\end{aligned}$$

where, $\tau = \sqrt{t^2 - z^2}$ is the longitudinal proper time and
$\eta = \ln((t+z)/(t-z))/2$ is the space-time rapidity.
The basis vectors satisfy the following normalization and orthogonal conditions:

$$\begin{aligned}
& U\cdot U = 1\\
X\cdot X &= Y\cdot Y = Z\cdot Z = -1,\\
X\cdot U &= Y\cdot U = Z\cdot U = 0,\\
X\cdot Y &= Y\cdot Z = Z\cdot X = 0.
\end{aligned}$$

Boost-invariant form for the spin polarization tensor:

We use the following decomposition of the vectors κ^μ and ω^μ,

$$\begin{aligned}
\kappa^\alpha &= C_{\kappa U} U^\alpha + C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \\
\omega^\alpha &= C_{\omega U} U^\alpha + C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.
\end{aligned}$$

Here the scalar coefficients are functions of the proper time (τ) only due to boost invariance. Since $\kappa \cdot U = 0, \quad \omega \cdot U = 0$, therefore

$$\begin{aligned}
\kappa^\alpha &= C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \\
\omega^\alpha &= C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha.
\end{aligned}$$

$\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta$ can be written as,

$$\begin{aligned}
\omega_{\mu\nu} &= C_{\kappa Z}(Z_\mu U_\nu - Z_\nu U_\mu) + C_{\kappa X}(X_\mu U_\nu - X_\nu U_\mu) + C_{\kappa Y}(Y_\mu U_\nu - Y_\nu U_\mu) \\
&\quad + \epsilon_{\mu\nu\alpha\beta} U^\alpha (C_{\omega Z} Z^\beta + C_{\omega X} X^\beta + C_{\omega Y} Y^\beta)
\end{aligned}$$

In the plane $z = 0$ we find:

$$\omega_{\mu\nu} = \begin{bmatrix} 0 & C_{\kappa X} & C_{\kappa Y} & C_{\kappa Z} \\ -C_{\kappa X} & 0 & -C_{\omega Z} & C_{\omega Y} \\ -C_{\kappa Y} & C_{\omega Z} & 0 & -C_{\omega X} \\ -C_{\kappa Z} & -C_{\omega Y} & C_{\omega X} & 0 \end{bmatrix}$$

Boost-Invariant form of fluid dynamics with spin:

Using the equations,

$$S^{\alpha,\beta\gamma}_{\Delta GLW} = \mathcal{A}_{(0)} U^\alpha U^\delta U^{[\beta}\omega^{\gamma]}{}_{\delta}$$
$$+\mathcal{B}_{(0)}\left(U^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_{\delta} + U^\alpha\Delta^{\delta[\beta}\omega^{\gamma]}{}_{\delta} + U^\delta\Delta^{\alpha[\beta}\omega^{\gamma]}{}_{\delta}\right),$$

and

$$S^{\alpha,\beta\gamma}_{GLW} = \cosh(\xi)\left(n_{(0)}(T)U^\alpha\omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta GLW}\right)$$

in

$$\partial_\alpha S^{\alpha,\beta\gamma}_{GLW}(x) = 0$$

Contracting the final equation with $U_\beta X_\gamma, U_\beta Y_\gamma, U_\beta Z_\gamma, Y_\beta Z_\gamma, X_\beta Z_\gamma$ and $X_\beta Y_\gamma$.

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3,$

$\mathcal{P}(\tau) = \mathcal{A}_1,$

$\mathcal{Q}_1(\tau) = -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{Q}_2(\tau) = -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right),$

$\mathcal{R}_1(\tau) = -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{R}_2(\tau) = -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right).$

$$\mathcal{A}_1 = \cosh(\xi)\left(n_{(0)} - \mathcal{B}_{(0)}\right),$$

$$\mathcal{A}_2 = \cosh(\xi)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right),$$

$$\mathcal{A}_3 = \cosh(\xi)\,\mathcal{B}_{(0)}$$

Spin polarization of particles at the freeze-out:

Average spin polarization per particle $\langle \pi_\mu(p) \rangle$ is given as:

$$\langle \pi_\mu \rangle = \frac{E_p \frac{d\Pi_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}}$$

where, the total value of the Pauli-Lubański vector for particles with momentum p is:

$$E_p \frac{d\Pi_\mu(p)}{d^3p} = -\frac{\cosh(\xi)}{(2\pi)^3 m} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p} \, \tilde{\omega}_{\mu\beta} p^\beta$$

momentum density of all particles is given by:

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4\cosh(\xi)}{(2\pi)^3} \int \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p}$$

and freeze-out hypersurface is defined as:

$$\Delta\Sigma_\lambda = U_\lambda dxdy \, \tau d\eta$$

Assuming that freeze-out takes place at a constant value of τ and parameterizing the particle four-momentum p^λ in terms of the transverse mass m_T and rapidity y_p, we get:

$$\Delta\Sigma_\lambda p^\lambda = m_T \cosh\left(y_p - \eta\right) dxdy \, \tau d\eta$$

Boost to the local rest frame (LRF) of the particle:

Polarization vector $\langle \pi^{\star}_{\mu} \rangle$ in the local rest frame of the particle can be obtained by using the canonical boost. Using the parametrizations $E_p = m_T \cosh(y_p)$ and $p_z = m_T \sinh(y_p)$ and applying the appropriate Lorentz transformation we get,

$$\langle \pi^{\star}_{\mu} \rangle = -\frac{1}{8m} \begin{bmatrix} 0 \\ \left(\frac{\sinh(y_p) p_x}{m_T \cosh(y_p)+m}\right) \left[\chi \left(C_{\kappa X} p_y - C_{\kappa Y} p_x\right) + 2 C_{\omega Z} m_T\right] + \frac{\chi\, p_x \cosh(y_p)\left(C_{\omega X} p_x + C_{\omega Y} p_y\right)}{m_T \cosh(y_p)+m} + 2 C_{\kappa Z} p_y - \chi C_{\omega X} m_T \\ \left(\frac{\sinh(y_p) p_y}{m_T \cosh(y_p)+m}\right) \left[\chi \left(C_{\kappa X} p_y - C_{\kappa Y} p_x\right) + 2 C_{\omega Z} m_T\right] + \frac{\chi\, p_y \cosh(y_p)\left(C_{\omega X} p_x + C_{\omega Y} p_y\right)}{m_T \cosh(y_p)+m} - 2 C_{\kappa Z} p_x - \chi C_{\omega Y} m_T \\ -\left(\frac{m \cosh(y_p)+m_T}{m_T \cosh(y_p)+m}\right) \left[\chi \left(C_{\kappa X} p_y - C_{\kappa Y} p_x\right) + 2 C_{\omega Z} m_T\right] - \frac{\chi\, m \sinh(y_p)\left(C_{\omega X} p_x + C_{\omega Y} p_y\right)}{m_T \cosh(y_p)+m} \end{bmatrix}$$

where,

$\chi\left(\hat{m}_T\right) = \left(K_0\left(\hat{m}_T\right) + K_2\left(\hat{m}_T\right)\right) / K_1\left(\hat{m}_T\right)$,

$\hat{m}_T = m_T / T$